Non-linearities in quantum circuits

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"A gentle journey into not-so-gentle quantum many-body problems"

Organization of the lectures

Superconducting nanocircuits

- Lecture 1 : i) General circuitry ; ii) From Josephson junctions chains to spin-boson model
- Lecture 2 : i) Scattering theory with dynamical emitter ; ii) Frequency conversion beyond RWA ; iii) Renormalization from many-body entanglement ; iv) Many-body inelastic scattering theory

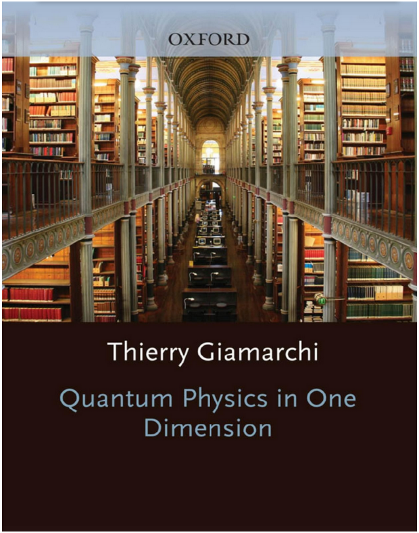
Metallic nanocircuits

- Lecture 3 : i) Anderson and Kondo models ; ii) DC Transport ; iii) Non Fermi liquid extensions
- Lecture 4 (closing the loop) : i) Boson-fermion connection in first quantization (using quantum optics concepts) ; ii) Insights into the Kondo wave-function ; iii) Emergent Majorana fermions

Today's menu

- Anderson orthogonality catastrophe
- Derivation of bosonization identities from first quantized electronic wavefunctions
- Equivalence of spin-boson and Kondo models
- Another look at the Kondo resonance (Toulouse solution)
- Emergent Majoranas for the 2-channel Kondo model (Emery-Kivelson solution)
- Some experimental signatures of 2-channel Kondo

Almost all you want to know about bosonization



Almost all you want to know about bosonization

APPENDIX D

BOSONIZATION DICTIONARY

This is a summary of most of the useful abelian bosonization formulas.

D.1 Spinless fermions

The single fermion operator and the density for a model with strictly linear relation are given by

$$\begin{aligned} \psi(x) &= \psi_R(x) + \psi_L(x) \\ \rho(x) &= \rho_R(x) + \rho_L(x) + (\psi_R^\dagger(x)\psi_L(x) + \text{h.c.}) \end{aligned} \tag{D.1}$$

where $\rho_r = \psi_r^\dagger(x)\psi_r(x)$ and where the right and left going fermions are

$$\psi_r(x) = U_r \lim_{\alpha\to 0} \frac{1}{\sqrt{2\pi\alpha}} e^{i(rk_F - \frac{\pi}{L})x} e^{-i(r\phi(x) - \theta(x))} \tag{D.2}$$

The operators U_r are operators that commute with the bosons. U_r of different species anticommute and U_r of the same species commute. To determine the *sign* of correlations containing the ψ operators the U_r can be replaced by Majorana (real) fermion operators. The fields ϕ and θ obey

$$[\phi(x), \frac{1}{\pi}\nabla\theta(x')] = i\delta(x - x') \tag{D.3}$$

thus the canonically conjugate momentum $\Pi(x)$ is

$$\Pi(x) = \frac{1}{\pi}\nabla\theta(x) \tag{D.4}$$

The fields are related to the densities by

$$\begin{aligned} \nabla\phi(x) &= -\pi[\rho_R(x) + \rho_L(x)] \\ \nabla\theta(x) &= \pi[\rho_R(x) - \rho_L(x)] \end{aligned} \tag{D.5}$$

The fields obey the Hamiltonian

$$H = \frac{1}{2\pi}\int dx\; uK(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2 \tag{D.6}$$

where u is a velocity and K a dimensionless parameter. For non-interacting fermions $u = v_F$ and $K = 1$. Quite generally $K < 1$ for repulsive interactions and

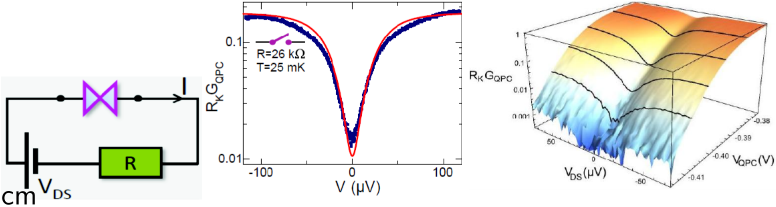
On the board : bosonization in first quantization

On the board : mapping between different models

Dynamical Coulomb blockade of tunneling

Tunneling in a large impedance circuit :

- Energetic cost of exciting electromagnetic modes
- Suppression of conductance : Zero Bias Anomaly
- The effect works if transmission $T < 1$

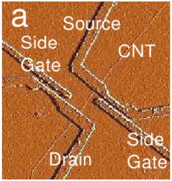


[Parmentier *et al.* Nat. Phys. 2011]

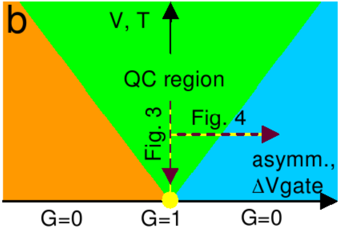
Dynamical Coulomb blockade with quantum dots

One step further : resonant tunneling with dissipation

- Spin polarized CNT quantum dot
- Strongly resistive contacts tuned to $Z = h/e^2$ [Mebrahtu *et al.* Nat. 2012 & Nat. Phys. 2013]

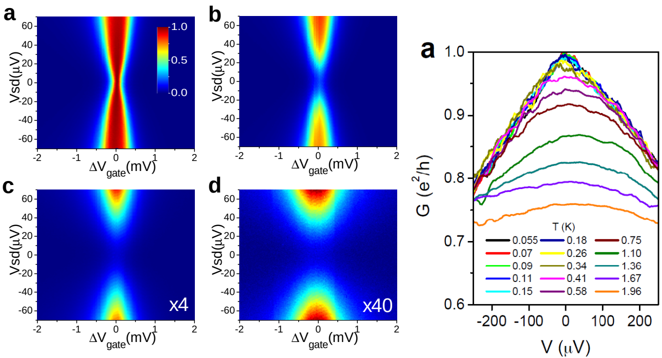


Expectation : tunneling survives only on-resonance and $t_L = t_R$



Zero temperature phase diagram

On resonance : emergent Majorana state



Non-Fermi liquid conductance : $G(V) = \frac{e^2}{h}\left(1 - V/V_0\right)$

On the board : scattering rate reveals partially hybridized Majorana fermions ! [Zheng *et al.* PRB 2014]

A word of conclusion

On the theory side :

- Many-body wavefunctions are a blossoming field
- Tough challenges for the quantum dynamics in 1D

Metallic quantum circuits :

- Some pretty funky quantum many-body states can be realized experimentally in quantum dots
- It was a long way (almost 30 years) between theoretical predictions and experimental realization

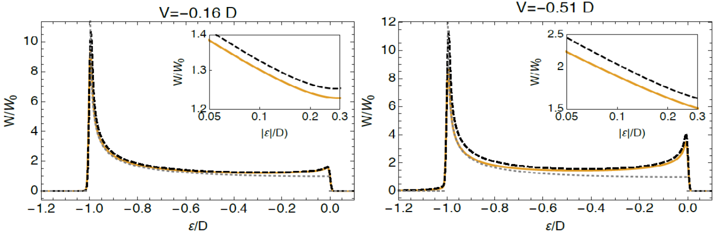
Superconducting quantum circuits :

- Will it hold its promises in terms of many-body physics ?


That's all Folks!
SUPERB wallpapers

Extra slides

X-ray edge problem : emission properties of a core electronic level that Coulomb interacts with a Fermi sea



Black dashed curve : brute force numerics (with cosine dispersion)

Continuous orange curve : variational coherent state