Low Noise Measurements

(for mesoscopic physics, quantum transport & circuits)

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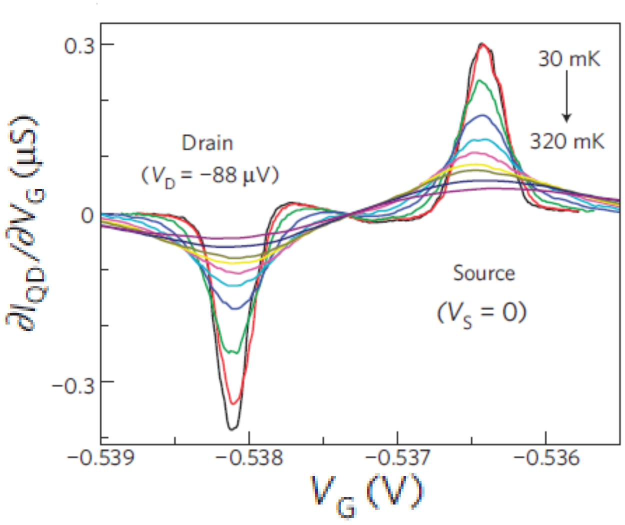
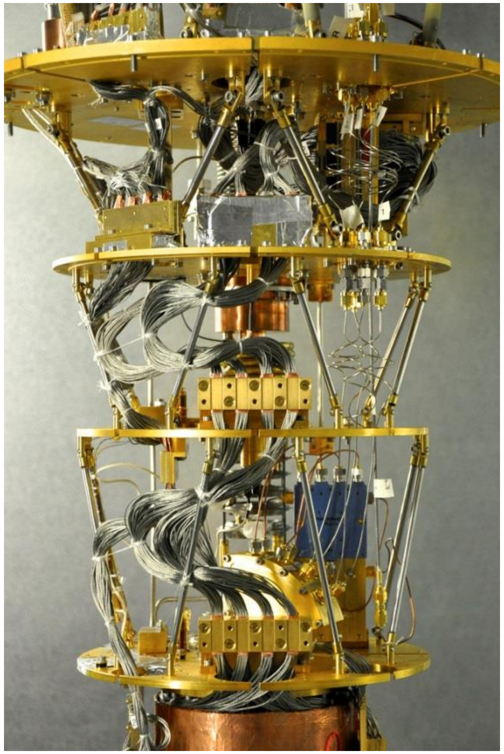




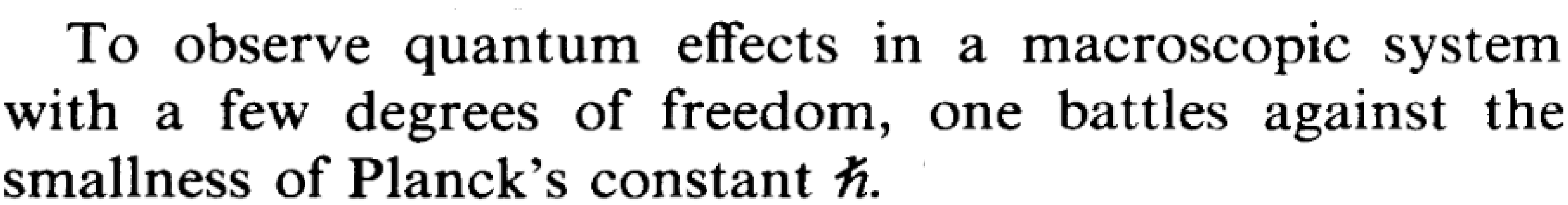
'a gentle journey in a world of many ~~body~~ problems & soldering irons'

Low Noise Measurements

Why this course?

1. They asked me…
2. low noise measurements for experimentalists
3. low noise measurements for theorists?

Low Noise Measurements



To observe quantum effects in a macroscopic system with a few degrees of freedom, one battles against the smallness of Planck's constant $\hbar$.

Martinis, Devoret, Clarke, PRB **35,** 4682 (1987)

(incomplete) Bibliography

- Books:

Lounasmaa – *Experimental principles and methods below 1K* (1974)
Pobell – *Matter and Methods at Low Temperatures* (1992)
Horowitz & Hill – *The Art of Electronics* (1989)
White – *Experimental Techniques in Low-Temperature Physics* (1979)
Ventura & Risegari – *The Art of Cryogenics* (2008)

- PhD theses:

H. Pothier (1991)
B. Huard (2006)
J. Gabelli (2006)
H. le Sueur (2007)
F. D. Parmentier (2010)
S. Jezouin (2014)
T. Jullien (2014)

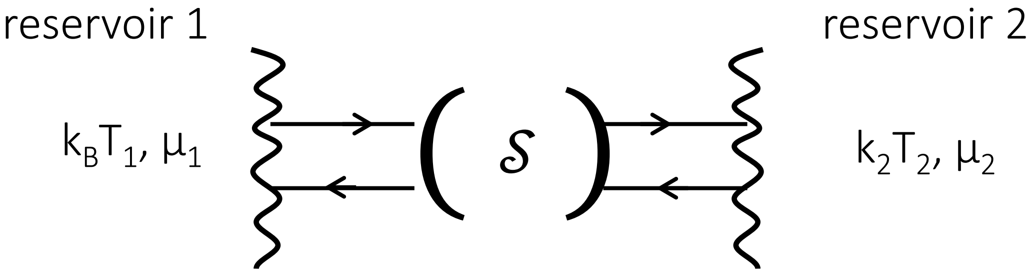
- review papers:

F. Giazotto *et al.,* Rev. Mod. Phys., **78,** 217 (2006)
D.C. Glattli, Eur. Phys. J. Special Topics **172,** 163–179 (2009)
A.A. Clerk et al., Rev. Mod. Phys. **82,** 1155 (2010)

+ references mentioned in the slides (and references therein)

Low Noise Measurements

Landauer Buttiker formalism:
electron quantum transport = coherent conductor (≡scattering matrix $\mathcal{S}$) + ideal reservoirs

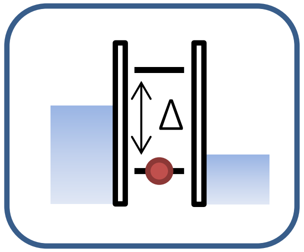


$$\mathcal{S} = \begin{pmatrix} r & t \\ t & -r \end{pmatrix}$$

- how does one realize ideal reservoirs, with well controlled $k_B T$ & μ?
- how does one measure $\mathcal{S}$ precisely in both linear & non-linear regimes?

transmission=conductance $\propto |t|^2$

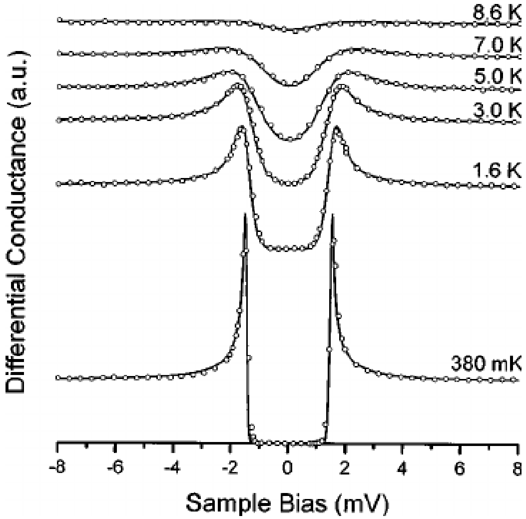
Energy scales in mesoscopic transport



“intrinsic”:

- quantum dots: charging energy E_c / level spacing Δ
- Kondo temperature $k_B T_K$
- superconducting gap Δ / Andreev bound states energy E_A
- …

STM tunneling conductance Nb / Au



APL **73**, 2992 - 2994 (1998)

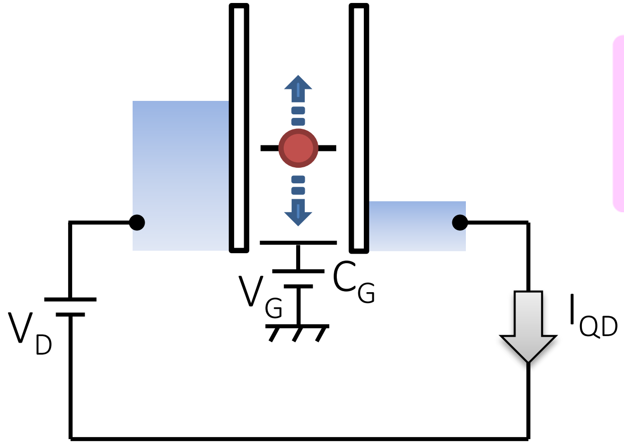
“external probes”:

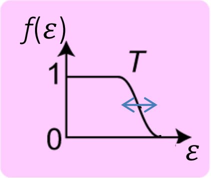
- dc/ac voltage V_{dc}/V_{ac}
- e-mag field / photons $\hbar\omega$
- temperature $k_B T$
- …

need to be tunable from << $\{E_c , \Delta , \ldots\}$ to ≥ $\{E_c , \Delta , \ldots\}$

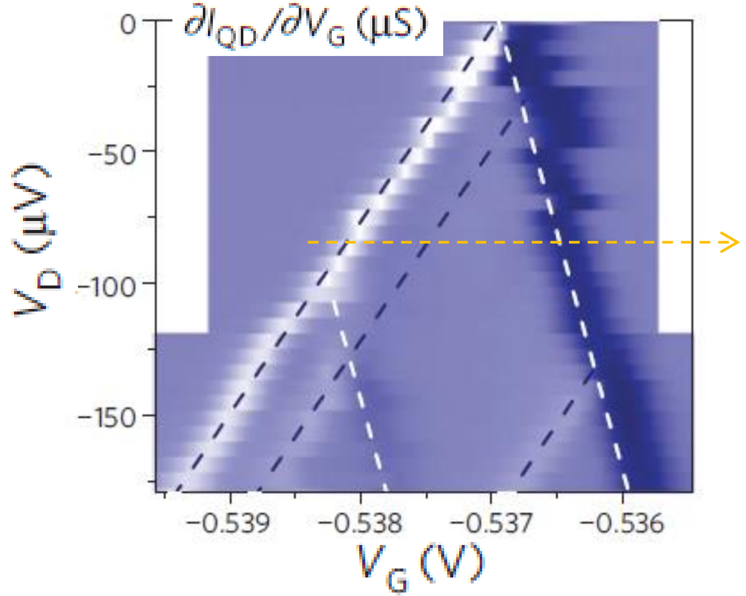
1 K ⇔ 86 µV ⇔ 20 GHz

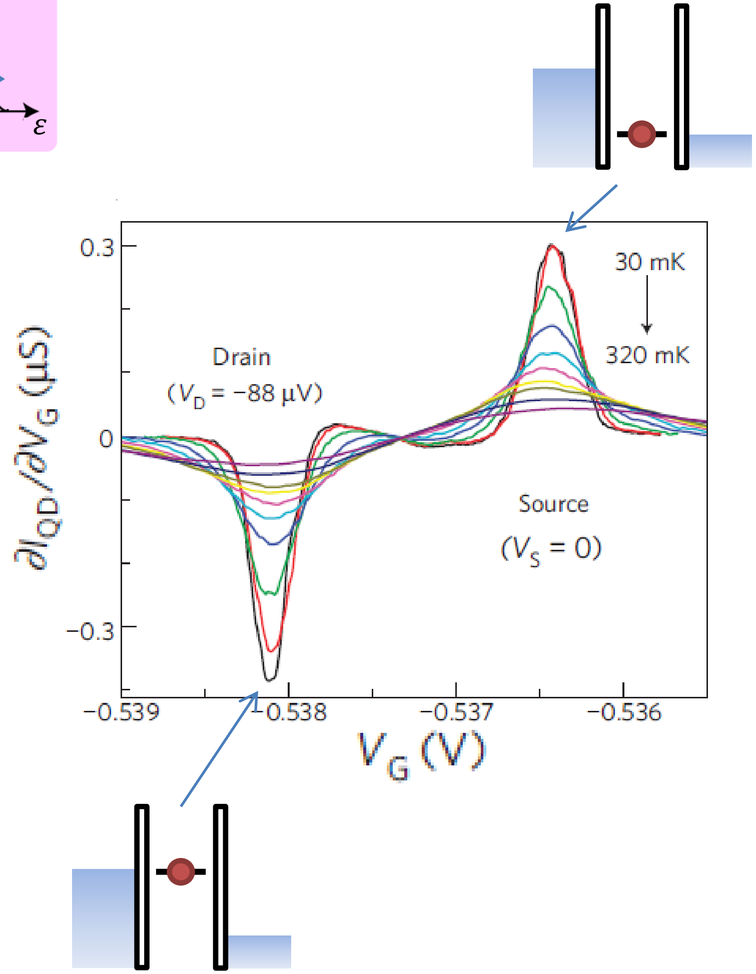
Example: transport in a quantum dot





Altimiras *et al.*, Nat. Phys. **6**, 34 - 39 (2010)

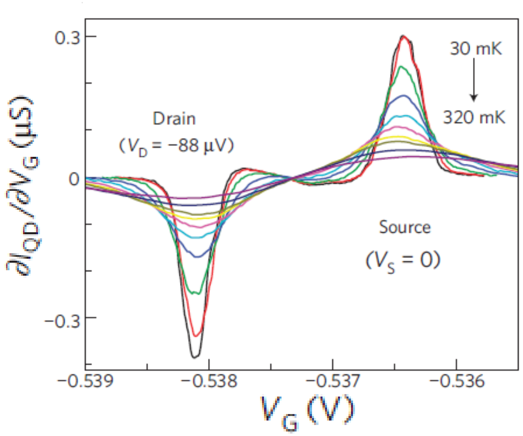




resonance width affected by temperature

Outline

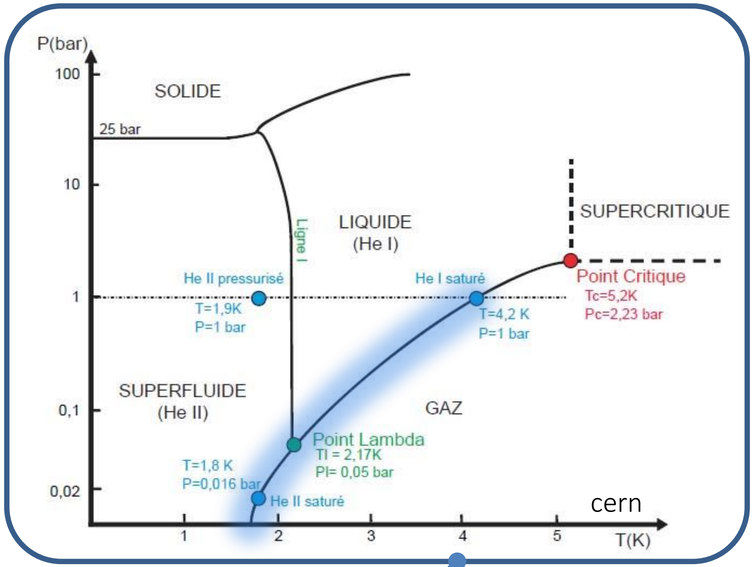
1. low temperature experiments
   - cryogenic systems
   - lattice vs electron temperature
   - filtering & shielding

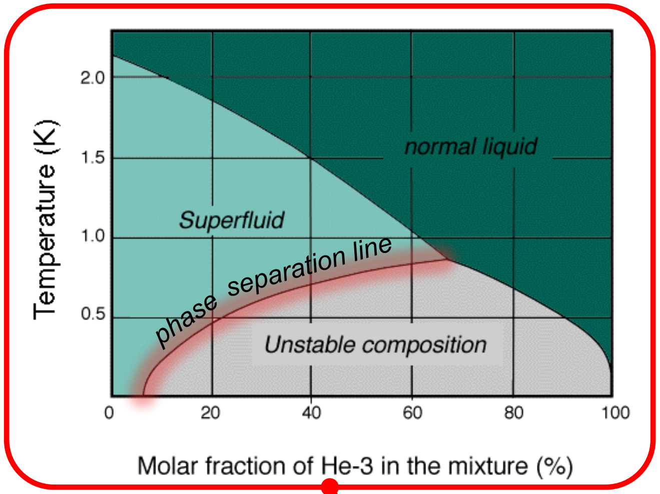


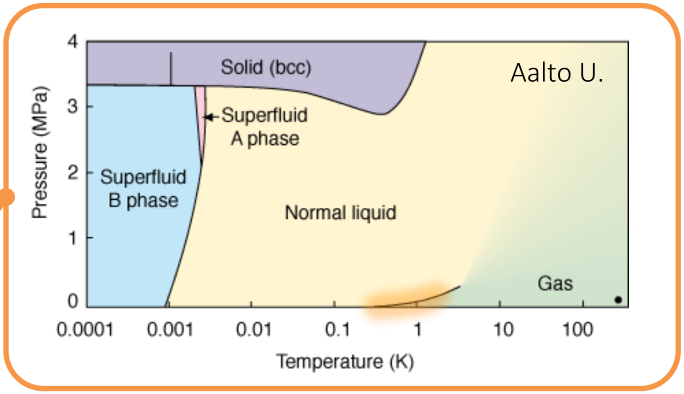
2. low noise cryoelectronics
   - signal vs noise
   - lock-in measurements
   - measurement configurations

3. beyond dc conductance
   - microwave measurements
   - shot noise

He-based cryogenic systems







pumped liqu. ^{4}He

pumped liqu. ^{3}He

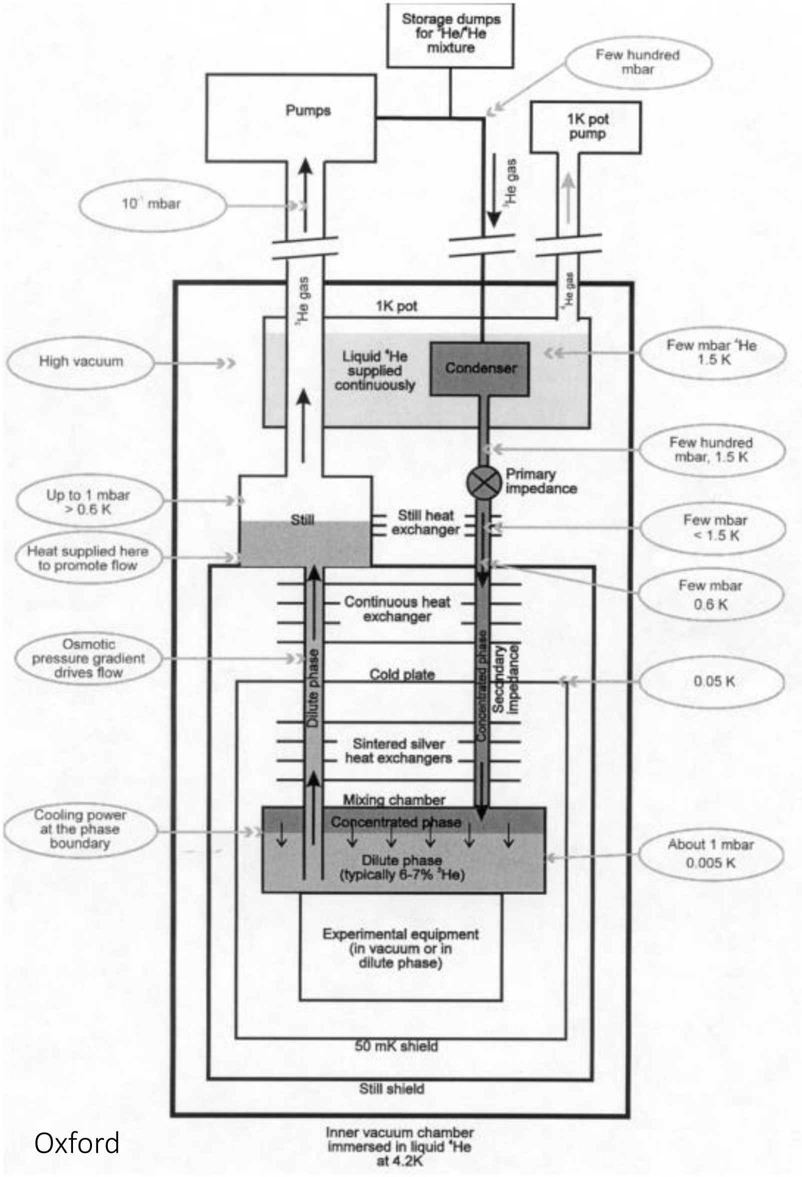
superfl. ^{4}He

^{4}He / ^{3}He
phase separation

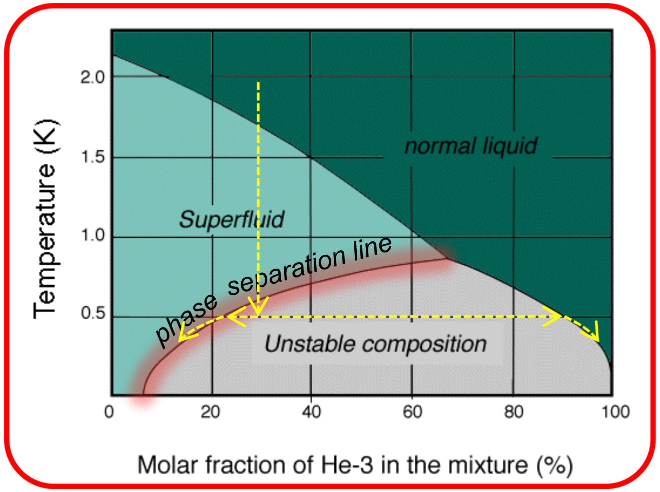
^{4}He / ^{3}He dilution refrigerators

4.2 K 2.17 K 1 K 0.87 K 0.3 K 0.01 K

^{4}He / ^{3}He dilution refrigerators



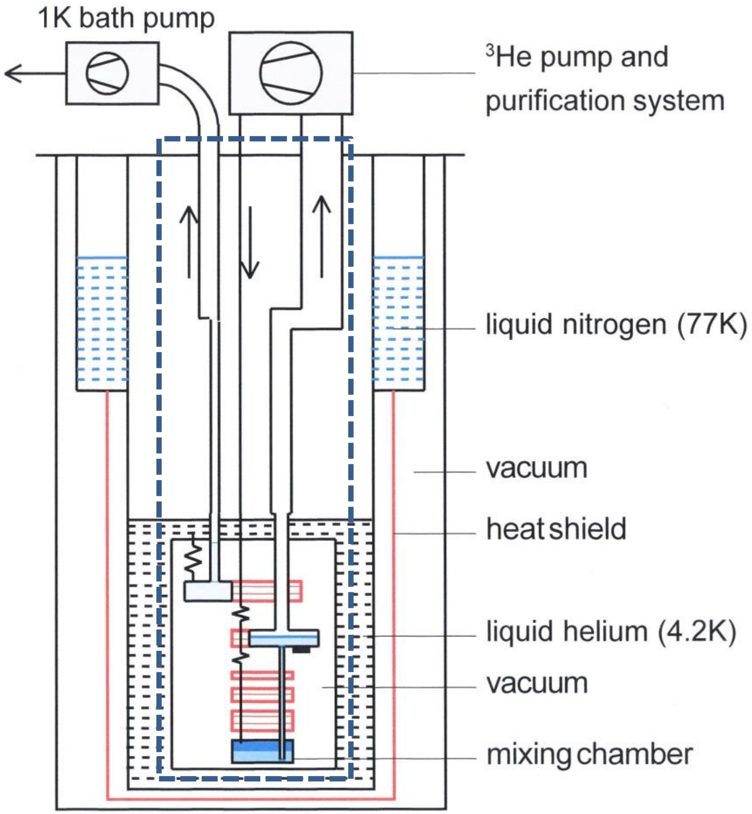
Oxford



- circulated ^{4}He / ^{3}He mixture
- continuous operation down to **1 mK**
- needs ^{4}He bath + cooling power at ~1 K

  (1 K pot or Joule-Thomson expansion)

Wet vs *dry* dilution refrigerators



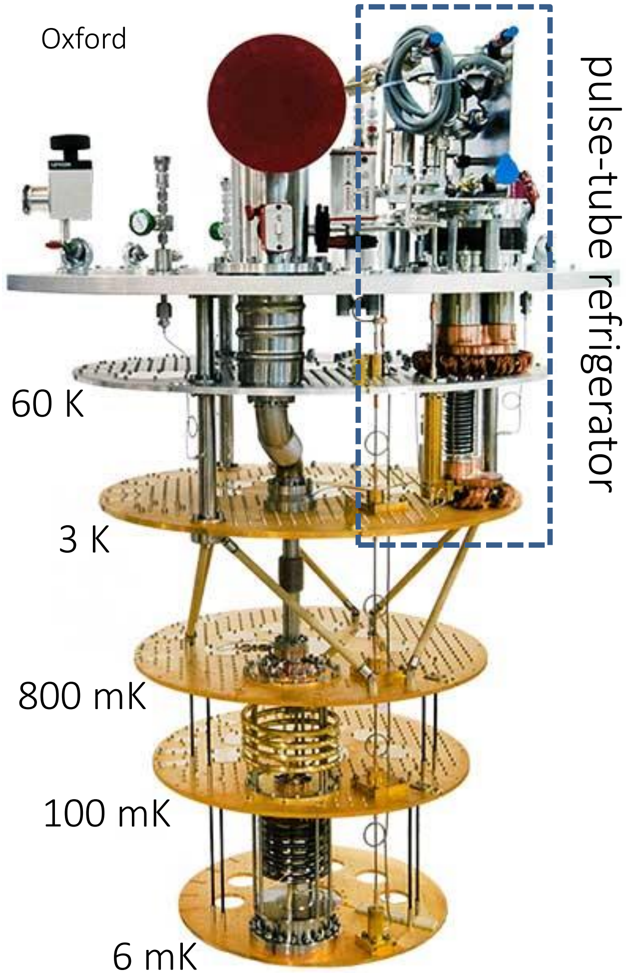
‘Wet’ fridge:
dilution unit dipped in liquid ^{4}He bath

+ reliable
+ (relatively) fast cooldowns

- liquid ^{4}He consumption expensive
- requires regular refills (week-end…)
- limited experimental space
- vacuum isolation = low temperature seal
- cryogenic liquids hazards

Das *et al.,* Low Temperature Physics LT9, 1253-1255 (1965)

Wet vs *dry* dilution refrigerators



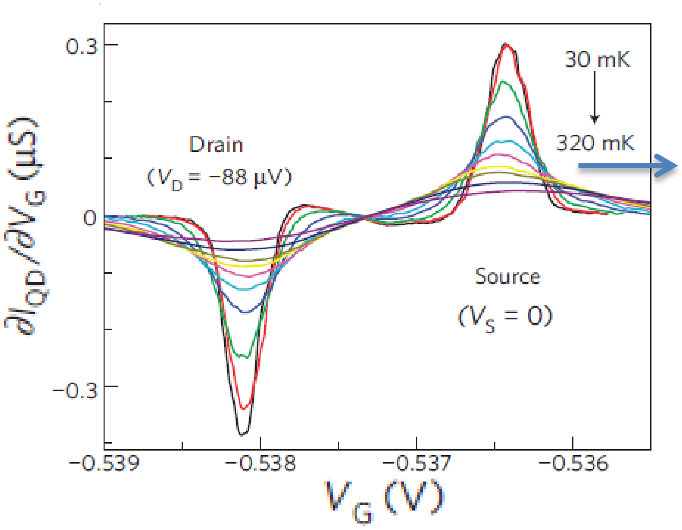
‘dry’ fridge:
precooling down to 4 K by mechanical refrigerator
(compression-expansion cycles in “pulse-tubes”)

+ automatized & autonomous
+ no more liquid ^{4}He transfers!
+ huge experimental space

- mechanical vibrations
- high electricity & cooling water consumption
- VERY POOR efficiency
  (5 kW electrical power → <1 W cooling @ 4K)
- sensitive to electrical failure

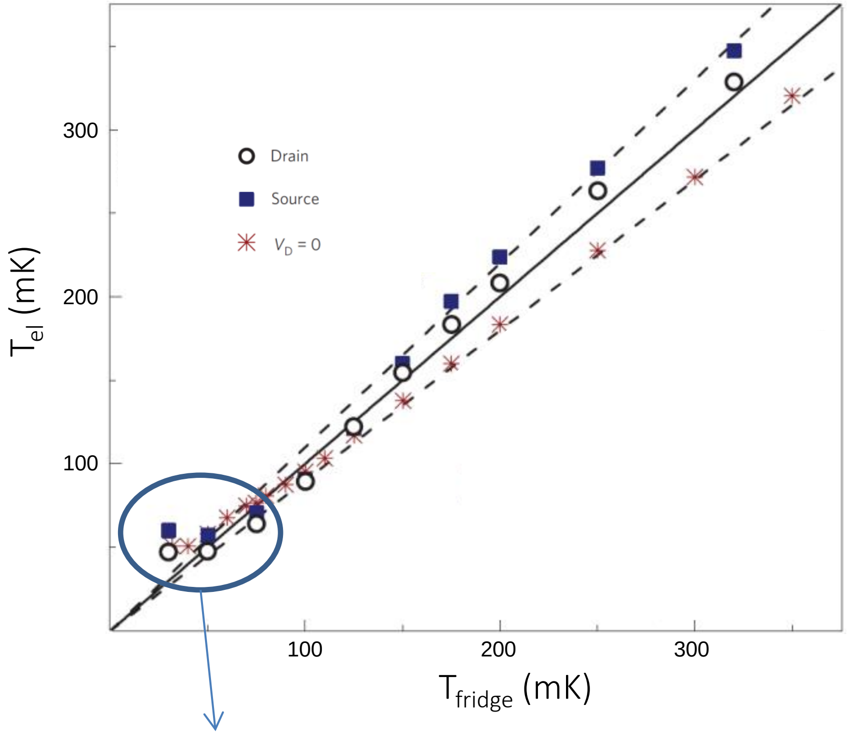
game-changer for labwork / labs funding / cryogenics industry

So you got a fridge… then what?



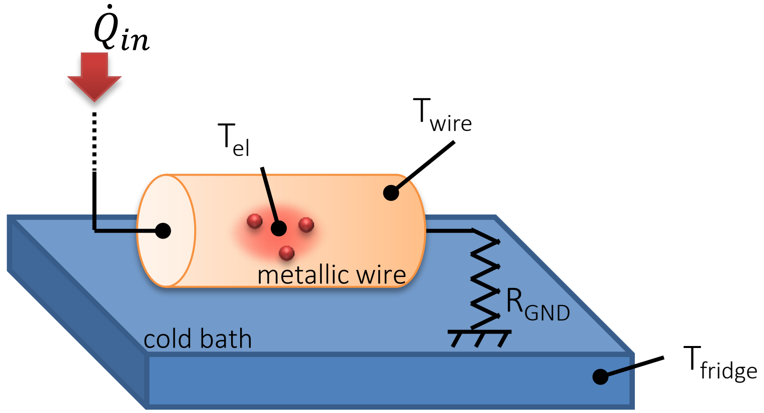
$$\propto \frac{\partial f_{\mathrm{Fermi}}(\varepsilon)}{\partial \varepsilon}$$

peak width ∝ sample temperature T_{el}

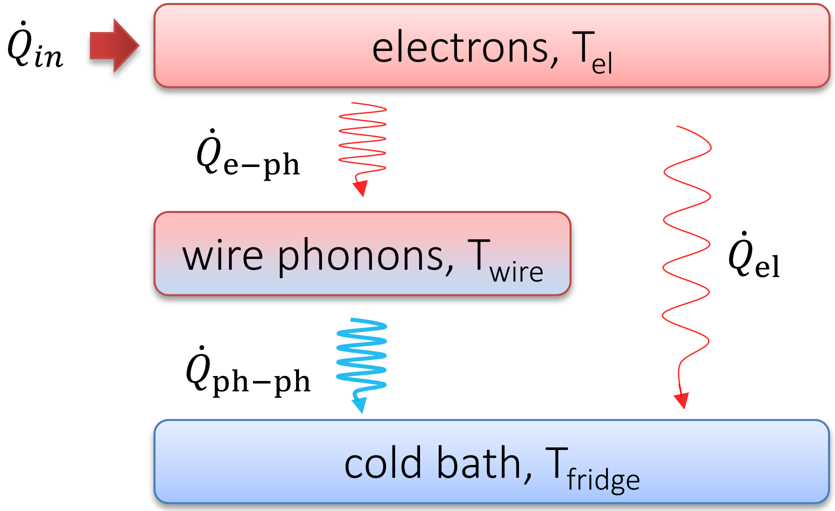


discrepancy T_{el}/T_{fridge} at low temperature?

Electron vs *lattice* temperature



thermal transport model:


$\dot{Q}_{in}$

electrons, T_{el}

$\dot{Q}_{\mathrm{e-ph}}$

wire phonons, T_{wire}

$\dot{Q}_{\mathrm{el}}$

$\dot{Q}_{\mathrm{ph-ph}}$

cold bath, T_{fridge}

Rev. Mod. Phys., **78,** 217 (2006)

electron-phonon cooling:

$$\dot{Q}_{\mathrm{e-ph}} = \Sigma U (T_{\mathrm{el}}^5 - T_{\mathrm{wire}}^5)$$

Σ: coupling constant (depends on material)
U: wire volume

phonon-phonon cooling (Kapitza resistance):

$$\dot{Q}_{\mathrm{ph-ph}} = KA (T_{\mathrm{wire}}^4 - T_{\mathrm{fridge}}^4)$$

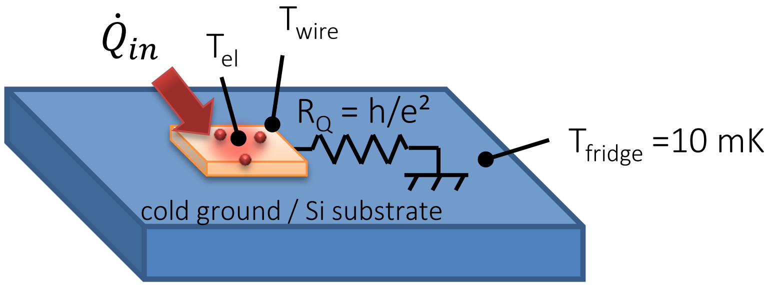
K: coupling constant (depends on materials)
A: contact area

electronic heat transport (Wiedemann-Franz law):

$$\dot{Q}_{\mathrm{el}} = L_0\,(T_{\mathrm{wire}}^2 - T_{\mathrm{fridge}}^2)/R_{\mathrm{GND}}$$

$L_0 = \pi^2 k_B^2 / 3e^2$: Lorenz number

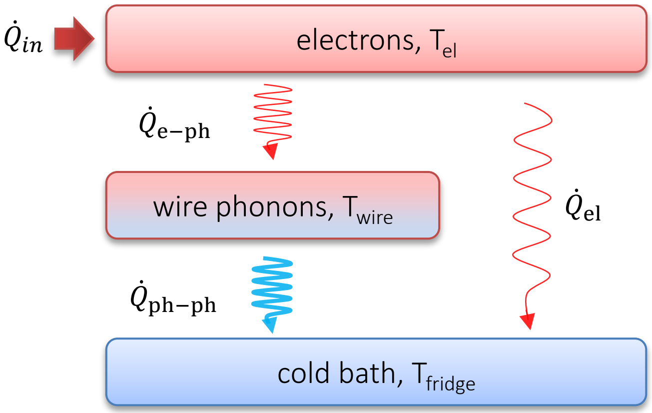
Example: metal contact on meso. conductor



300 µm x 300 µm x 100 nm Cu contact

Σ_{cu}=2x10^9 Wm^{-3}K^{-5}

$K_{cu/Si}$=166 Wm^{-2}K^{-4}



$\dot{Q}_{in}$

electrons, T_{el}

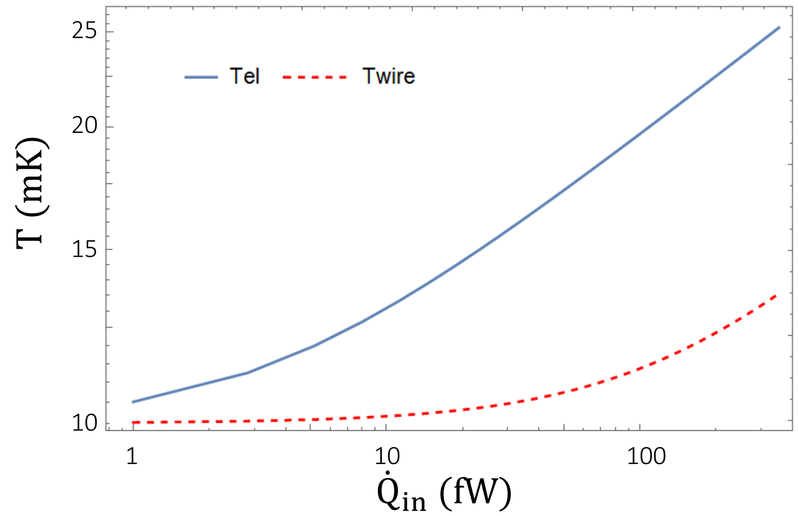
$\dot{Q}_{e-ph}$

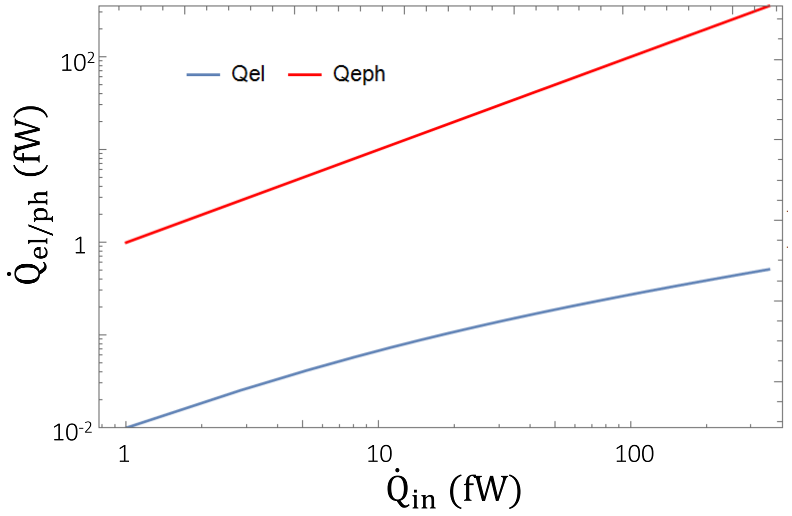
wire phonons, T_{wire}

$\dot{Q}_{el}$

$\dot{Q}_{ph-ph}$

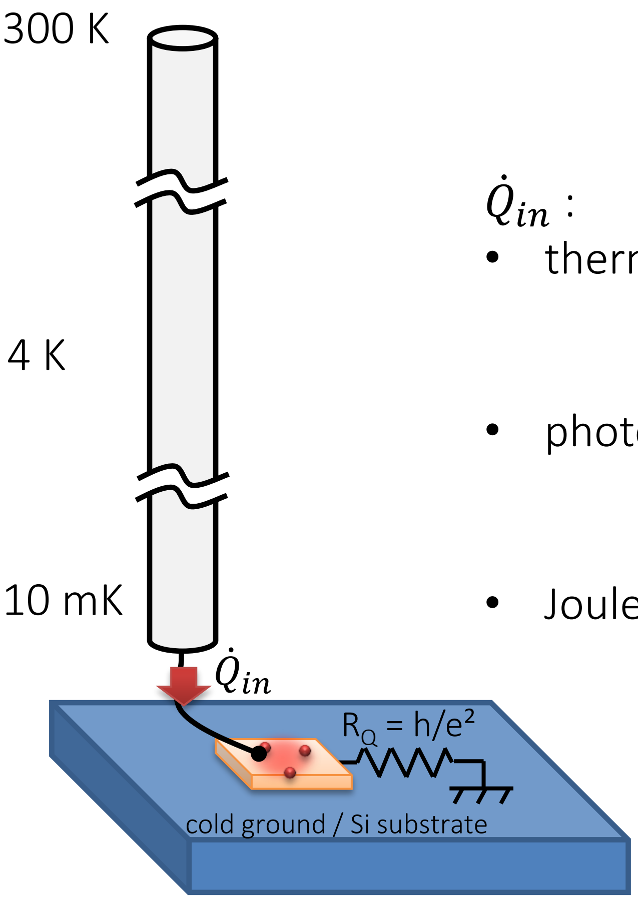
cold bath, T_{fridge}





minimize power $\dot{Q}_{in}$ incoming on sample

Example: metal contact on meso. conductor



$\dot{Q}_{in}$:

- thermal conduction from warmer parts

➡ **wire thermalization**

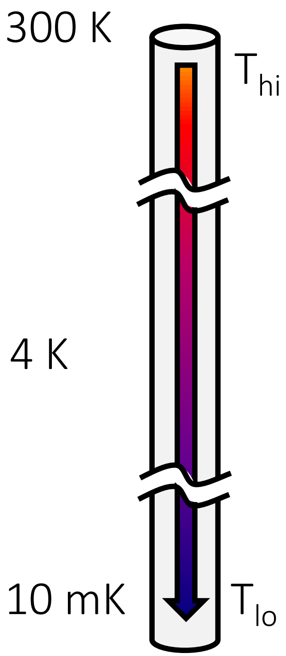
- photons (blackbody radiation)

➡ **shielding & filtering**

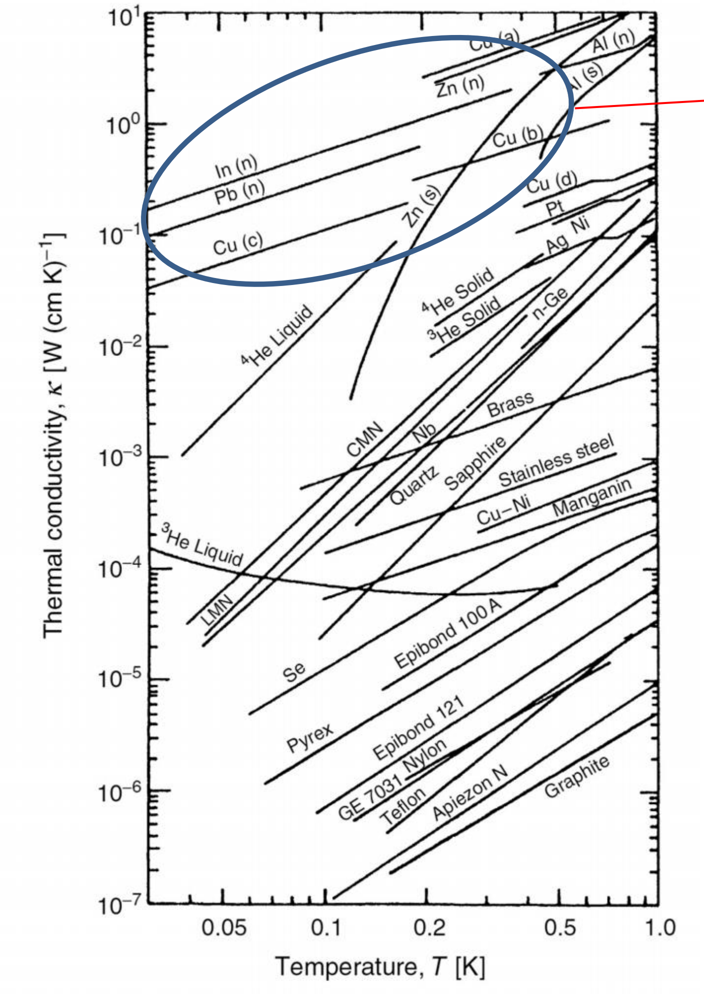
- Joule power dissipated in the contact

➡ **small electrical signals**

Wire thermalization



thermal conductivity κ @ low temp



metals: $\kappa \propto T$

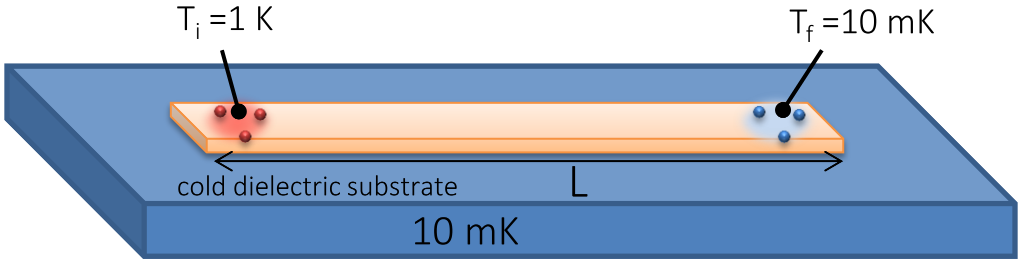
- heat mainly carried by electrons (Wiedemann-Franz law):

$$\dot{Q}_{\mathrm{el}} = L_0\,(T_{\mathrm{hi}}^2 - T_{\mathrm{lo}}^2)/R_{\mathrm{wire}}$$

- use resistive wire

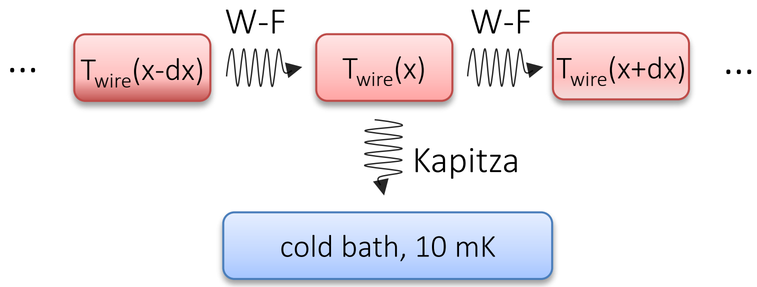
Lounasmaa - Experimental principles and methods below 1K (1974)

Wire thermalization



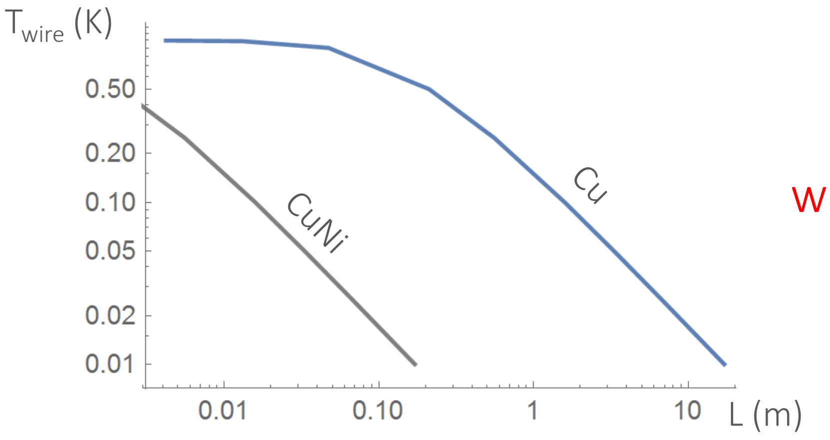
wire length L to thermalize electrons from 1 K to 10 mK ?

hyp.: electrons well thermalized to wire phonons



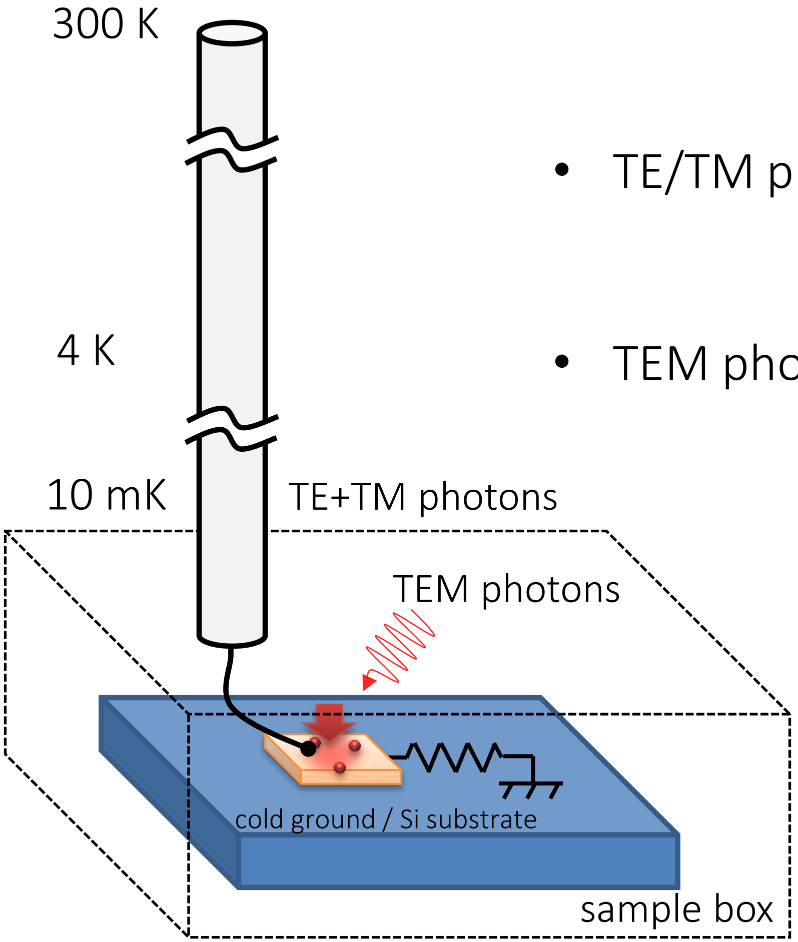
electrical conductivity

$$L = \sqrt{\frac{2\,L_0 \sigma}{K}\int_{T_f}^{T_i}\frac{dT}{\sqrt{T^4 - T_f^4}}}$$



wire material is crucial!

Photons impinging on the sample



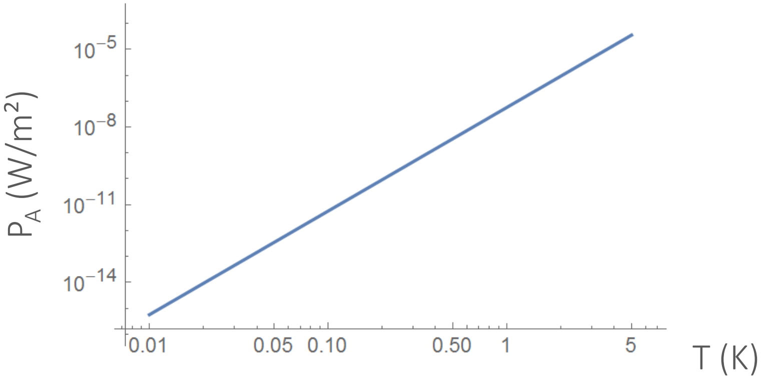
- TE/TM photons propagating in the wires
- TEM photons modes of the sample box

⇨ **shielding!**

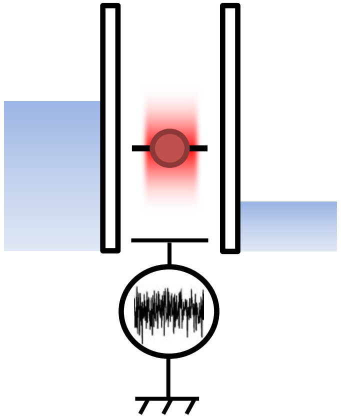
non-thermalized photons

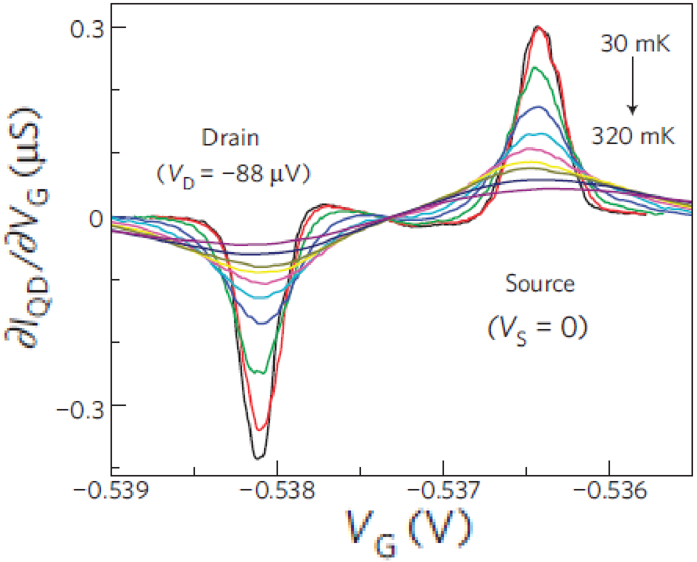
- radiative heat transfer:
  Stefan-Boltzmann law

$$P_A\ [\mathrm{W/m^2}] = \frac{2\pi^5 k_B^4}{15h^3c^2} T_{\mathrm{photons}}^4$$



- electromagnetic noise on the gates

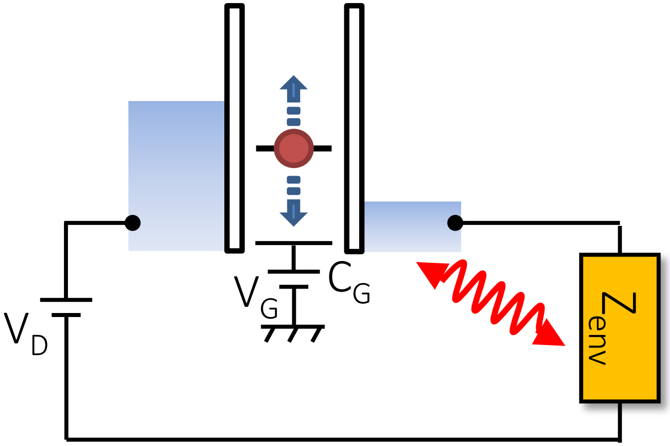




non-thermalized photons II

- charge transfer in coherent conductor coupled to electromag. env^t:

Dynamical Coulomb Blockade



tunneling rate Γ affected by prob. $P(E)$
for env^t to absorb photon with energy E :

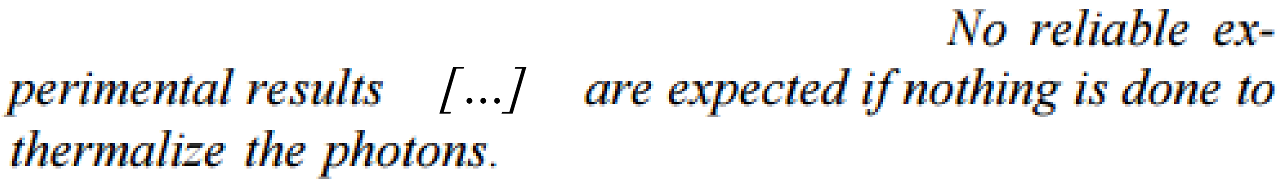
$$\Gamma(\varepsilon) = \int dE\, \Gamma_0(\varepsilon - E) P(E)$$

$$Z_{\text{env}} \ll R_K \rightarrow \quad P(E) \approx \frac{1}{2\pi\hbar} \int dt e^{\frac{iEt}{\hbar}} \times \exp\left[\frac{2\pi}{\hbar R_K} \int d\omega \frac{S_V(\omega)}{\omega^2} (\cos\omega t - 1) \right]$$

$$S_V(\omega) \propto \frac{\mathrm{Re}[Z_{\text{env}}]\hbar\omega}{\exp\left(\hbar\omega / k_B T_{\text{ph}}\right) - 1}$$ (Planck's law)

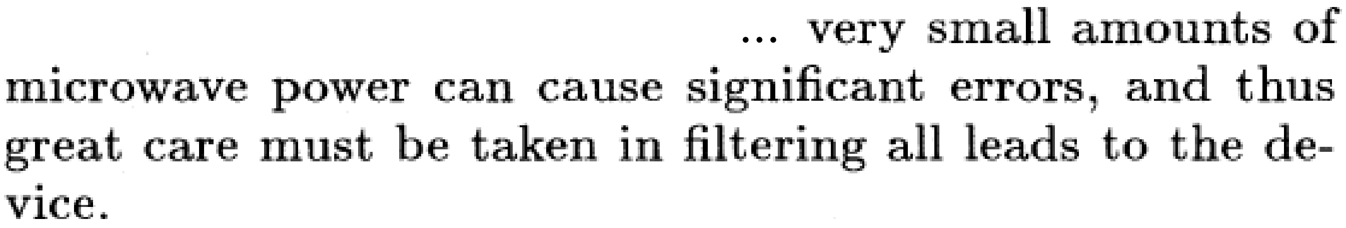
Devoret *et al.*, Phys. Rev. Lett. **64,** 1824-7 (1990)
Nazarov & Ingold, *Single Charge Tunneling* (1992)
Martinis & Nahum, PRB **48,** 18316 (1993)

non-thermalized photons



No reliable experimental results [...] are expected if nothing is done to thermalize the photons.

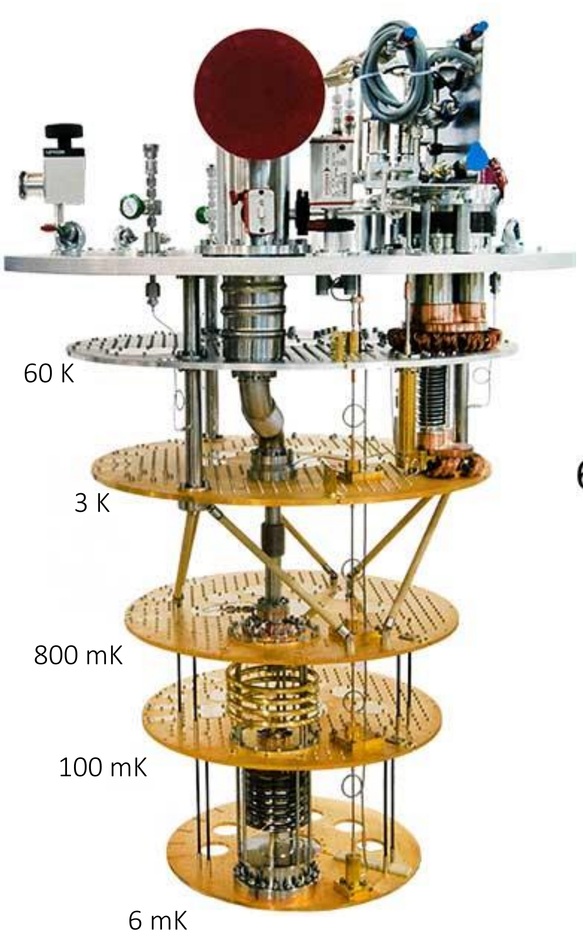
D. C. Glattli, P. Jacques, A. Kumar, P. Pari, & L. Saminadayar, Journal of Applied Physics **81**, 7350 (1997)

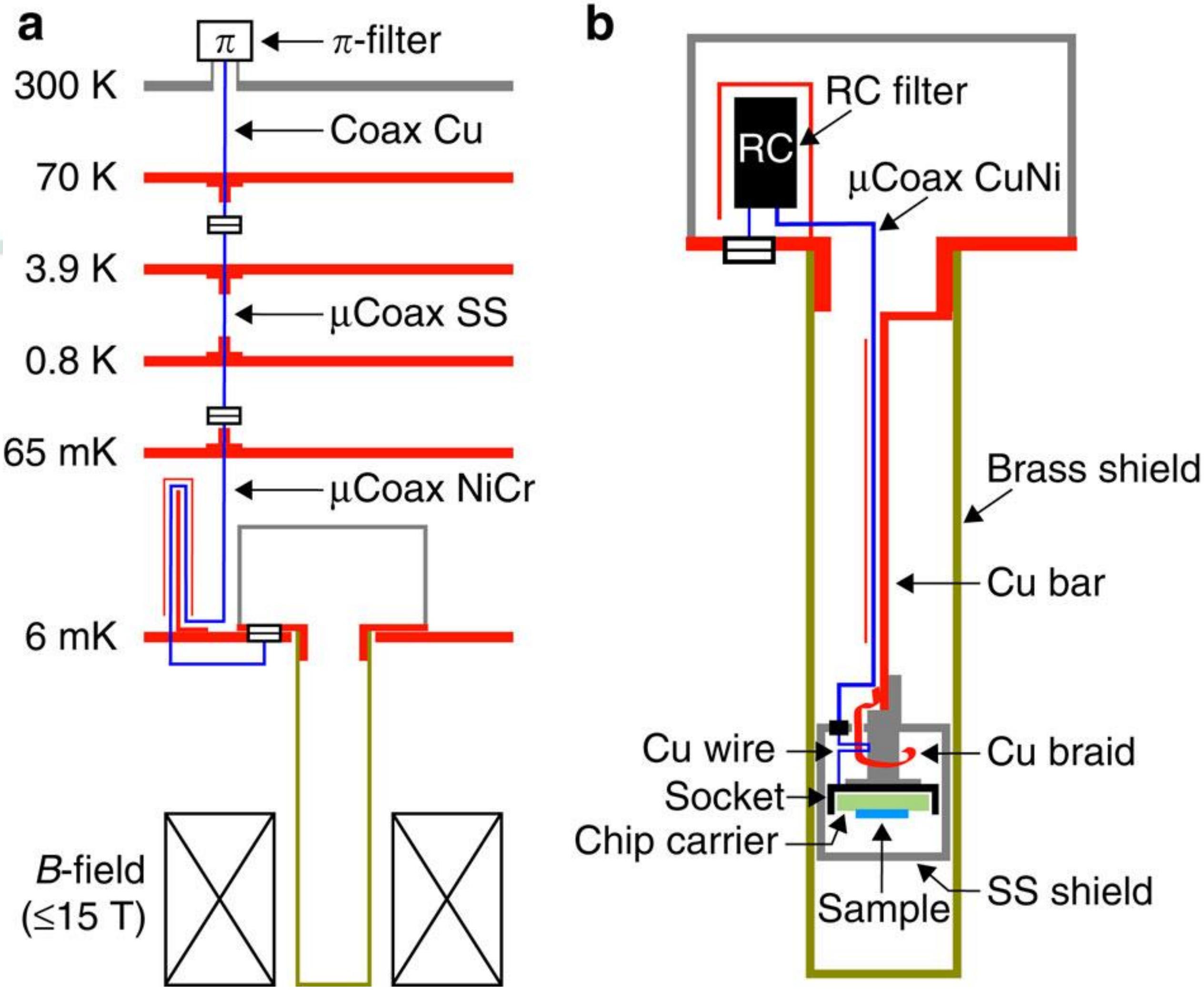


... very small amounts of microwave power can cause significant errors, and thus great care must be taken in filtering all leads to the device.

Martinis & Nahum, PRB **48,** 18316 (1993)

Filtering, wiring and shielding



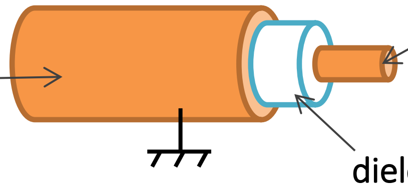


Z. Iftikhar *et al.,* Nature Communications **7,** 12908 (2016)

Filtering: coax. wires

resistive wires needed for electron thermalisation ➡ just add capacitance!

➡ **cryo coaxial wire: lossy transmission line**

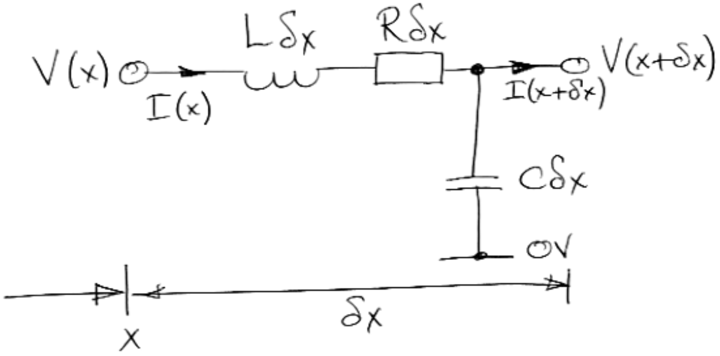


outer conductor (ground)
- minimize heat load on fridge
- define a good ground
- thermal anchor to fridge
  → thin and slightly resistive

inner conductor (signal)
- electron thermalization
- suppress thermal photons
  → resistive
- minimize heat load
  → thin

dielectric
- defines good capacitance
- good thermal transfer between inner & outer conductors

→ thin

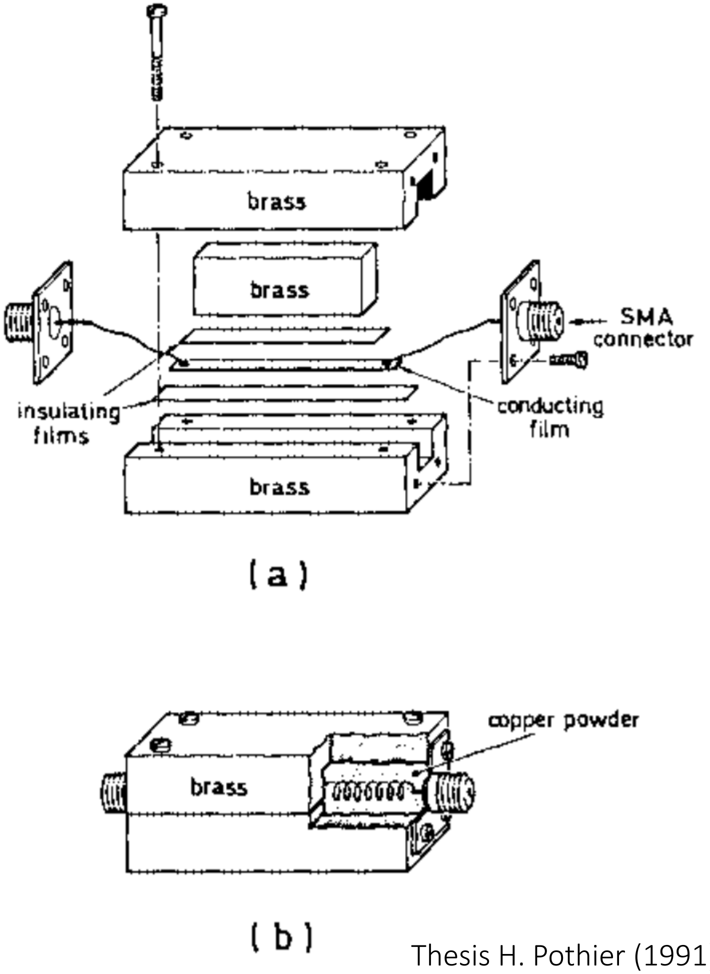


typ. R=50-100 Ω/m
C=100 pF/m

- commercial cryo coax
- thin resistive wire threaded in small CuNi tube

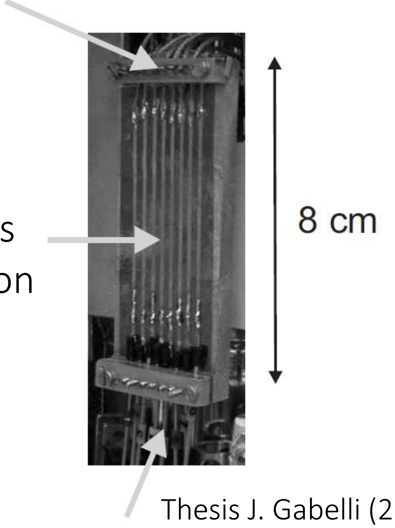
Filtering: discrete elements

- planar copper lines:
  not good below 1 K!



Thesis H. Pothier (1991)

copper clamp



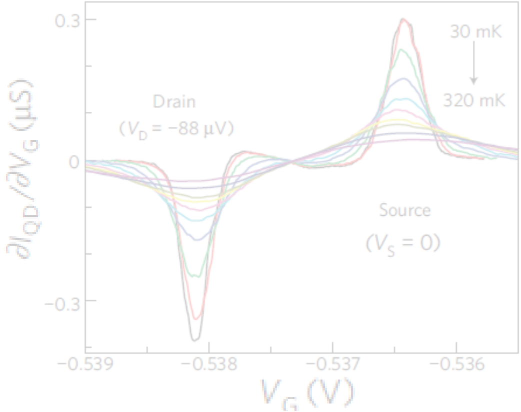
Cu lines on kapton

Thesis J. Gabelli (2006)

coax

Outline

1. low temperature experiments
    - cryogenic systems
    - lattice vs electron temperature
    - filtering

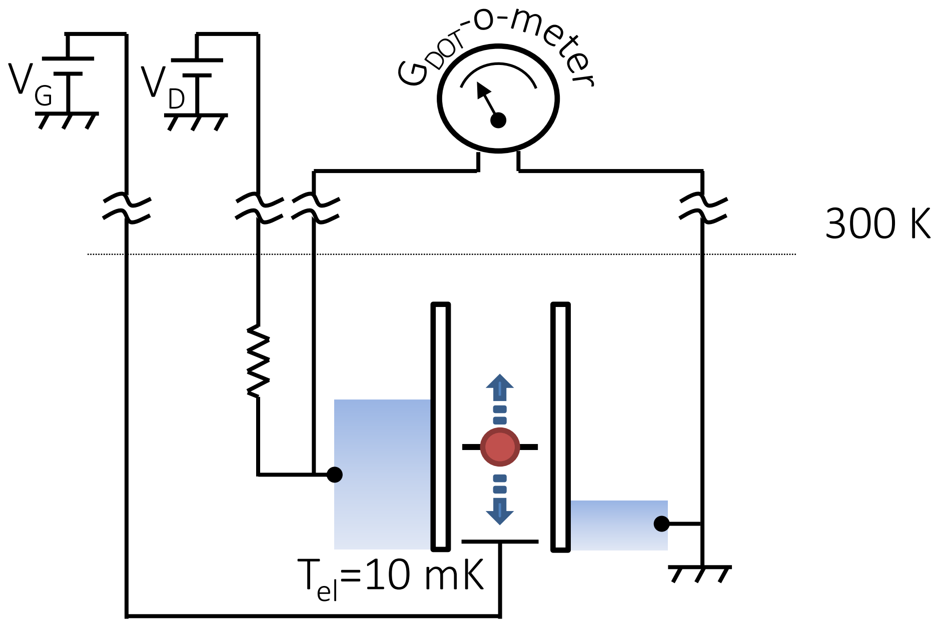


2. low noise cryoelectronics
    - signal vs noise
    - DC & AC meas^{t}, lock-in
    - measurement configurations

3. beyond dc conductance
    - microwave measurements
    - noise

Low noise cryoelectronics

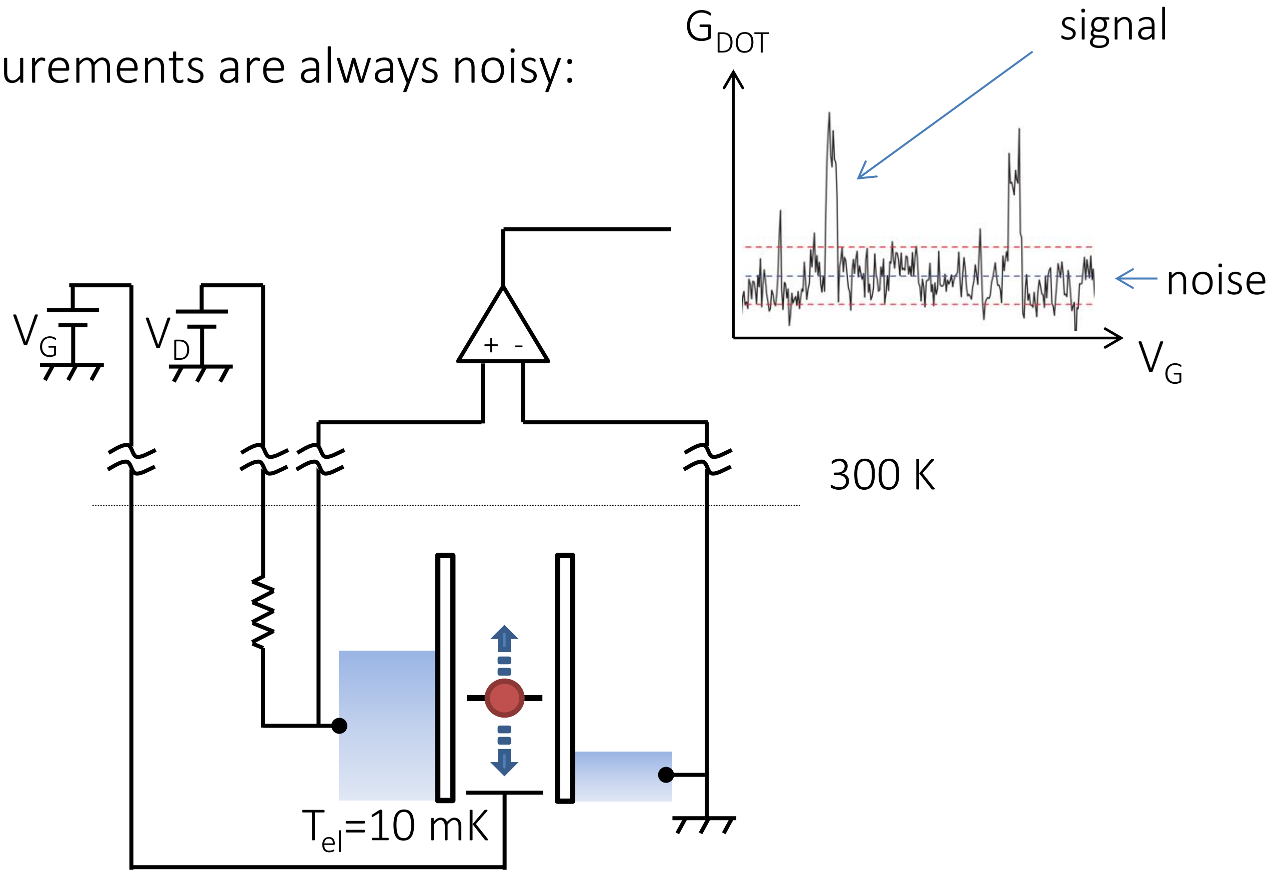
- ✓ electrons are cold
- ✓ photons are filtered out



measurement of $G_{DOT}(V_D,V_G)$ at low temperature ?

Signal vs noise

real measurements are always noisy:



Maximize signal / noise ratio (SNR):

A) noise sources

B) which signals?

Noise sources

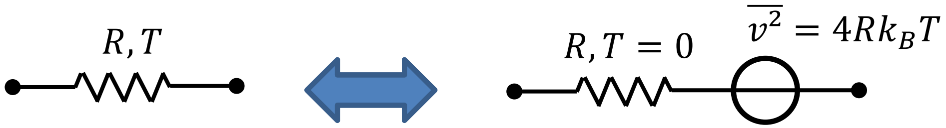
- thermal noise
- 1/f noise
- amplification noise
- parasitics, room T:
  - flux
  - ground loops
- parasitics, cryogenics
  - thermovoltages
  - triboelectrics
  - microphonics

Thermal (Johnson-Nyquist) noise

thermal agitation of charge carriers in a conductor (resistance *R*, temperature *T*)

voltage fluctuations $\overline{v^2} = 4Rk_BT$

units: V^2/Hz
(or V/√Hz)

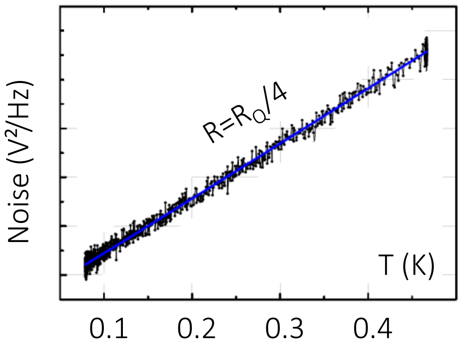


Johnson, Phys. Rev. **32,** 97 (1928) (experiment)
Nyquist, Phys. Rev. **32,** 110 (1928) (theory)

50 Ω @ 300 K → 0.9 nV/√Hz

Electrical example of fluctuation-dissipation theorem!

good primary thermometer
for low (and high!) temperatures
(also a good way to calibrate amps)

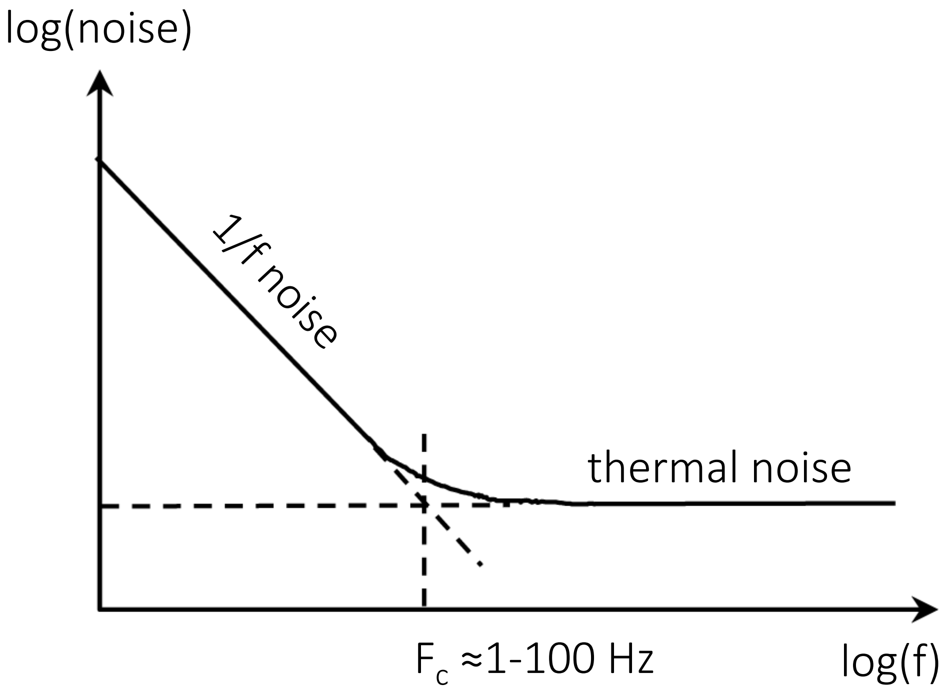


1/f (flicker) noise

sample in electrostatic environment:

surrounding trapped charges = fluctuating two-level systems

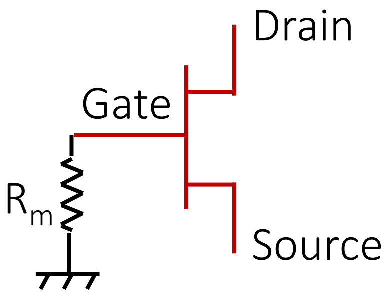
⇨ low frequency fluctuations!

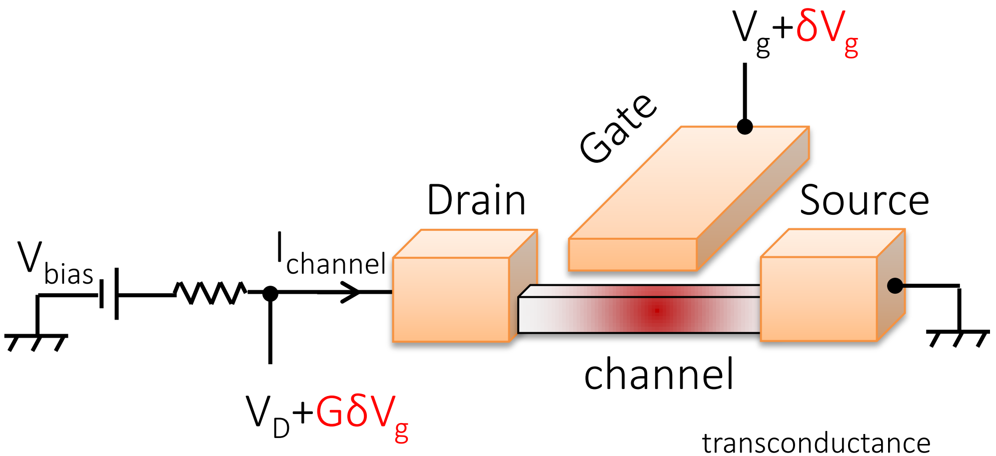


very unfavorable for DC measurements, thermally activated

Amplification noise: field-effect transistors

field-effect transistor (FET)





$$G \approx -\frac{g_{\mathrm{m}}}{g_{\mathrm{c}}}$$

transconductance

$$g_{\mathrm{m}} = {\partial I_{\mathrm{channel}}}/{\partial V_g}$$

channel conductance

$$g_{\mathrm{c}} = {\partial I_{\mathrm{channel}}}/{\partial V_D}$$

Main noise sources:

- 1/f noise
- thermal noise of the channel
- shot noise of the gate leakage current

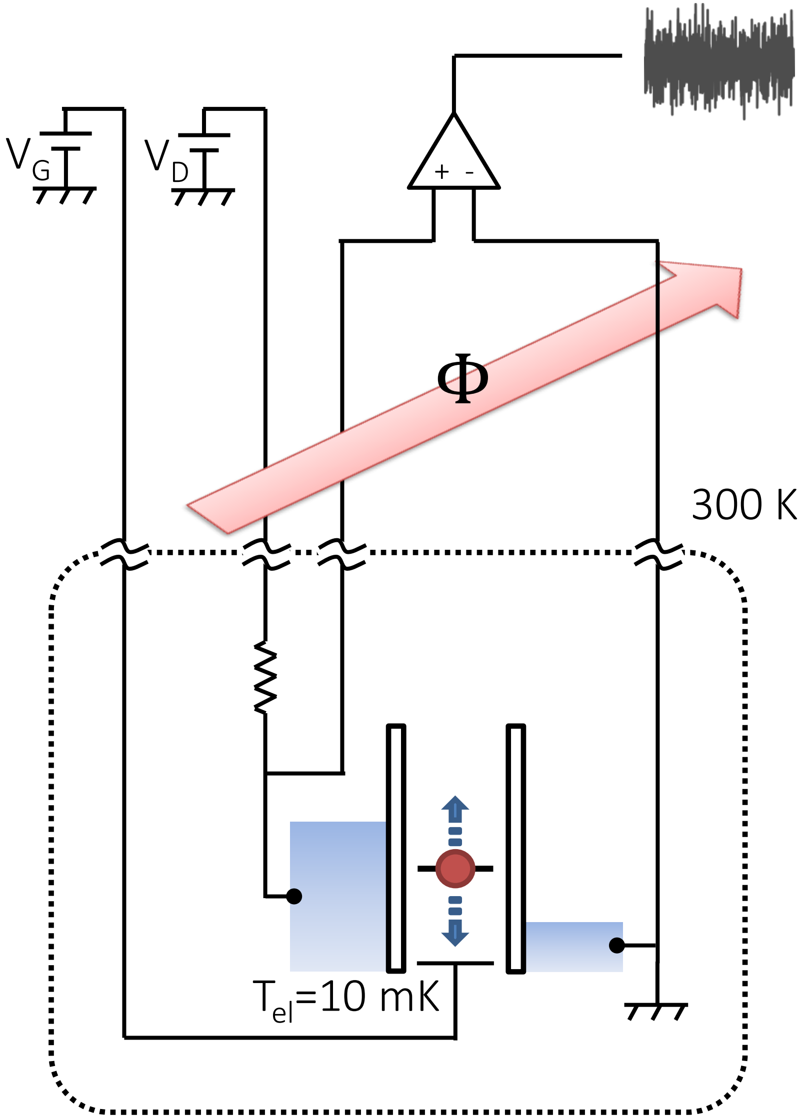
sources independent of measured impedance R_m: *voltage noise*

sources dependent of R_m: *current noise*

typical noise: $\sqrt{\boldsymbol{\delta}\mathbf{v}^2} \sim \mathbf{1\ nV}/\sqrt{\mathbf{Hz}}$

Bordoni *et al.,* Rev. Sci. Instrum. **52,** 1079 (1981)

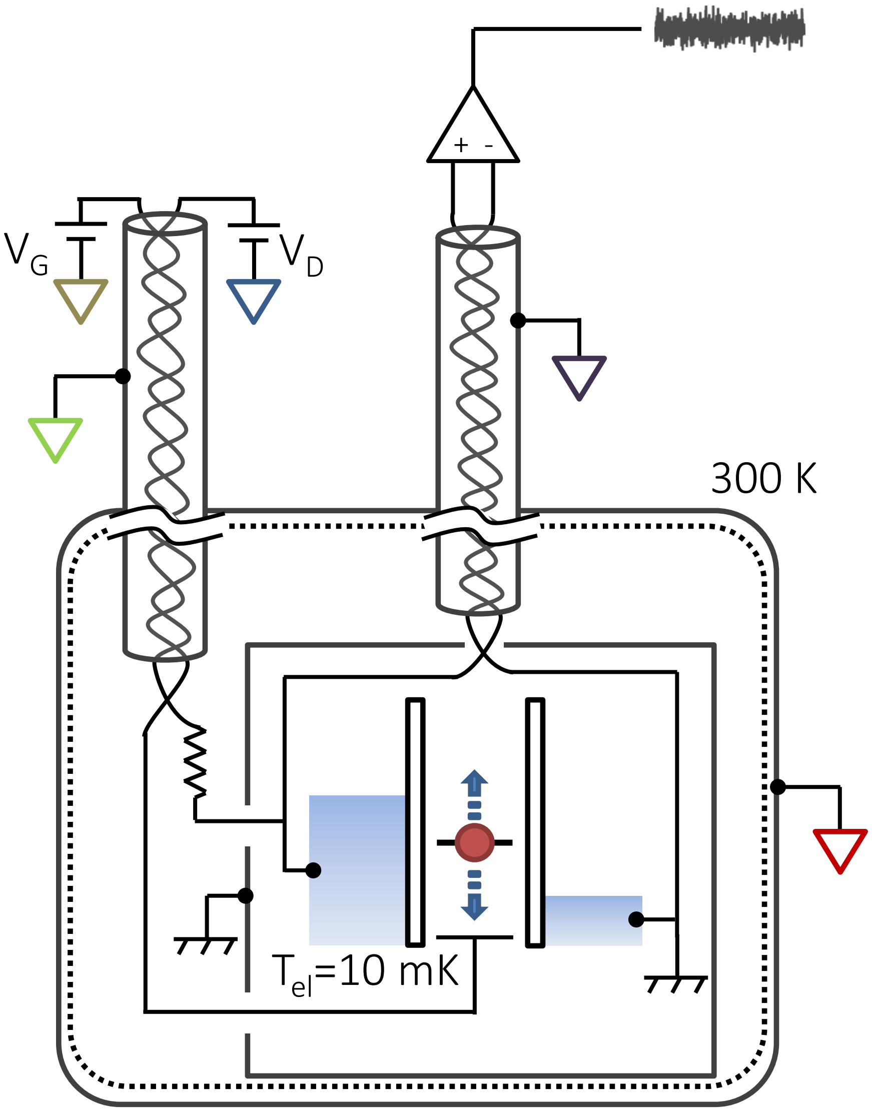
Room T: flux & ground loops



Fluctuating magnetic flux (50 Hz, …) in the loop induces noise

➡ get rid of loops!

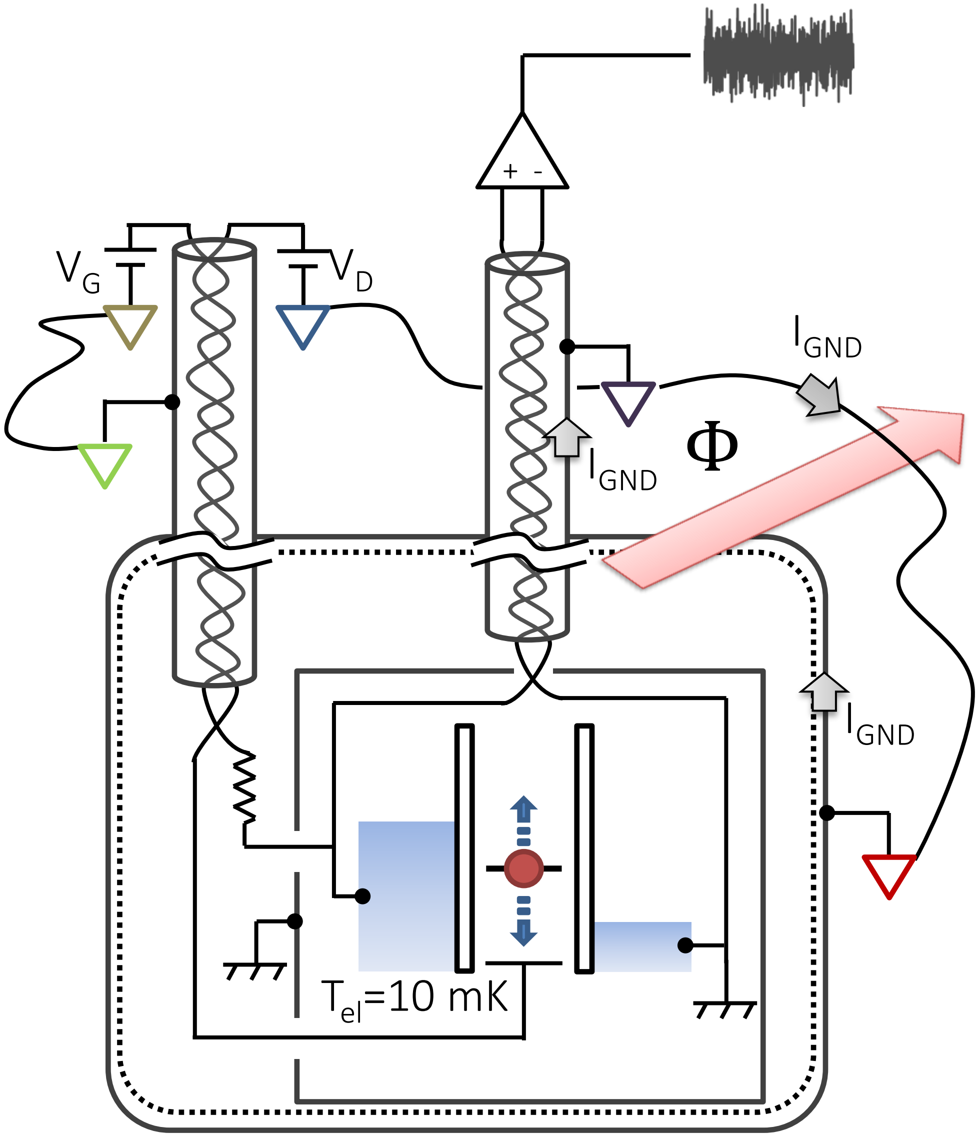
Room T: flux & ground loops



get rid of loops:

- twisted pairs
- shielding

Room T: flux & ground loops



get rid of loops:

- twisted pairs
- shielding

beware of ground loops!

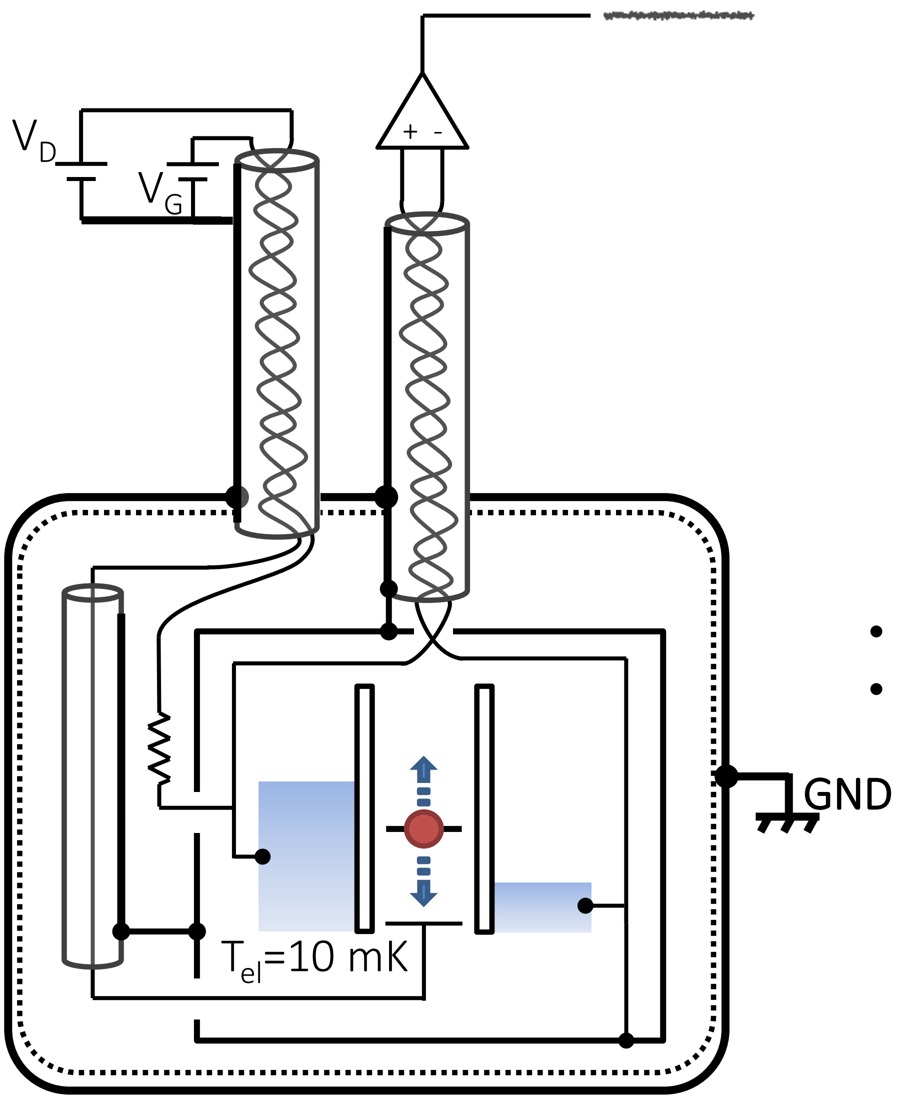
long, poorly conducting GND wires

➡ pick up flux

➡ parasitic fluctuating I_{GND}

➡ **noise!!**

Room T: flux & ground loops



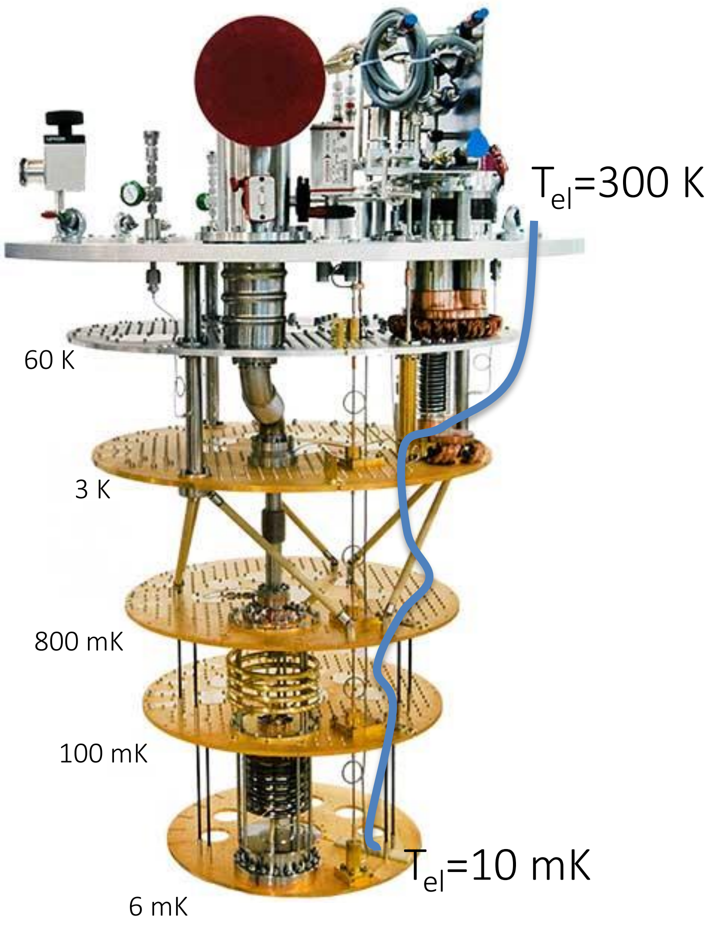
get rid of loops:

- twisted pairs
- shielding

beware of ground loops!

- single well defined GND (close to fridge)
- short, low resistance wires to the GND

Thermoelectric voltages



Seebeck effect: $V_{\text{therm}} = S\Delta T \quad (\frac{\Delta T}{T} \ll 1)$

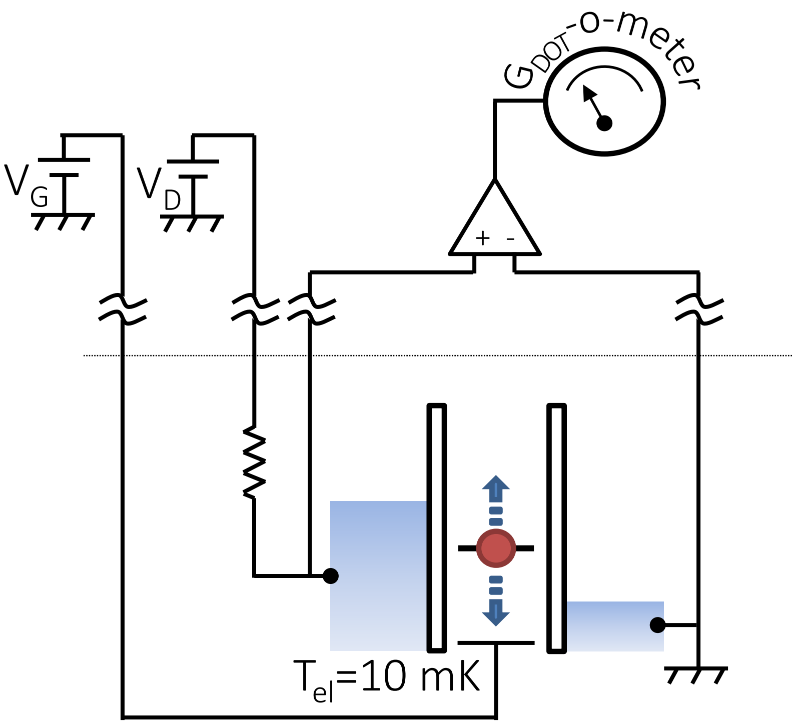
thermoelectric dc voltage between 300 K and 10 mK

$$V_{\text{therm}} \sim 1 - 100\ \mu V$$

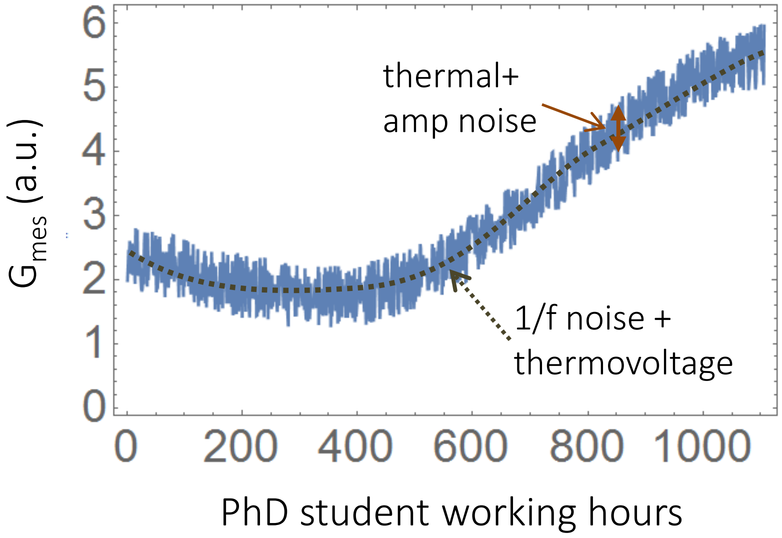
slow fluctuations with temperature!

unfavorable for DC measurements

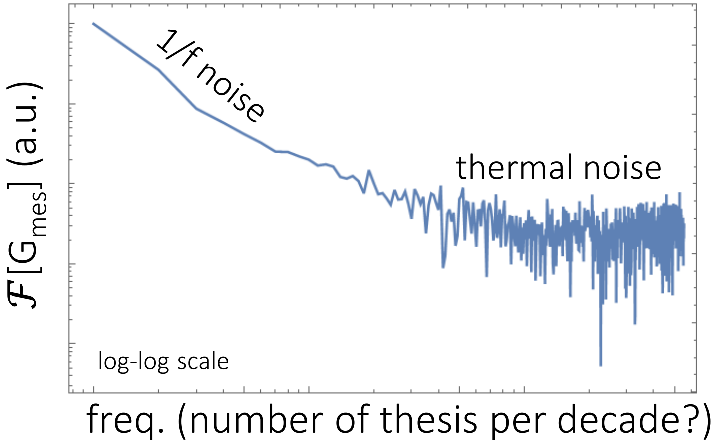
DC vs finite frequency meas^t



V_D=0 : G_{DOT}=0 (no signal from sample)

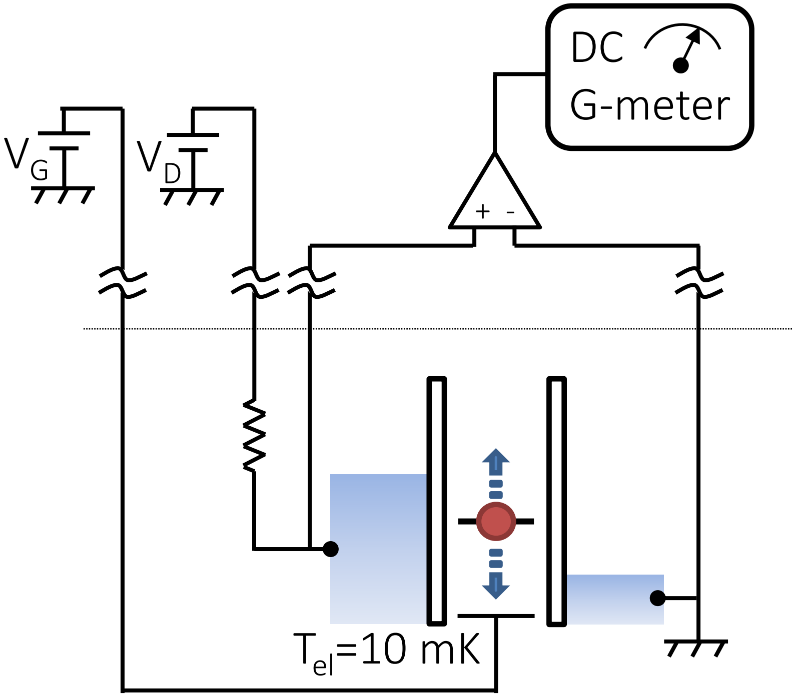


FFT

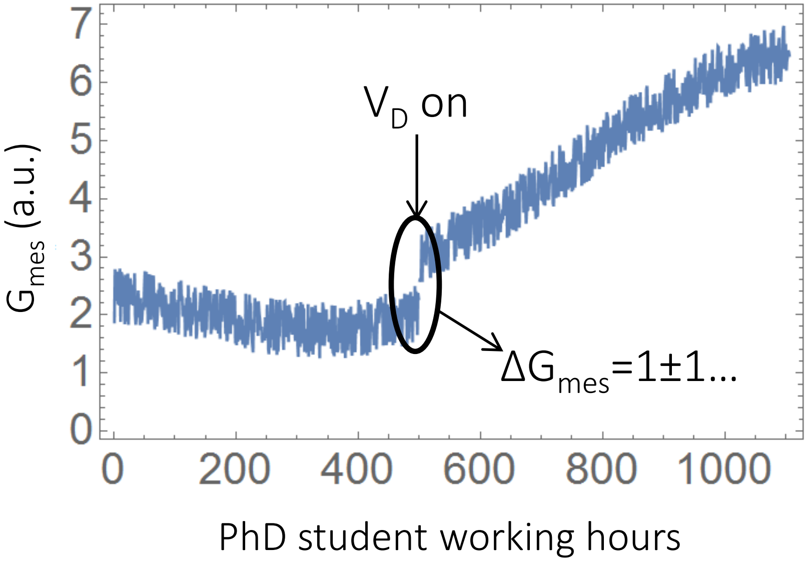


DC vs finite frequency meas^t

after working ~500 hrs, student A remembers to turn V_D on.



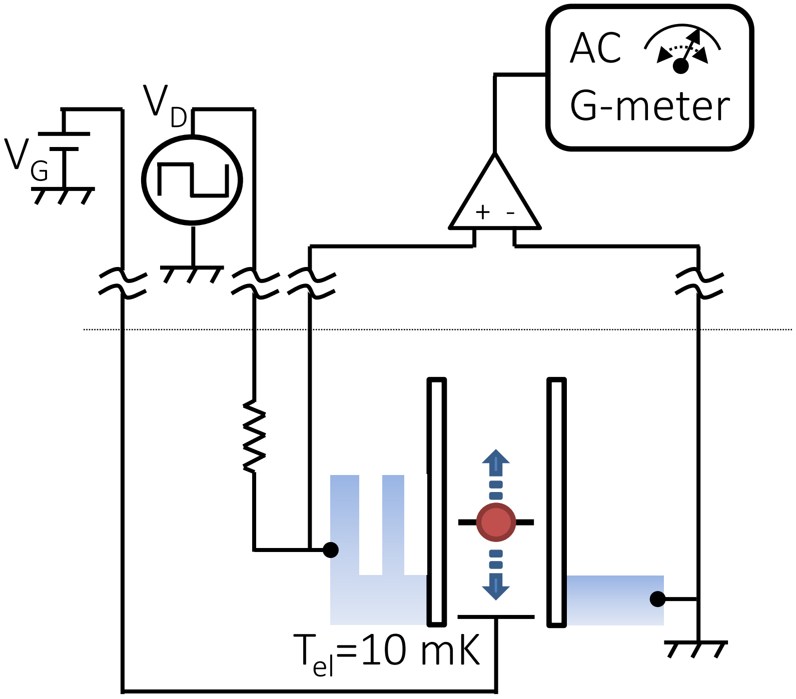
V_D=1 V: G_{DOT}=1 a.u.



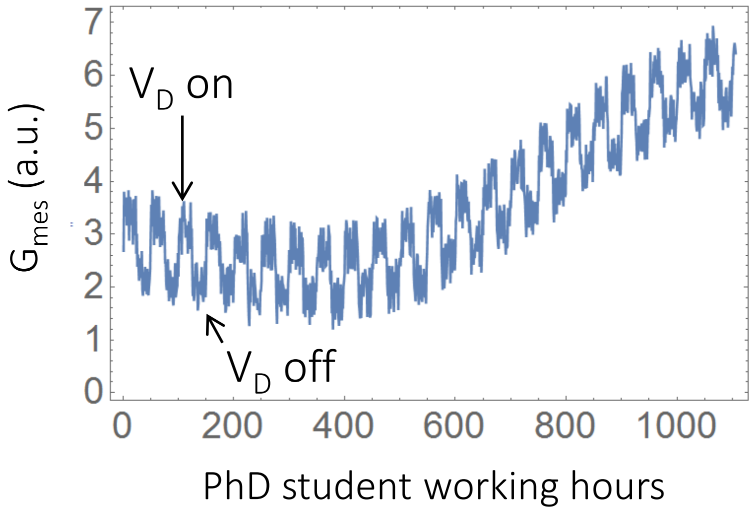
- 'instant' DC meas^t → very noisy!
- averaged DC meas^t → drift!!

DC vs finite frequency meas^t

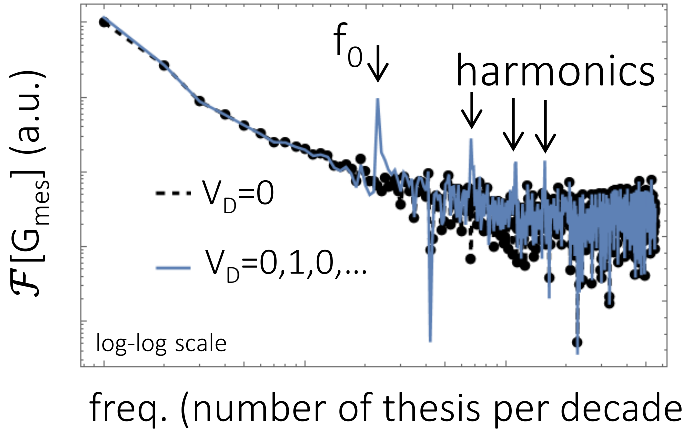
Solution: turn V_D on periodically



V_D=0,1,0,... V → G_{DOT}=0,1,0,... a.u.

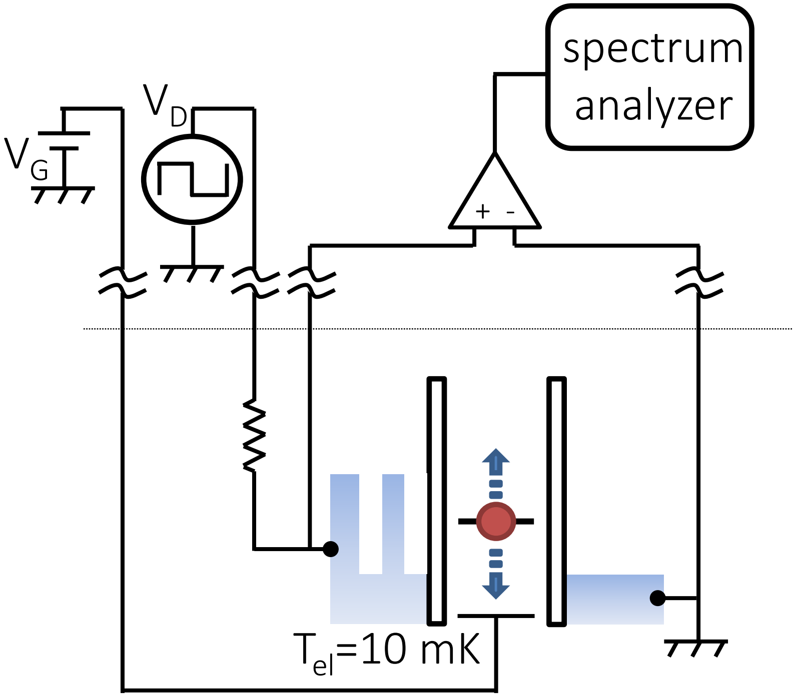


FFT

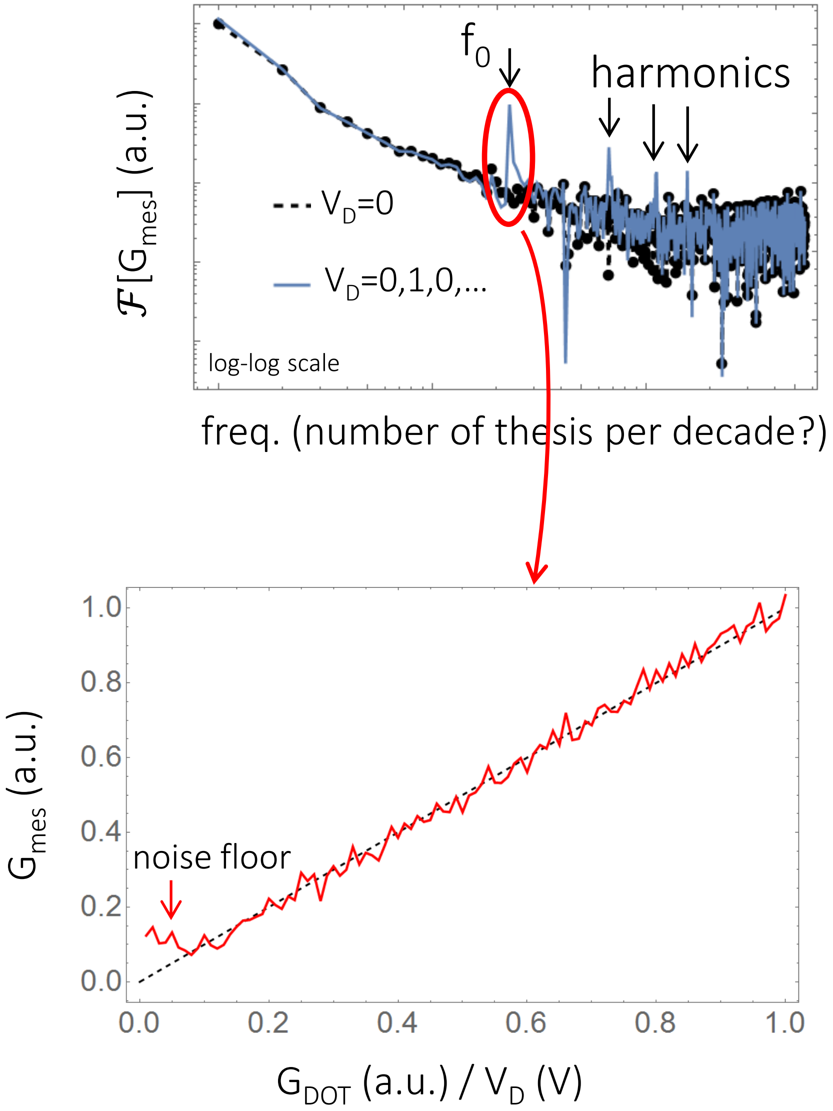


DC vs finite frequency meas^t

Solution: turn V_D on periodically
+ measure power at f_0



V_D=0,1,0,... V → G_{DOT}=0,1,0,... a.u.

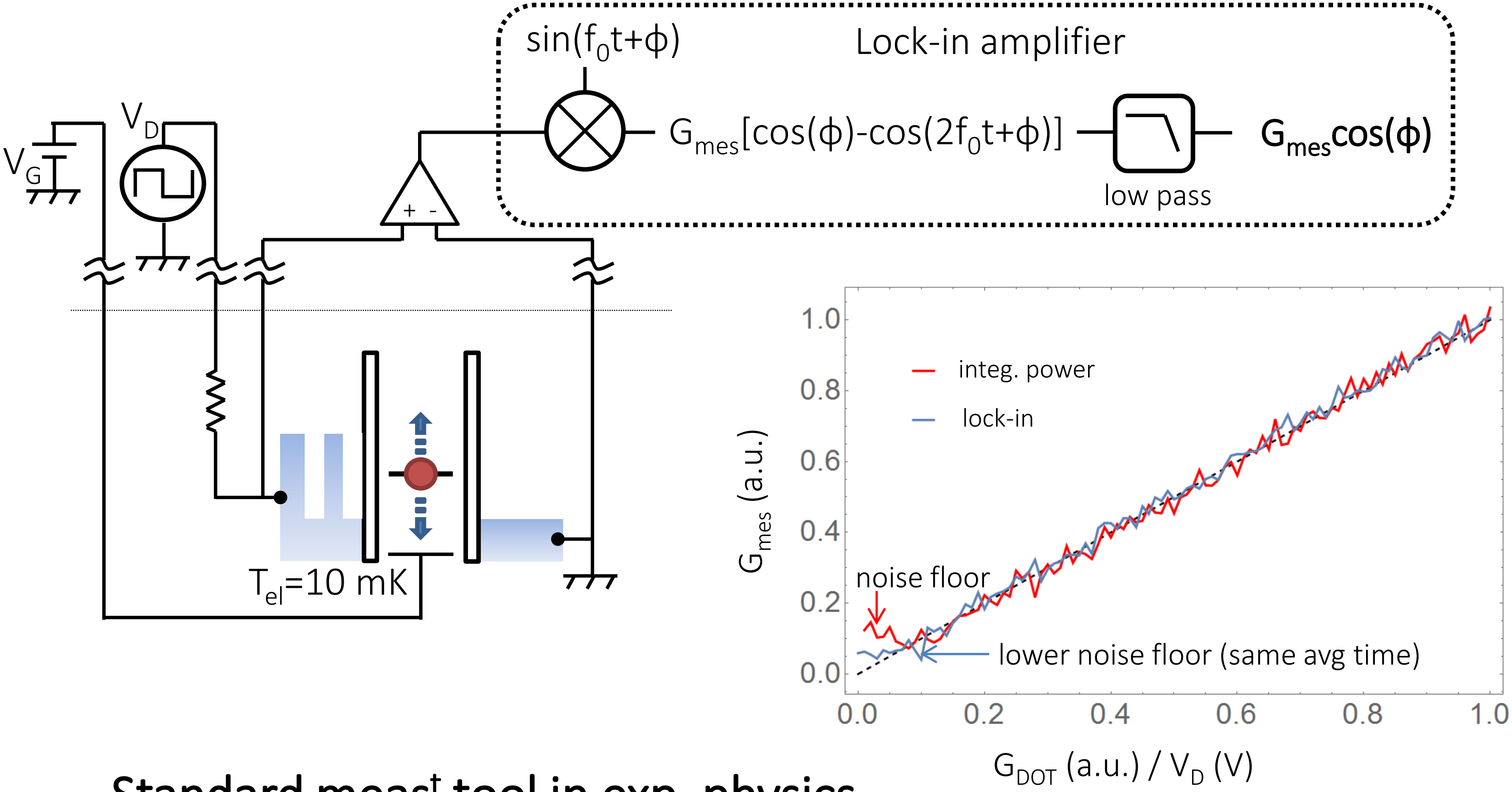


AC meas^t: **less noise**

Lock-in measurement

turn V_D on periodically + multiply output signal by sine wave at same period

http://www.thinksrs.com/



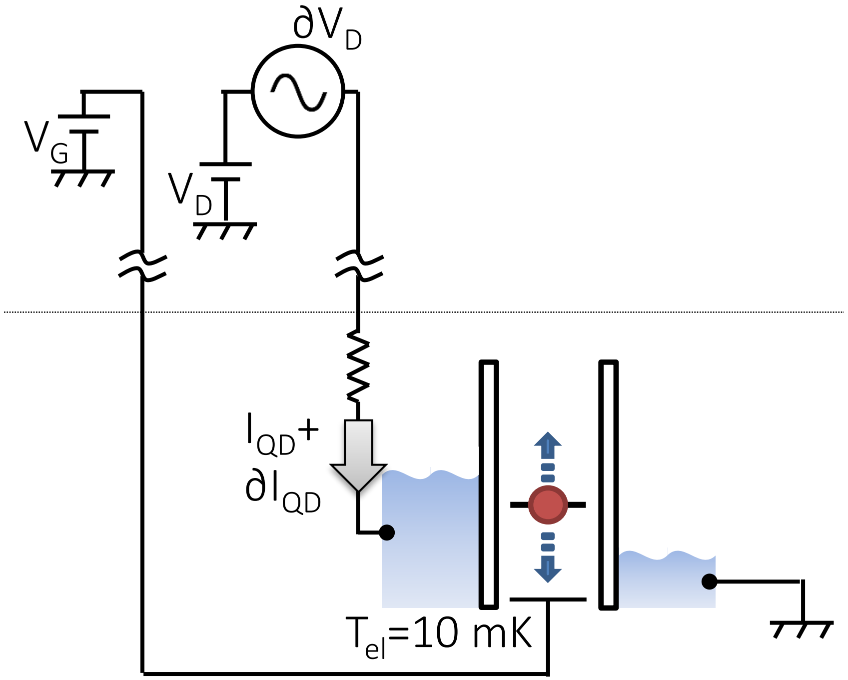
Standard meas^t tool in exp. physics

Differential conductance meas^t

lock-in: great for small AC signals detection

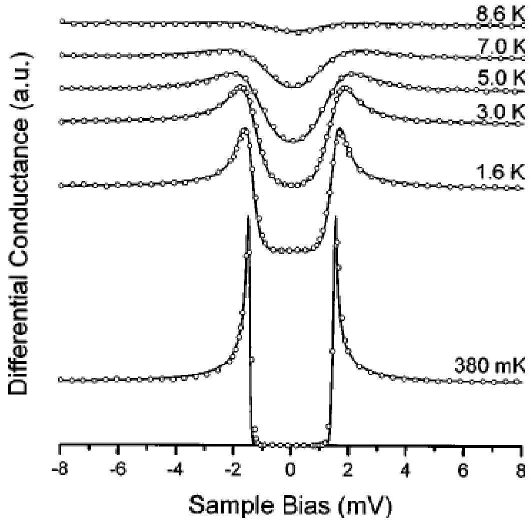
➡ drive sample with small (linear) AC signal ∂V on top of DC signal

+ lock-in measurement of diff. conductance $g_{DOT}(V_D)=\partial I_{QD}/\partial V_D$



- **proportional to D.O.S.!**
- access to the linear (V_D=0) regime
- emphasizes non-linear features

STM tunneling conductance Nb / Au



APL **73,** 2992 - 2994 (1998)

Differential conductance meas^t

lock-in: great for small AC signals detection

➡ drive sample with small (linear) AC signal ∂V on top of DC signal
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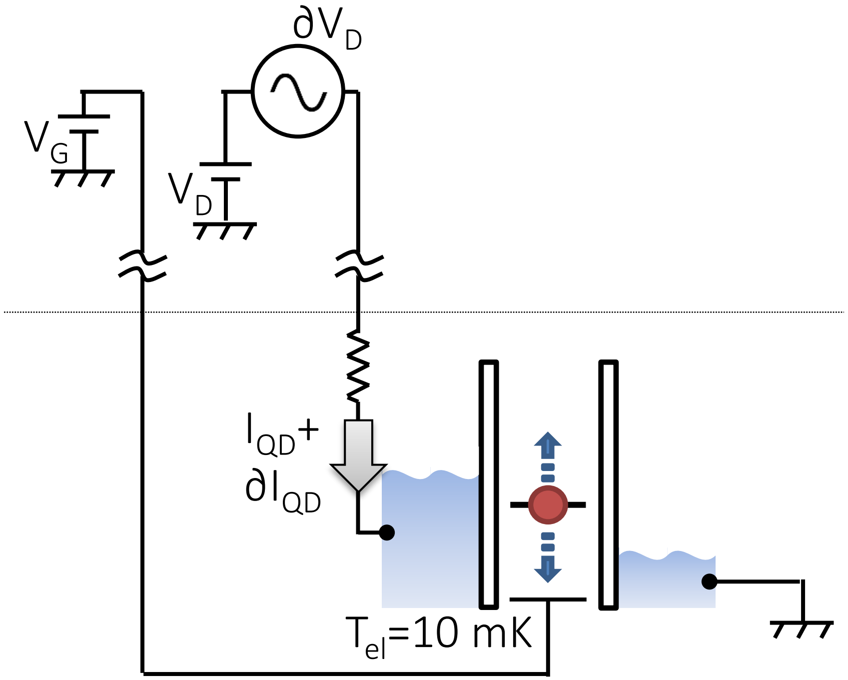
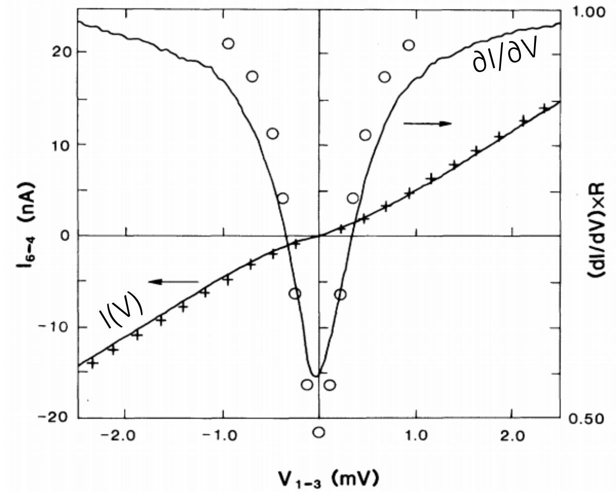


- proportional to D.O.S.!
- access to the linear (V_D=0) regime
- emphasizes non-linear features

Dynamical Coulomb Blockade
in ultrasmall tunnel junctions

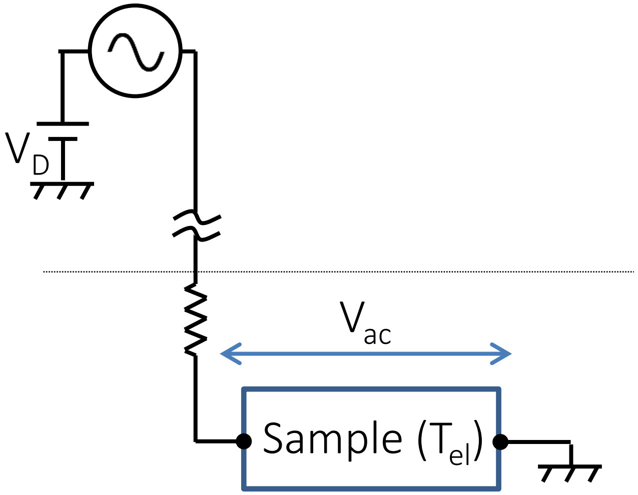


Delsing *et al.*, PRL **63,** 1180 (1989)

Differential conductance meas^t

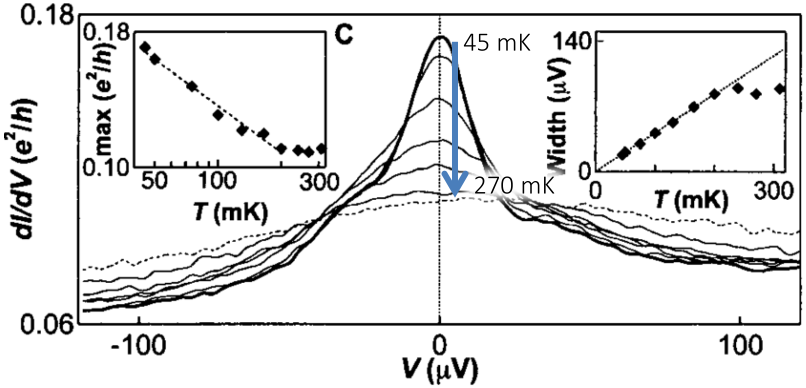
lock-in: great for small AC signals detection

➡ drive sample with small (linear) AC signal ∂V on top of DC signal
+ lock-in measurement of diff. conductance $g_{DOT}(V_D)=\partial I_{QD}/\partial V_D$



- proportional to D.O.S.!
- access to the linear (V_D=0) regime
- emphasizes non-linear features

Kondo effect in semiconductor quantum dots



Cronenwett *et al.*, Science **280**, 540 (1998)

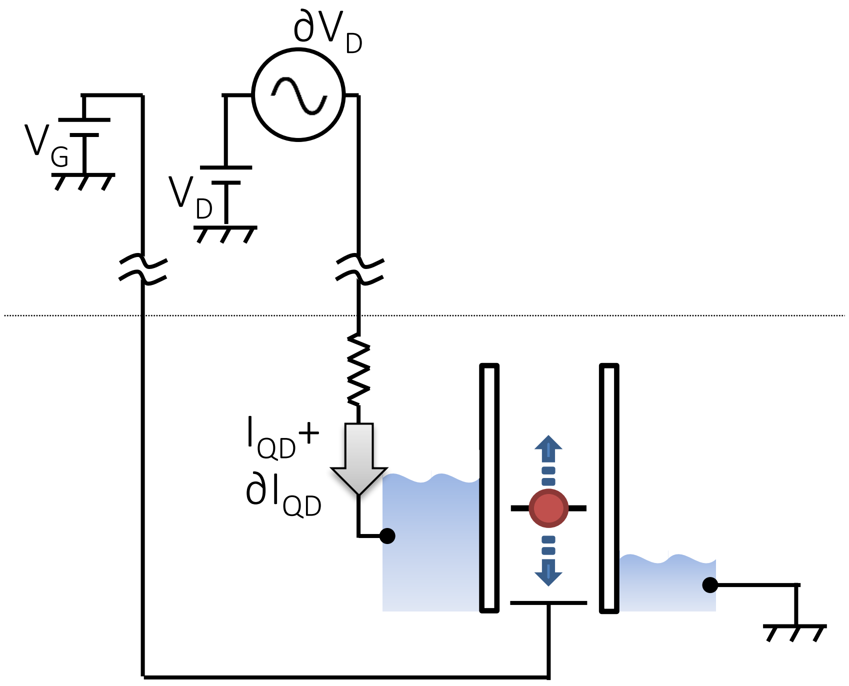
$$\boxed{eV_{ac}<k_BT_{el}}$$

1 μV ↔ 11.5 mK

AC voltage scale set by T_{el}

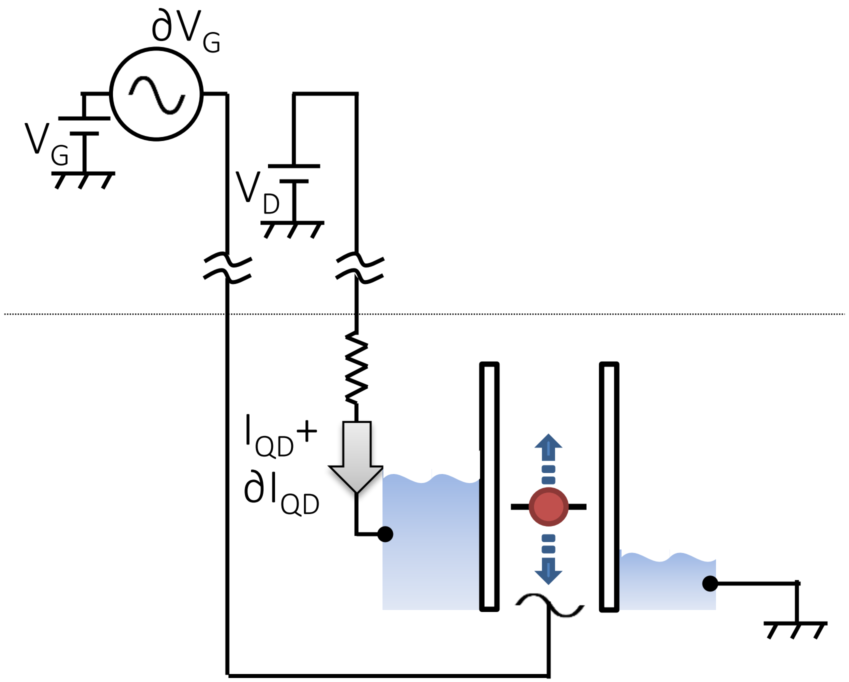
Lock-in meas^t configurations

- differential conductance $\partial I_{QD}/\partial V_D$



straightforward , quantitative
extraction of G_{DOT} / $\mathcal{S}$-matrix

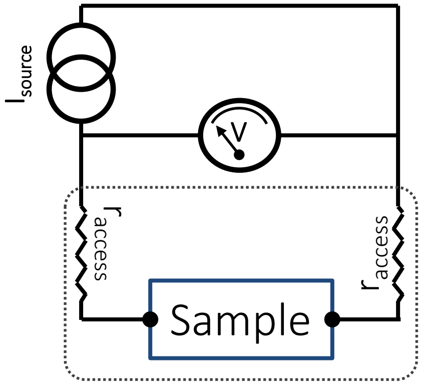
- transconductance $\partial I_{QD}/\partial V_G$



out-of-equilibrium meas^t:
$V_{D1,2,\dots}$ applied to various terminals
(requires calibration of lever arm $V_G \leftrightarrow \varepsilon_{DOT}$)

Meas^t configs.: 2/4 points

2 points resistance/conductance meas^t

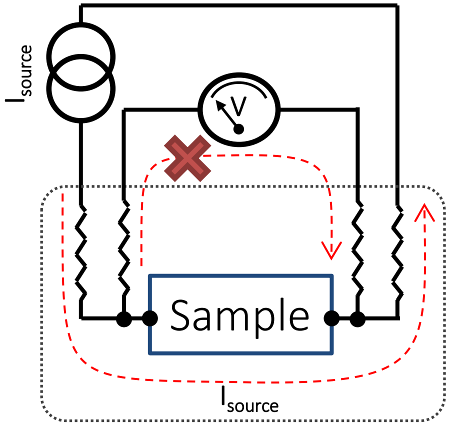


2 points: $V/I=R_{sample}+2r_{access}$

r_{access}= wires, contacts, …

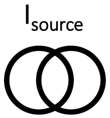
➡ ‘false’ reading of R_{sample}

4 points resistance/conductance meas^t



4 points: $V/I=R_{sample}$

➡ no contribution from r_{access}

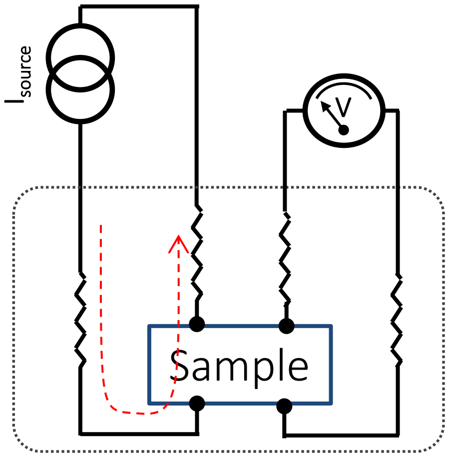


current source: $R_{int} \ll R_{sample}$

volt-meter: $R_{int} \gg R_{sample}$

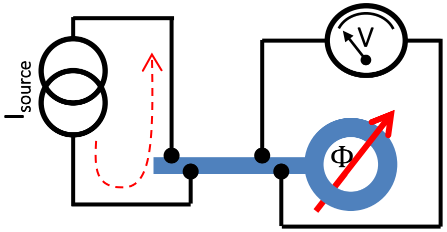
➡ be careful when measuring very high R_{sample}!

4 pts & more: non-local meas^t

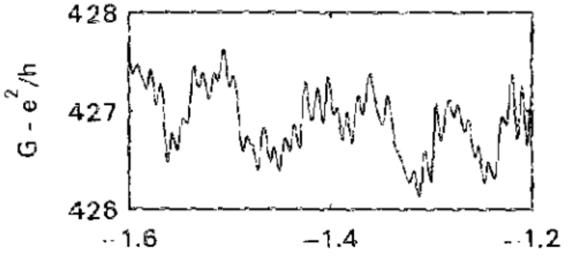


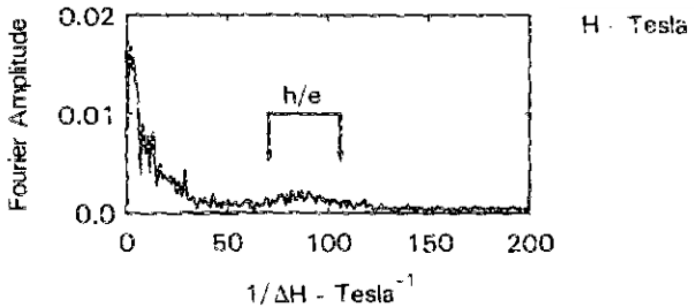
drive current in one side of sample
& measure non-local voltage on other side

A) unveils non-local effects

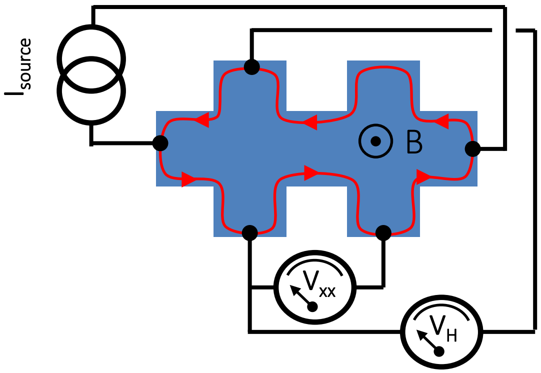


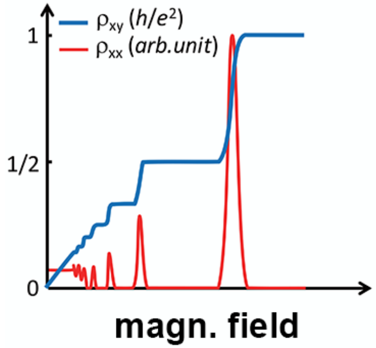
Umbach *et al.*, APL **50,** 1289 (1987)





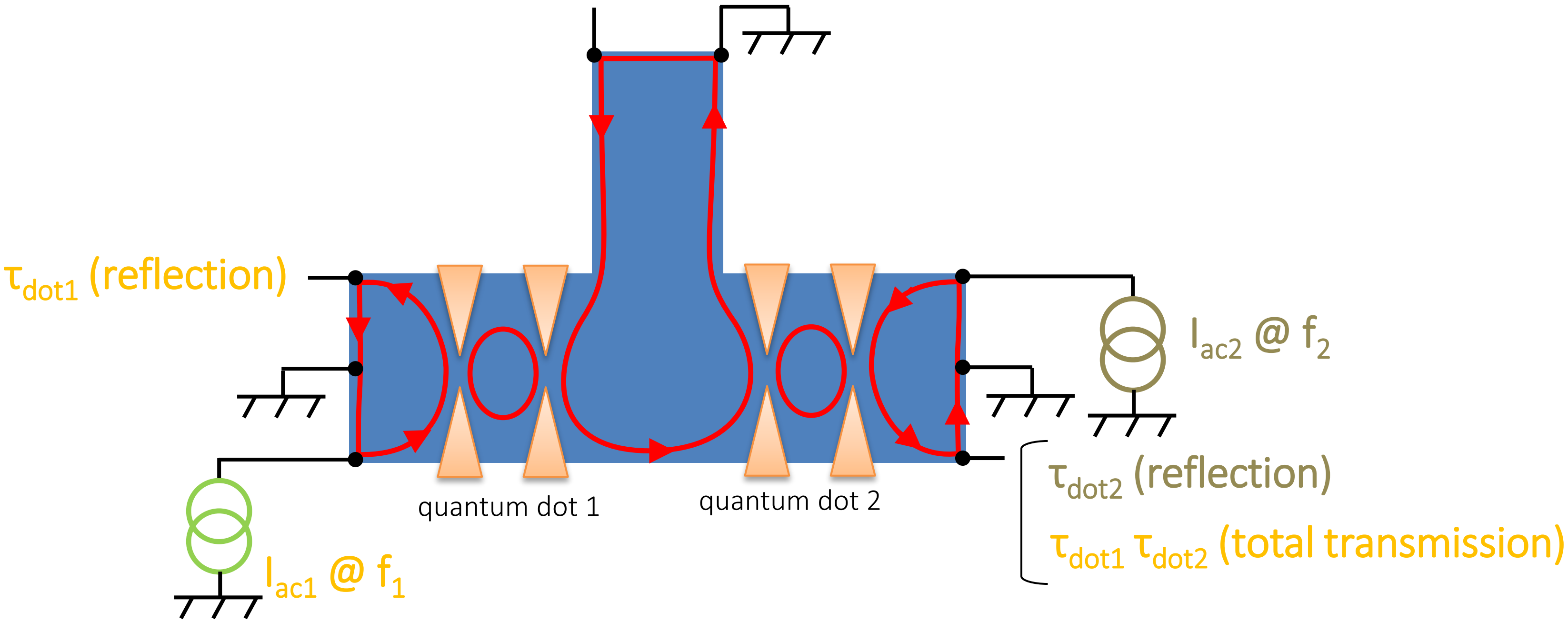
B) probes chirality: quantum Hall effect





Even more complex (chiral) geometries

complex quantum circuits:
→ several simultaneous lock-in measurements at various frequencies!

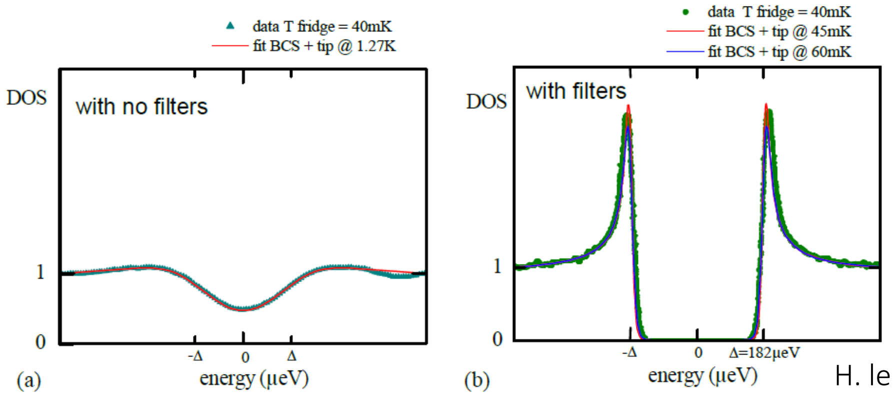


R. Rodriguez' PhD experiment, SPEC

use chirality and lock-in measurement to independently extract all parameters

Conclusions of parts 1 & 2

1) electron thermalization & filtering crucial for low T quantum transport experiments



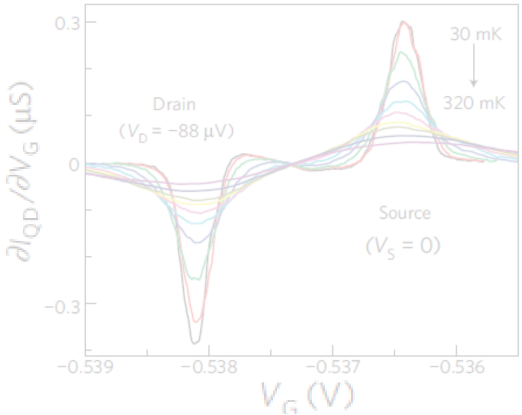
H. le Sueur, PhD thesis (2008)

2) small signals in cryo env^t:

- beware of noise sources & signal/GND loops
    - lock-in meas^t: how large is V_{ac}?

Outline

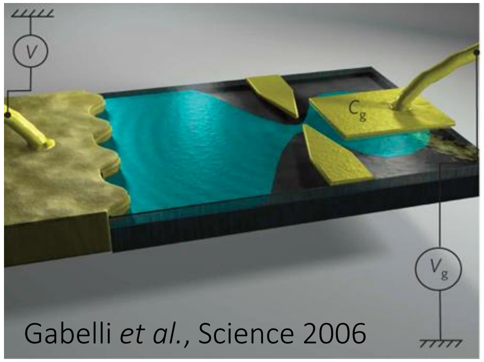
1. low temperature experiments
    - cryogenic systems
    - lattice vs electron temperature
    - filtering



2. low noise cryoelectronics
    - signal vs noise
    - DC & AC meas^t, lock-in
    - measurement configurations

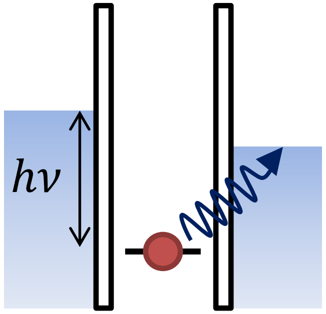
3. beyond dc conductance
    - microwave measurements
    - noise

Microvave signals in meso. physics

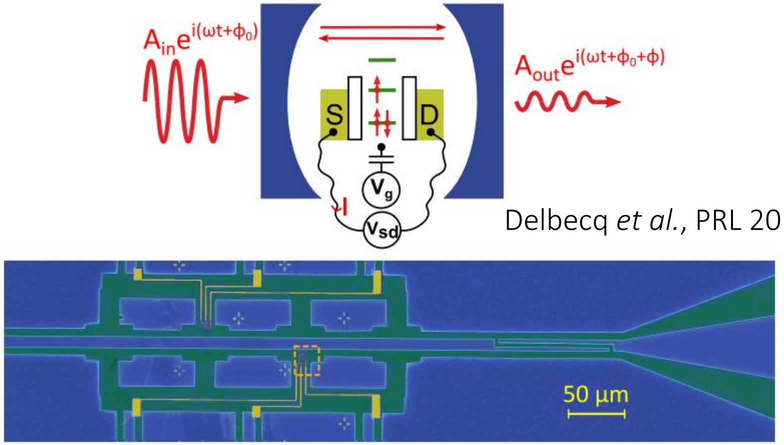


Gabelli *et al.*, Science 2006

time dependent transport
& quantum capacitances

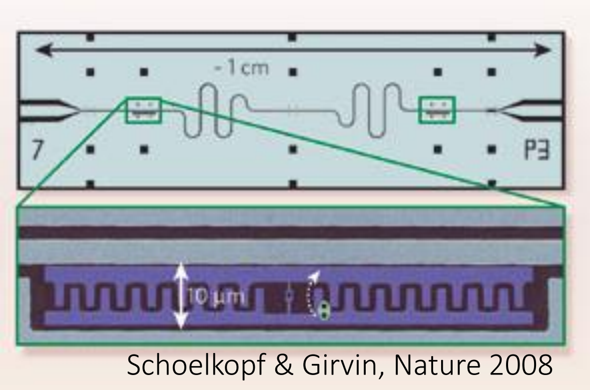


photon-assisted
tunneling



Delbecq *et al.*, PRL 2011

open quantum system dispersive readout



Schoelkopf & Girvin, Nature 2008

control & spectroscopy:
circuit quantum electrodynamics

Microvave signals in meso. physics

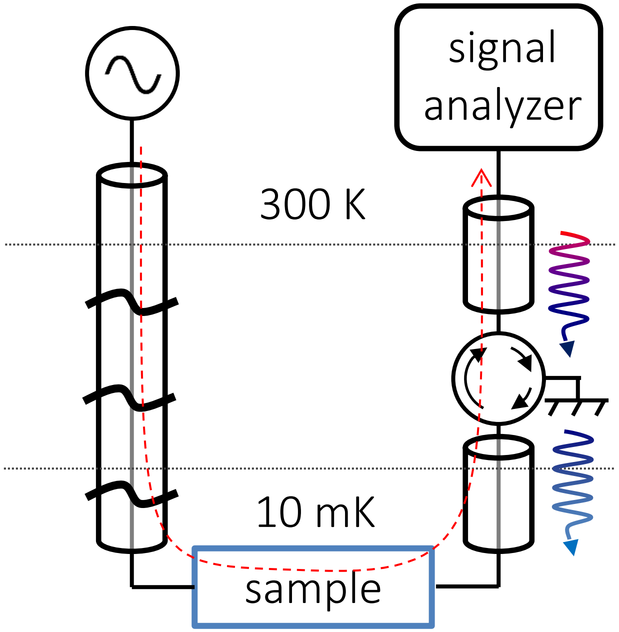
typical frequencies $hf > k_B T_{\mathrm{el}}$ → $f > 1$ GHz

→ can’t use usual filtered dc lines !

solution: differentiate input and output lines

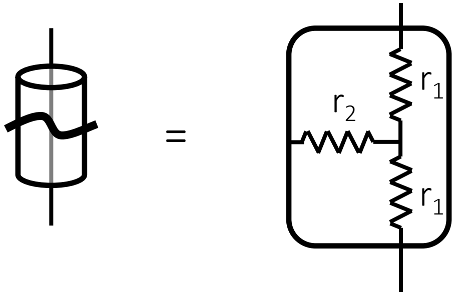
Microvave signals in meso. physics

solution: differentiate input and output lines



input:

use attenuators



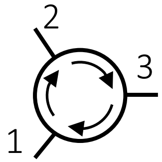
- attenuates (yes!) µwave power
- res. bridge:

electron thermalization + decoupling

- reduces multiple reflections

output:

use circulators

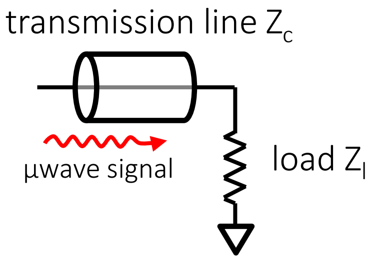


$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

induces non-reciprocity
in µwaves propagation
(*e.g.* using ferrites)

BUT: bulky + limited bandwidth

micro**waves**: impedance matching



wave propagation:

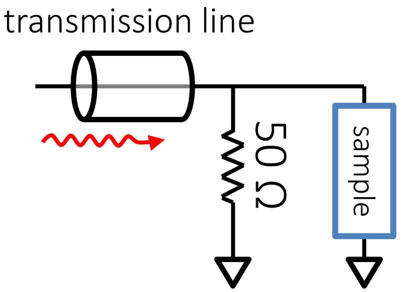
reflection coefficient on the load $r = \frac{Z_l - Z_c}{Z_l + Z_c}$

impedance matching: $r = 0$ for $Z_l = Z_c$

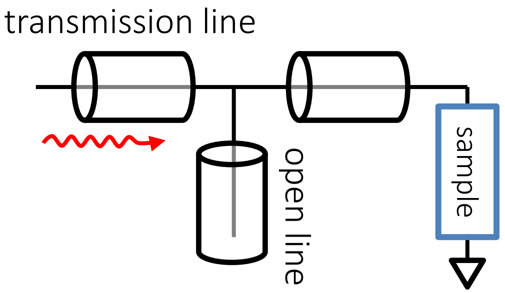
problem:

- Z=50 Ω for most commercial µ-wave electronics
- but $Z \approx R_K = 25813\ \Omega$ for typical meso. circuits…

hard to have good coupling & large bandwidth!



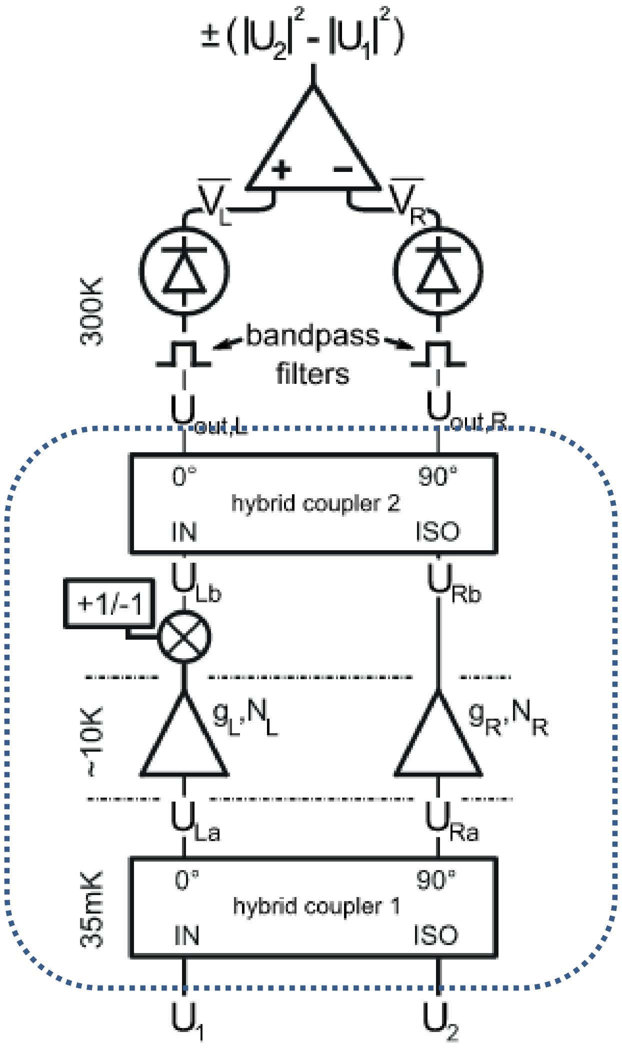
shunt sample with 50 Ω load:
poor coupling, good bandwidth



'stub tuner': good coupling, small bandwidth

T. Hasler, et al., Phys. Rev. Applied 4, 054002 (2015)

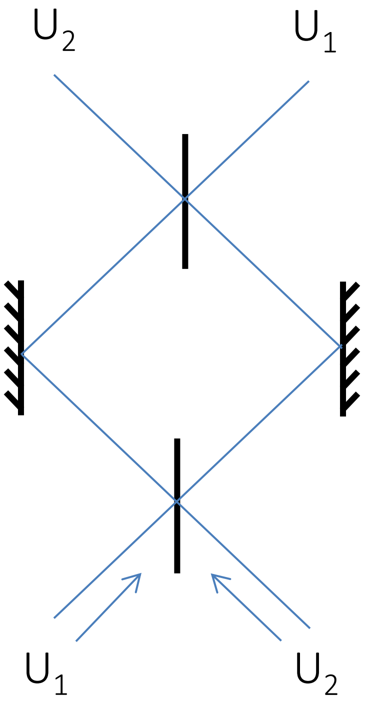
(complicated) example: double balanced amplifier



F.D. Parmentier et al., Rev. Sci. Inst. **82,** 013904 (2011)

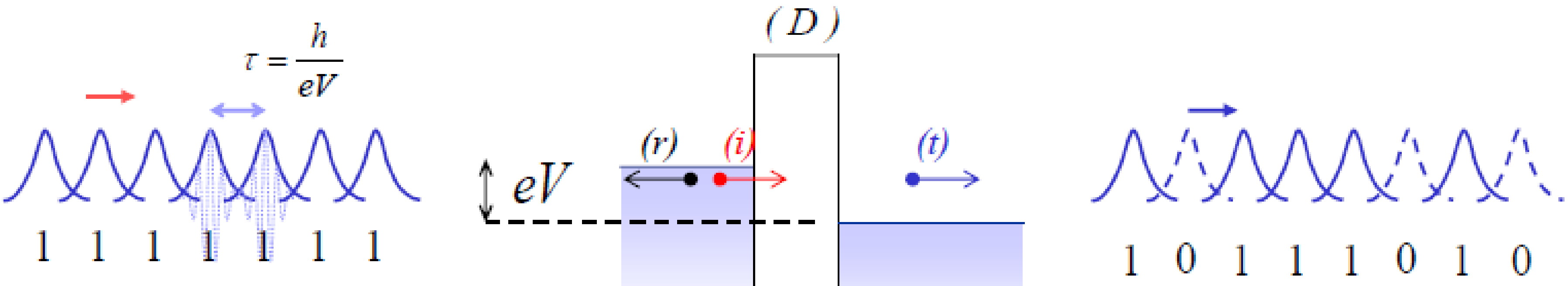
µwave Mach Zehnder interferometer

hybrid coupler
= beam splitter



(quantum) noise measurements

shot noise



$$I_{incident} = e.\frac{eV}{h}$$

$$I = D\,\frac{e^2V}{h}$$

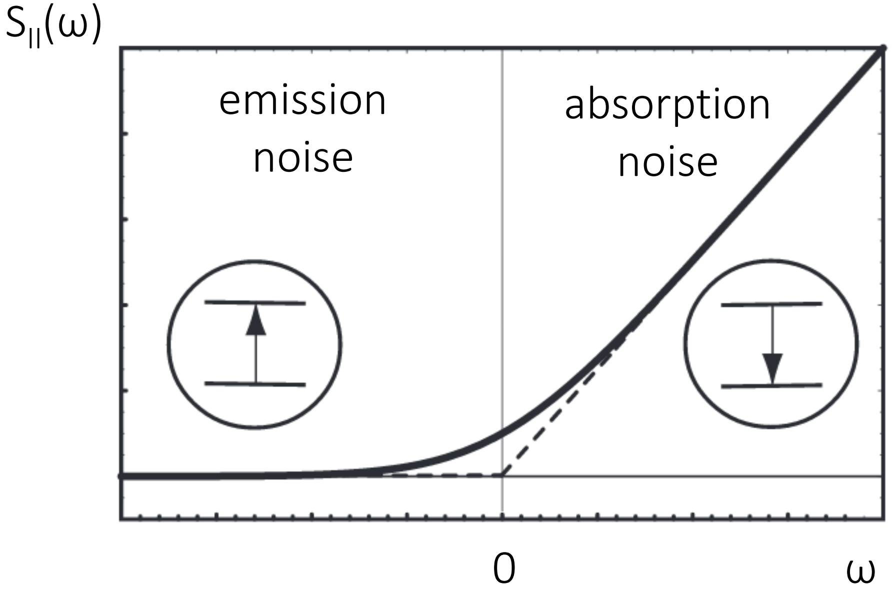
$$S_{II} = 2eI\underbrace{(1-D)}_{\text{Fano factor}}$$

G. Lesovik 89, M. Büttiker 91
Th. Martin, R. Landauer 92

shot noise: granularity of charge transfers + partitioning

zero & finite frequency quantum noise

current correlator: $S_{II}[\omega] = \int dt \langle \hat{I}(t)\hat{I}(0)\rangle e^{i\omega t}$



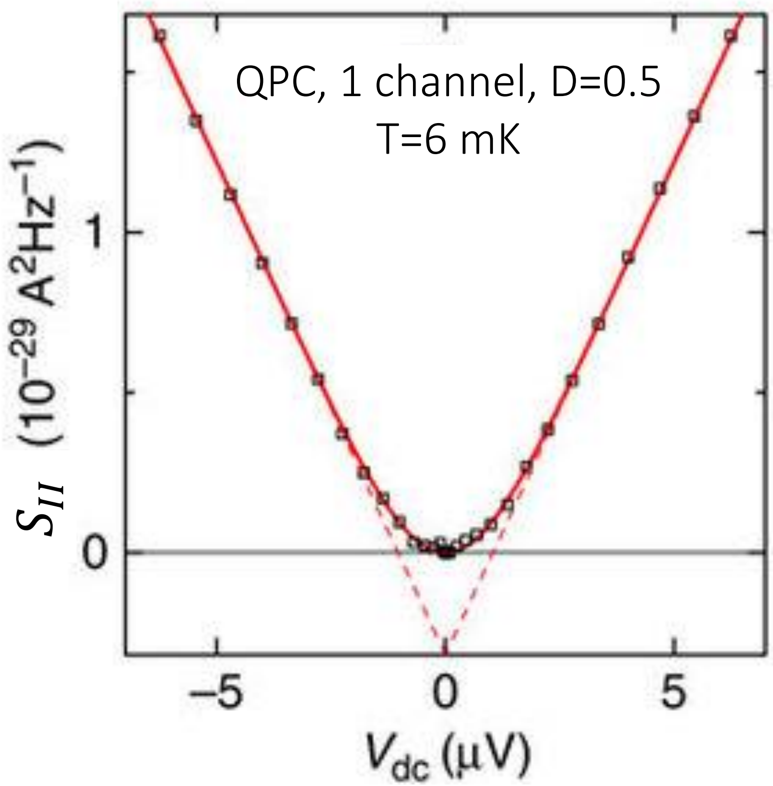
A.A. Clerk et al., Rev. Mod. Phys. **82,** 1155 (2010).

noise meast = measuring a power over some freq. bandwidth

very (very very) small signals

$$S_{II} \approx 2eI(1 - \mathrm{D})$$

$$\Rightarrow S_{II} \lesssim 2e \times 1\ \mathrm{nA} \sim 10^{-30} - 10^{-28}\ \mathrm{A^2/Hz}$$


QPC, 1 channel, D=0.5
T=6 mK

Z. Iftikhar *et al.,* Nature Communications **7,** 12908 (2016)

very (very very) small signals

$$S_{II} \approx 2eI(1-\mathrm{D})$$

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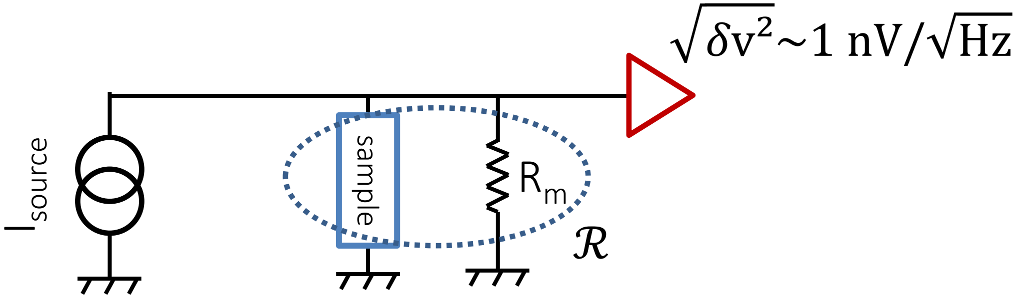

signal to noise ratio:

$$\mathrm{SNR} = \frac{\mathcal{R}^2 S_{II}}{\delta \mathrm{v}^2} \frac{1}{\sqrt{N_{\mathrm{mes}}}} \approx 10^{-10} \frac{\mathcal{R}^2}{\sqrt{N_{\mathrm{mes}}}}$$

$$N_{\mathrm{mes}} = \Delta f \times t_{\mathrm{mes}}$$

needs a long averaging time!

noise meas^t with tank circuits