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FONDATION



Non-linearities in quantum circuits

Serge Florens [Néel Institute - CNRS/UGA Grenoble]

"A gentle journey into not-so-gentle quantum many-body problems"

Organization of the lectures

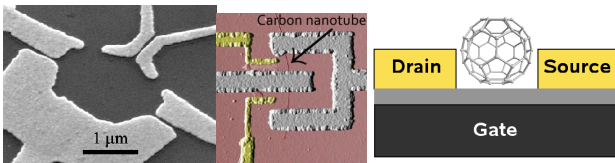
Superconducting nanocircuits

- ▶ Lecture 1 : i) General circuitry ; ii) From Josephson junctions chains to spin-boson model
- ▶ Lecture 2 : i) Scattering theory with dynamical emitter ; ii) Frequency conversion beyond RWA ; iii) Renormalization from many-body entanglement ; iv) Many-body inelastic scattering theory

Metallic nanocircuits

- ▶ **Lecture 3** : i) Anderson and Kondo models ; ii) DC Transport ; iii) Non Fermi liquid extensions
- ▶ Lecture 4 (closing the loop) : i) Boson-fermion connection in first quantization (using quantum optics concepts) ; ii) Insights into the Kondo wave-function ; iii) Emergent Majorana fermions

Various quantum dot systems



Rules of the game

What kind of experimental system does one need ?

- ▶ 1. Good tunability
- ▶ 2. Reasonable energy scales

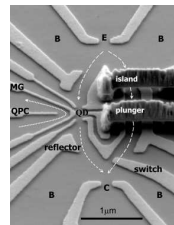
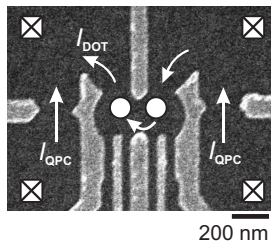
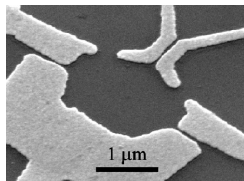


Reminder : Helium dilution fridges work for $T \gtrsim 20\text{mK}$
($1\text{K} \simeq 0.1 \text{ meV} \simeq 20 \text{ GHz}$)

Semiconducting quantum dots

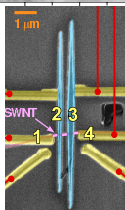
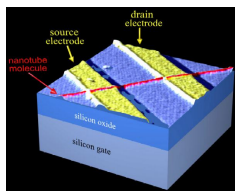
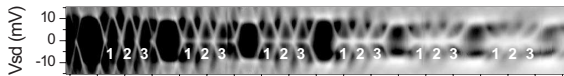
- ▶ System : 2D electron gas (GaAs, InAs, S, Ge...)
- ▶ ++ great tunability and scalability by electric gates
- ▶ -- small charging (Coulomb) energy :
 - $U = 40\text{K}$ for spin qbit experiments
 - $U < 10\text{K}$ for Kondo experiments

Ultra-small Kondo temperature $\implies T_K < 300\text{mK}$

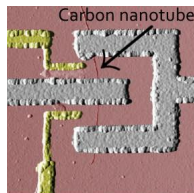


Carbon nanotube quantum dots

- ▶ System : 1D carbon molecule with gates on top
- ▶ ++ tunable & hybrid : metal, ferro or superconducting leads
- ▶ + intermediate charging energy $U = 10K - 100K$
- ▶ \pm orbital degeneracy : SU(4) vs SU(2) physics



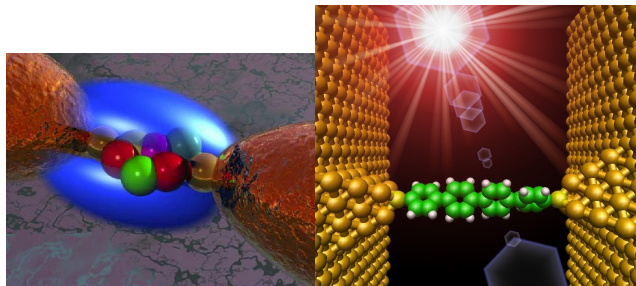
Ferro [ENS-Paris]



SQUID [Grenoble]

Molecular devices

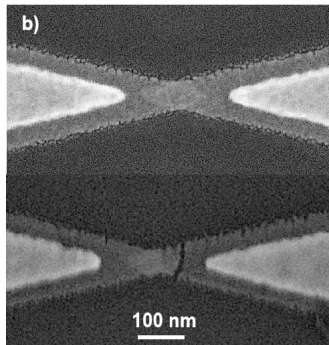
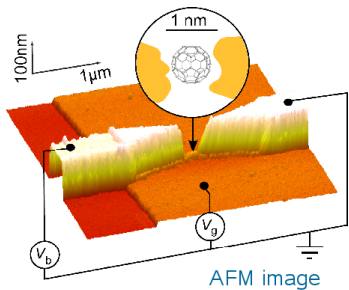
- ▶ System : nanometer size molecule inbetween metal contacts
- ▶ - limited tunability (no local gates, but some surprises...)
- ▶ ++ more tunability from chemistry
- ▶ -- lack of reproducibility
- ▶ ++ very large energy scales $U = 1000K - 10000K$
⇒ good playground for Kondo physics !



Molecular electronics HOWTO : electromigration

Basic idea :

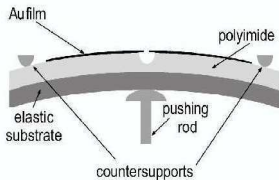
- ▶ E-beam lithography cannot reach single molecule size
- ▶ Solution : open a metal junction via electromigration
- ▶ Back gate : molecular transistor



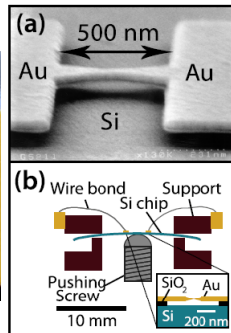
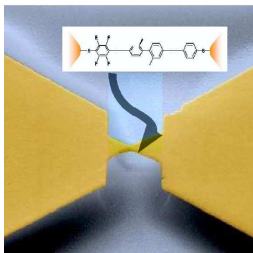
Molecular electronics HOWTO : break junctions

Basic idea :

- ▶ Solution : open mechanically a metal junction
- ▶ No back gate, but strain changes tunnel couplings



J. v. Ruitenbeek, Leiden '95



[D. Ralph (Cornell)]

Overview of regular (screened) Kondo effect

Kondo chronology

Textbook stuff in solid state physics :

- ▶ 1934 : observation of resistance anomalies in metals
- ▶ 1964 : Kondo proposes his model
- ▶ 1975 : Wilson solves the “Kondo problem”

The Kondo revival :

- ▶ 1988 : Glazman-Raikh & Ng-Lee predict Kondo anomalies in quantum dots
- ▶ 1998 : First observation by Goldhaber-Gordon & Kouwenhoven
- ▶ Now : it is routinely observed in all kinds of nanostructures !

Some good references to dig deeper :

- ▶ Coleman : “Heavy electron systems” cond-mat/0206003
- ▶ Glazman&Pustilnik : “Les Houches 2004” cond-mat/0501007

The essence of the Kondo problem

Question : what becomes a magnetic atom in a metal ?

Experimental fact : magnetic impurities **not always** show a moment

Simplified atom : $H_U = \epsilon_d(n_{d\uparrow} + n_{d\downarrow}) + Un_{d\uparrow}n_{d\downarrow}$

⇒ Moment stabilized by U , **high energy** excitations

Resonant level : $H_t = \sum_{k\sigma} [\epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + td_\sigma^\dagger c_{k\sigma} + h.c.]$

⇒ No moment, resonant level at **low energy**

Anderson model : $H = H_U + H_t$

⇒ Competition localized/delocalized

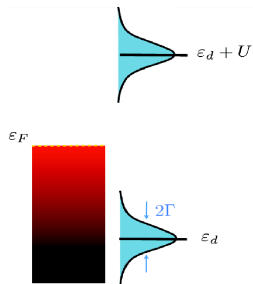
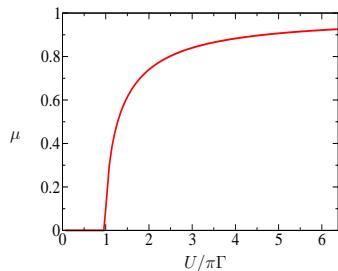
Captures the physics between the two above limits

On the board : Atomic limit $t = 0$

Local moment formation : $t \neq 0$ ($\epsilon_d = -U/2$ here)

Mean field theory [Anderson 1950] :

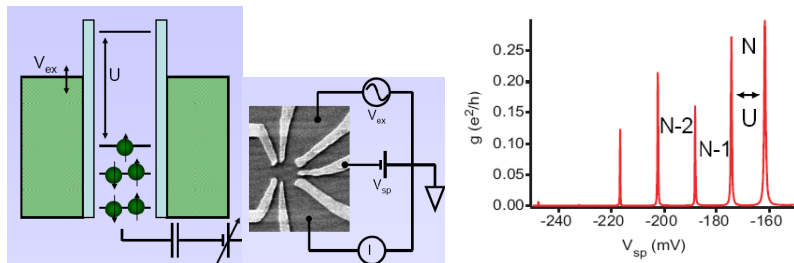
Assumes a frozen moment $\mu = \langle n_{d\uparrow} - n_{d\downarrow} \rangle$



Results :

- ▶ Local moment $\mu \neq 0$ for $U > \pi\Gamma = \pi t^2 \rho_0$
 \Rightarrow need relatively closed quantum dot
- ▶ Broadened **high energy** features
 (Hubbard bands \Leftrightarrow Coulomb blockade edges)

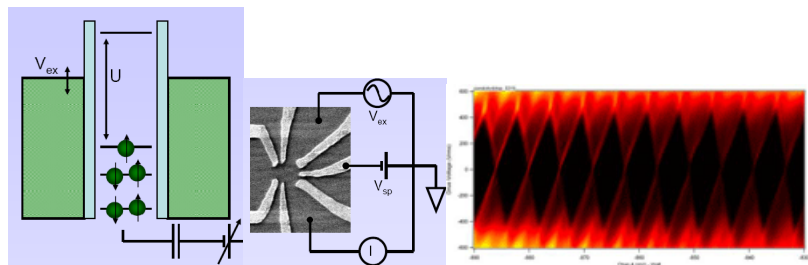
Coulomb blockade : weakly open quantum dot

Conductance vs. gate :

- ▶ Charging energy dominates : $H = \frac{U}{2}(Q_{\text{dot}} - \alpha eV_g/U)^2$
- ▶ Frozen charge fluctuations : transport suppressed
- ▶ Conductance has peaks at charge degeneracy

Coulomb blockade : weakly open quantum dot

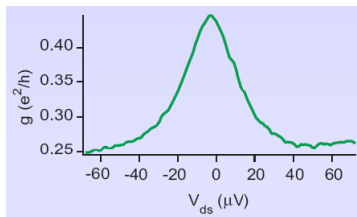
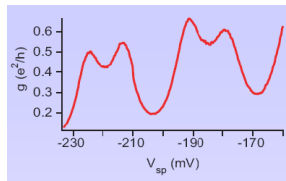
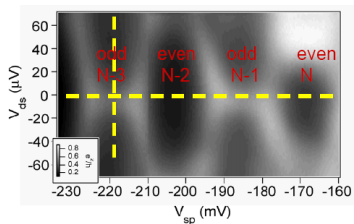
Conductance vs. gate and bias :



- ▶ Coulomb diamonds with low conductance
- ▶ Totally generic observation !
- ▶ Single charge transistor

Coulomb blockade suppression : Kondo anomaly

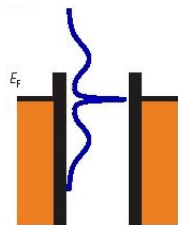
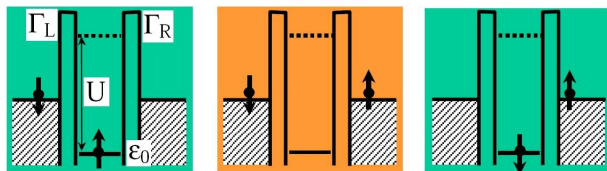
Moderately open quantum dot : [data from Goldhaber-Gordon group]



- Zero-bias transport is restored for odd charge

Physical origin of Kondo effect

- ▶ General idea : lifting of a degeneracy by a Fermi sea through exchange interaction
- ▶ Odd charge quantum dots : resonant spin flip processes



- ▶ Resonant binding of electrons near Fermi level
- ▶ Open channel for transport

Theory : Glazman&Raikh, Ng&Lee (1988)

Experiment : Goldhaber-Gordon, Kouwenhoven (1998)

On the board : perturbation at order U^2

Fermi liquid description [Langreth, Nozières, Yosida,...]

Small U : perturbation theory in U well-behaved

\Rightarrow Self-energy : $\Sigma_d(\omega) = (1 - Z^{-1})\omega + iA\omega^2$ for $\omega \rightarrow 0$

\Rightarrow Effective width : $T_K = Z\Gamma$ **Renormalized quasiparticles**

Conductance from linear-response : Landauer-like

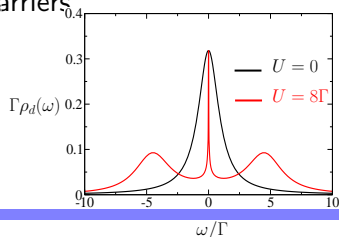
$$G(T) = \frac{2e^2}{h} \frac{4t_L^2 t_R^2}{(t_L^2 + t_R^2)^2} \int d\omega n'_F(\omega) \pi\Gamma \text{Im}[G(\omega)]$$

Friedel sum rule : $\text{Im}[G(0)] = \frac{1}{\Gamma}$ at $T = 0$ and $\epsilon_d = -U/2$

$\Rightarrow G(T = 0) = \frac{2e^2}{h}$ for symmetric barriers

Key question :

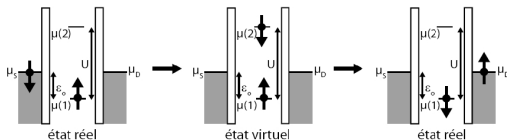
does that remain true for $U \gg \Gamma$?



On the board : Schrieffer-Wolff transformation

On the board : Schrieffer-Wolff transformation

Schrieffer-Wolff transformation : keep only spin states at large U
 \Rightarrow Antiferromagnetic coupling $J_K = 8 \frac{t^2}{U}$ to the Fermi sea



Kondo model :

$$H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \vec{S} \cdot \sum_{kk'\sigma\sigma'} c_{k\sigma}^\dagger \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{k'\sigma'}$$

$$H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J \vec{S} \cdot \sum_{\sigma\sigma'} c_\sigma^\dagger(x=0) \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{\sigma'}(x=0)$$

Kondo logarithms [Abrikosov, Suhl, Nagaoka, Kondo, Anderson...]

Perturbation in $j = J/D$: logarithmic divergent at low T

$$\chi_{spin}(T) = \frac{3}{4T} \left[1 - j - j^2 \log \frac{D}{T} + \dots \right] \simeq \frac{3}{4T} \left[1 - j_R(T) + \dots \right]$$

$$G(T) = \frac{2e^2}{h} \frac{3\pi^2}{16} \left[j^2 + j^3 \log \frac{D}{T} + \dots \right] \simeq \frac{2e^2}{h} \frac{3\pi^2}{16} \left[j_R^2(T) + \dots \right]$$

- ▶ Kondo temperature :

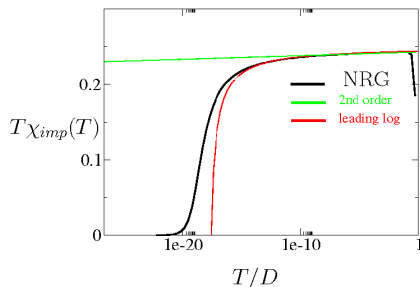
$$T_K = D e^{-D/J} = D e^{-\pi U/8\Gamma}$$

- ▶ Renormalized coupling :

$$j_R(T) = \frac{1}{\frac{D}{j} + \log(T/D)}$$

$$= \frac{1}{\log(T/T_K)} \text{ grows}$$

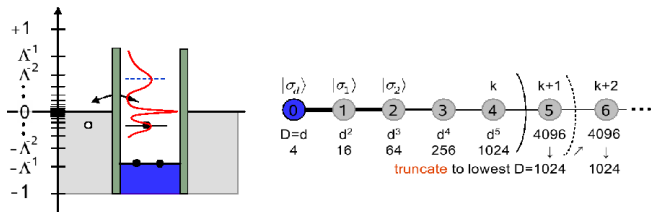
- ▶ Hints that local moment does not survive at $T \ll T_K$



Solution of the Kondo problem : an NRG primer

Algorithm : [Wilson (1975)]

- ▶ Logarithmic discretization of the Fermi sea : $[\Lambda^{-k-1}, \Lambda^k]$
- ▶ Mapping of a chain with exponential vanishing couplings
- ▶ Iterative solution : diagonalize + truncate Hilbert space



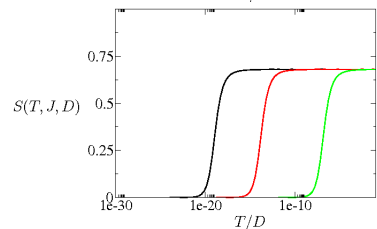
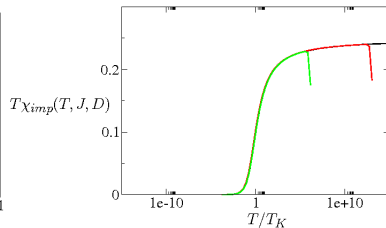
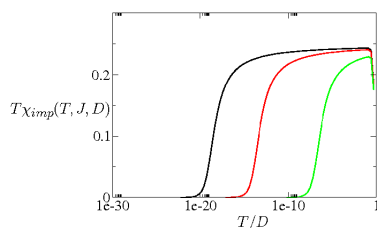
Remarks :

- ▶ Resolves low energy features (Kondo peak)
- ▶ Stability requires scale separation : $\Lambda > 2$
- ▶ Discretization errors $\simeq e^{-\pi^2/\log(\Lambda)} \rightarrow$ small !

Illustration of the universal crossover

Curie constant and entropy from NRG calculations :

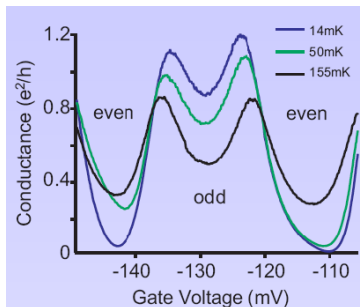
- ▶ Three large values of U/Γ



Perfect data collapse :
 all observables are functions
 of T/T_K only
 ⇒ **universality !**

Back to the experiments

- ▶ Conductance is large at $T \lesssim T_K$: Kondo prevails
- ▶ Coulomb blockade is recovered at $T \gg T_K$

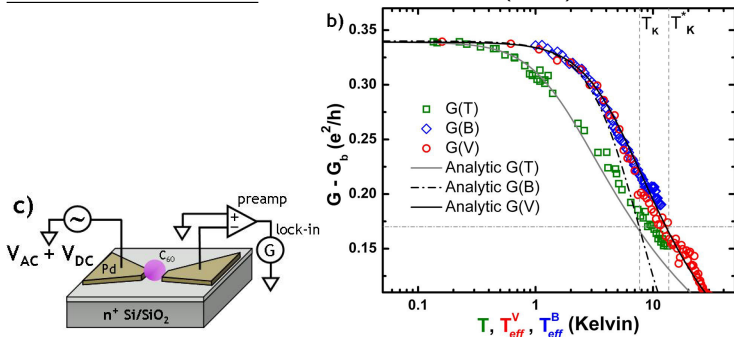


General Friedel sum rule at $T=0$:

$$G = \frac{2e^2}{h} \frac{4t_L^2 t_R^2}{(t_L^2 + t_R^2)^2} \sin^2 \left(\frac{\pi}{2} Q_{\text{dot}} \right)$$

Testing the theory with experiments : $G(B)$ and $G(T)$

C_{60} molecular device [Scott et al. PRB 2013] (Rice)

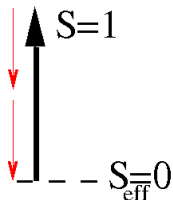


Successful fit of $G(T)$ and $G(B)$ with consistent $T_K \simeq 10K$

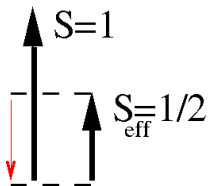
Non-Fermi liquid extensions

Spin $S = 1$: screening versus underscreening

Two screening channels : full quenching \implies regular Kondo effect

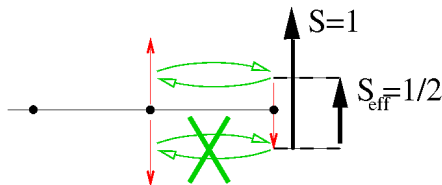


One screening channel : residual spin \implies anomalous Kondo effect



Under screening : Nozières-Blandin argument

Spin $S = 1$ and **single** screening channel :



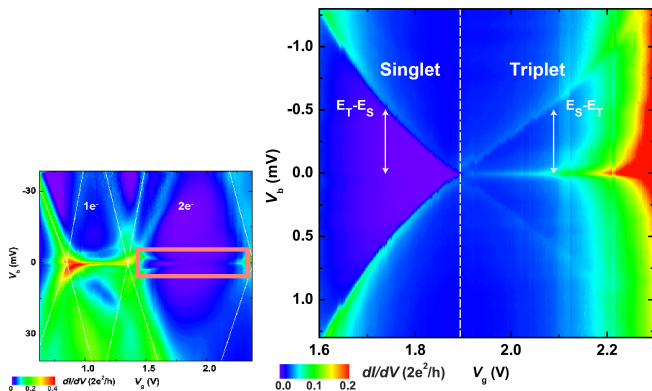
- ▶ Effective spin $S_{eff} = 1/2$ (residual entropy)
- ▶ Effective Kondo coupling $J_{eff} \propto -t^2/J$: ferromagnetic !
 \implies temperature dependence : $J_{eff}(T) \propto \frac{1}{-\frac{D}{J_{eff}} + \log(T/T_0)}$
 $\implies J_{eff}(T)$ vanishes at low T

Transport : logarithmic approach to unitarity

$$G(T) = \frac{2e^2}{h} \left[1 - \frac{c}{\log^2(T/T_K)} \right]$$

First experimental evidence of underscreening

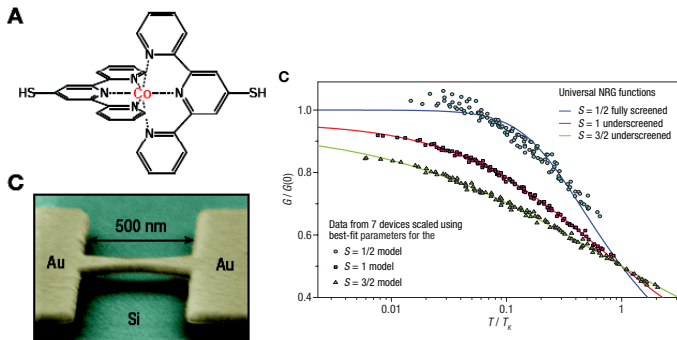
C₆₀ device : [Roch *et al.*, Nature (2008)] (Grenoble)



- ▶ Surprising gate effect on the singlet-triplet gap
- ▶ All transport signatures now well-understood

A second experiment : break junction device

Cobalt in a “cage” : [Parks *et al.* cond-mat (2010)] (Cornell)



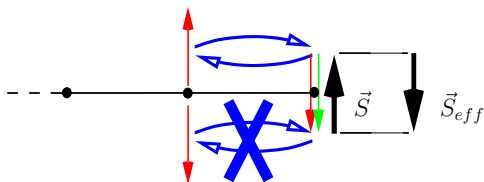
Underscreened Kondo anomaly : clearly logarithmic below T_K

Two-channel Kondo effect

Spin $S = 1/2$ but two independent Fermi seas ($m=1,2$) :

$$H = H_1 + H_2 + \sum_{kk'} \sum_{\sigma\sigma'} \sum_m J_m c_{k\sigma m}^\dagger \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{k'\sigma' m} \cdot \vec{S}$$

Strong coupling argument : at $J_1 = J_2$ large



- ▶ Effective spin $S_{eff} = 1/2$ (residual entropy)
- ▶ Effective Kondo coupling $J_{eff} \propto +t^2/J$: antiferromagnetic !
 \implies strong coupling fixed point is unstable !

Quick status of 2CK : theory and experiment

Some theoretical facts : (NRG, CFT, Bosonization...)

- ▶ $S(T=0) = \log(\sqrt{2}) = \log(2)/2$: emergent Majorana !
- ▶ $\chi(T) \propto \log(T_K/T)$
- ▶ $G(T)/G_0 \simeq 1 - a\sqrt{T/T_K}$: non Fermi liquid

Several experimental studies : (discussion beyond this course)

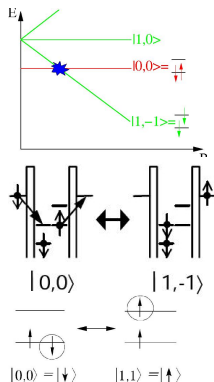
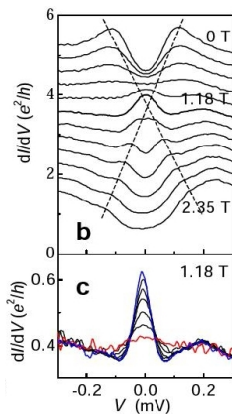
[Potok *et al.* Nature 2007 ; Keller *et al.* Nature 2015] (Stanford)
 [Mebrahtu *et al.* Nature 2012 & Nature Physics 2013] (Duke)
 [Ifthikar *et al.* Nature 2015] (LPN)

Extra slides

Nice illustration with “real” and “fake” spin 1/2

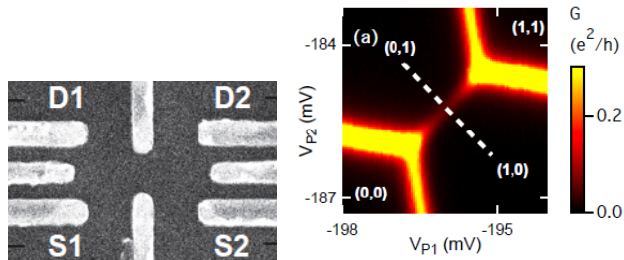
Even charge CNT quantum dot : [Nygard *et al.*, Nature (2000)]

- ▶ Magnetic-field induced degeneracy by crossing of singlet and lowest triplet



Another illustration with “fake” spin

Charge states in double dot setup : [Goldhaber-Gordon group (2012)]

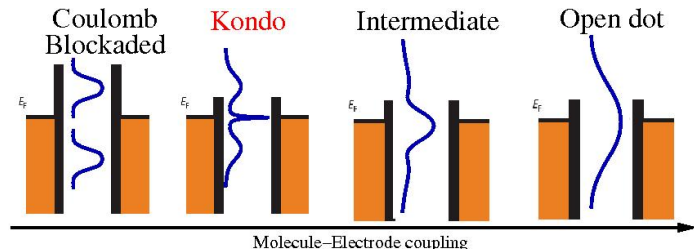
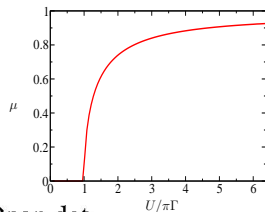


- ▶ Pseudospin : $(1,0) \Leftrightarrow \uparrow$ and $(0,1) \Leftrightarrow \downarrow$
- ▶ Kondo process takes place

How to reach the scaling limit ?

Universal (sharp) Kondo resonance requires :

- ▶ well-formed local moment i.e. large U/Γ



Consequence : **only** in the Kondo regime

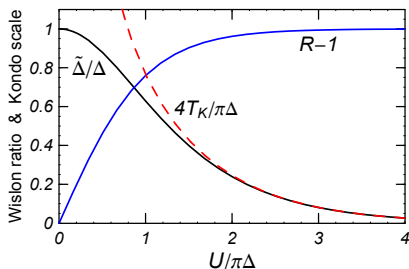
- ▶ single parameter scaling of physical quantities
Zero-bias Conductance : $G(T) = G_0 * f(T/T_K) + G_1$

Question for experiments : how to quantify the universal regime ?

Precise condition to be universal

Wilson ratio : $R = \frac{\chi}{\gamma} * \frac{\gamma}{\chi|_{U=0}}$

- ▶ Zero T spin susceptibility (screening) : $\chi(T) \propto \frac{1}{T_K}$
- ▶ Low T specific heat (Fermi liquid) : $C(T) = \gamma T \propto \frac{T}{T_K}$



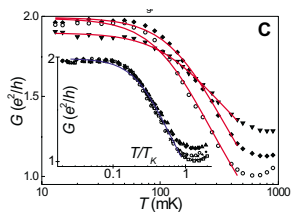
Effective width :
here $\tilde{\Delta} \equiv \tilde{\Gamma}$

Kondo scale :
 $T_K \propto e^{-\pi U/8\Gamma}$

- ▶ 3% deviation to universality : $U \gtrsim 2\pi\Gamma \implies T_K \lesssim U/40$
- ▶ Also constraints on level spacing

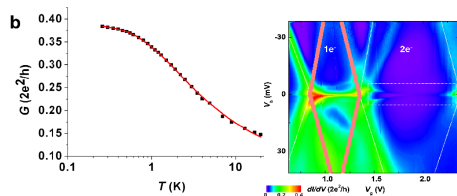
Testing the theory with experiments : $G(T)$

Semiconducting quantum dots : [van der Wiel *et al.* Nature (2000)]



Sizable deviations to scaling due to too small U/T_K

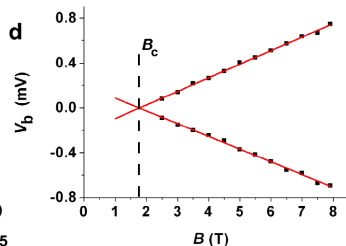
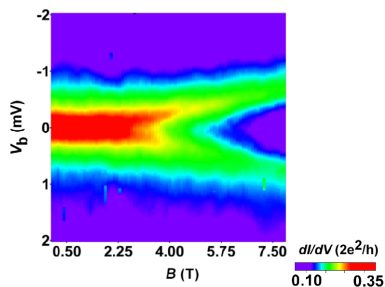
Molecular quantum dots : [Roch *et al.* PRL (2009)]



$T_K = 4K$, $U > 600K$
conditions are met !

Testing the theory with experiments : $G(B)$

Magnetotransport data in molecular quantum dot :

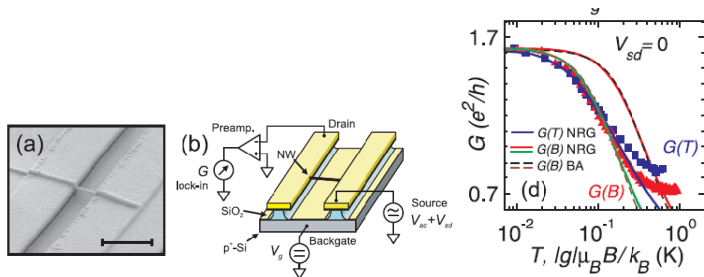


Fit of the Zeeman splitting :

- ▶ Kondo peak splits at $B_c = k_B T_K / (2g\mu_B) \Rightarrow T_K = 4.8K$ OK!

Testing the theory with experiments : $G(B)$

Attempt at scaling analysis : (InAs nanowire, Mahalu's group)



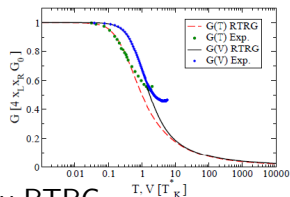
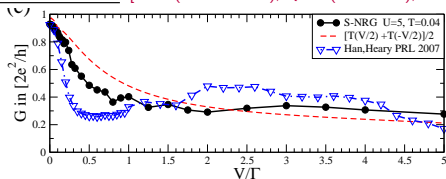
- ▶ $G(T)$ and $G(B)$ are different universal scaling functions
- ▶ The T_K and B_K scales are not consistent !
- ▶ The experiment is quantitatively not fully conclusive...

Out of equilibrium scaling

Finite-bias conductance $G(V)$:

- ▶ Main physical effect : enhanced scattering from large current density kills the Kondo resonance
- ▶ Tough problem for many-body theory
- ▶ Reliable experimental data are needed also

Methods : [NRG (Anders 2009), QMC (Han 2007), RTRG (Pletyukhov-Schoeller 2012)]



- ▶ T_K and V_K consistently predicted by RTRG
- ▶ Deviations to scaling (semiconducting dot)

Checking Fermi Liquid Theory

At low energy : $T \ll T_K$ and $eV/k_B \ll T_K$

$$G(T, V) = G_0 \left[1 - c_T \left(\frac{\pi T}{T_K} \right)^2 - c_V \left(\frac{eV}{k_B T_K} \right)^2 \right]$$

Out of equilibrium Fermi Liquid theory : [Oguri JPS (2004)]

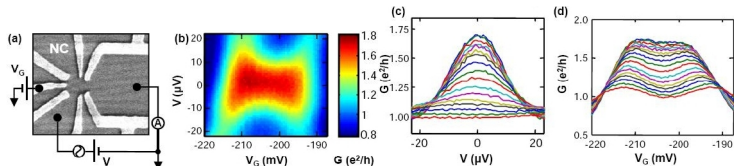
$$\begin{aligned} \alpha \equiv \frac{c_V}{\pi^2 c_T} &= \frac{3}{4\pi^2} \frac{1 + 5(R - 1)^2}{1 + 2(R - 1)^2} \\ &= \frac{3}{2\pi^2} \simeq 0.16 \text{ [Kondo regime } (R = 2)] \\ &= \frac{3}{4\pi^2} \simeq 0.08 \text{ [uncorrelated } (R = 1)] \end{aligned}$$

Note : α depends on Γ_L/Γ_R in general

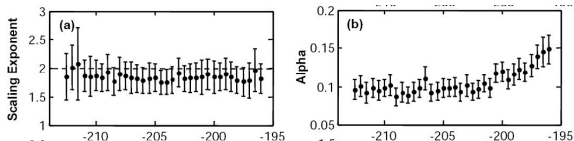
but not any more in the $R = 2$ limit (universality again !)

Hunt for α : 2DEG quantum dot data

Recent experiment : [Grobis *et al.* PRL (2008)]



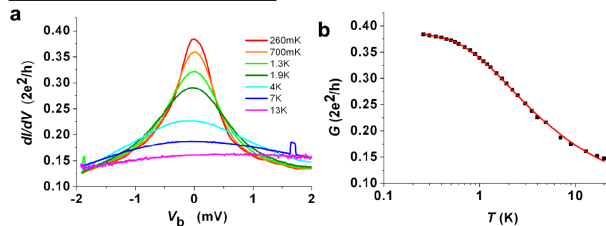
Fermi liquid coefficients :



- ▶ Nice systematics !
- ▶ $0.1 < \alpha < 0.15$: signature of intermediate correlations ? !

Hunt for α : molecular quantum dot data

Grenoble experiment : [Roch *et al.* PRL (2009)]



- ▶ Here $T_K = 4K \implies$ less noise!
- ▶ For this only sample : we extract $\alpha \simeq 0.15$, OK!
- ▶ See recent study by Mahalu's group

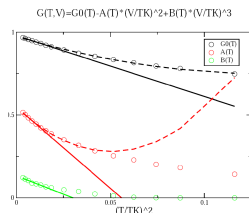
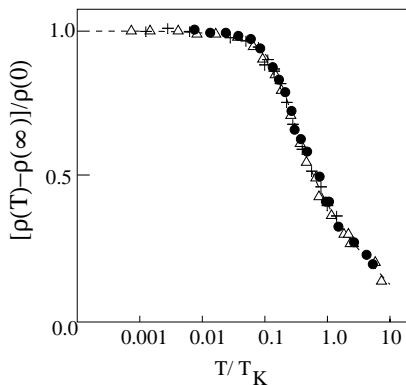


Illustration : whole crossover in diluted alloy

Resistivity of Kondo compound $\text{Mo}_x\text{Nb}_{1-x}$:

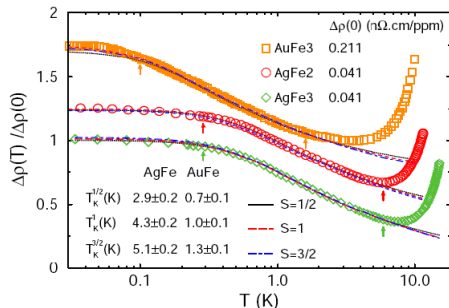


- Scaling on 4 orders of magnitude !!

Disgression : Kondo anomalies in $\text{Ag}_{1-x}\text{Fe}_x$ and $\text{Au}_{1-x}\text{Fe}_x$

Resistance of wires : [Costi *et al.* PRL 2009]

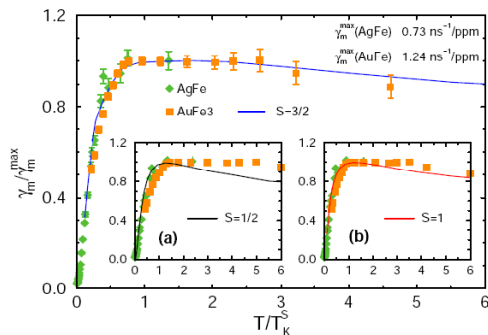
- ▶ LDA predicts spin $S = 3/2$ for Fe and 3 orbitals involved in screening process



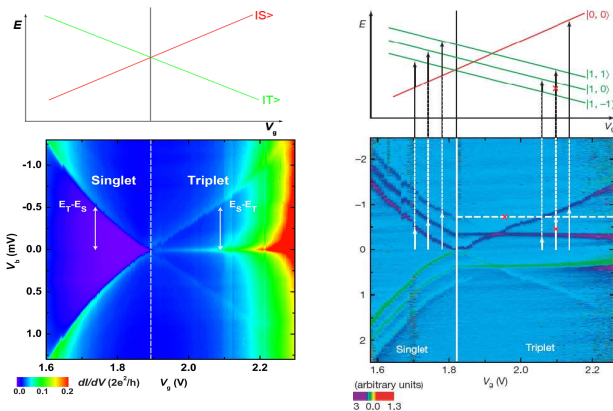
- ▶ Data compatible with full screening (no underscreening!)
- ▶ Theory cannot distinguish the spin value S

Disgression : Kondo anomalies in wires

Inelastic contribution to the resistivity :



- ▶ This allows to discriminate the spin value :
 $\implies S = 3/2$ and 3 orbital involved !
- ▶ This systems is thus not underscreened...

Identifying the spin states : gate voltage scan at $B = 3T$ 

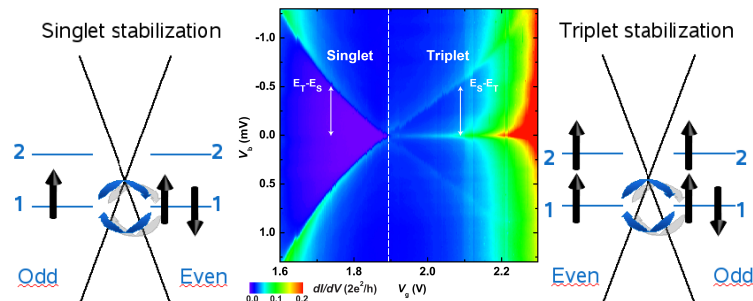
- ▶ Zeeman effect agrees with spin 0 or spin 1 ground state
- ▶ Gate-induced magnetic splitting (tunable Hund's rule!)

Origin for the gating effect : phenomenology

Role of the leads : energy gain by charge fluctuations

Hopping from level 1 or 2 $\implies \delta E_{1,2} = -\frac{t_{1,2}^2}{E_{add}}$

Hypothesis : hopping asymmetry $t_1 \gg t_2$ explains data

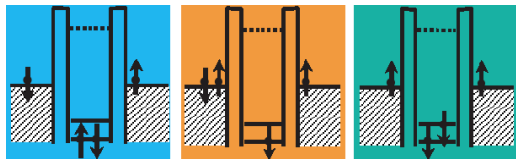
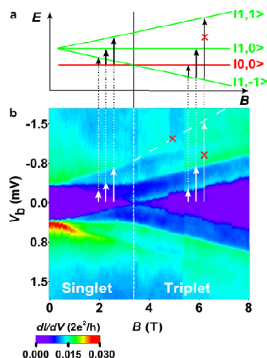


Crucial observation : **single** screening channel !!

Second evidence for single screening channel

Magnetic field effect :

- ▶ No Kondo anomaly at the singlet-triplet degeneracy point !



- ▶ Kondo coupling :

$$J_K \propto \frac{t_1 t_2}{E_c} \text{ small}$$

as $t_1 \gg t_2$

Modelization the gating effect

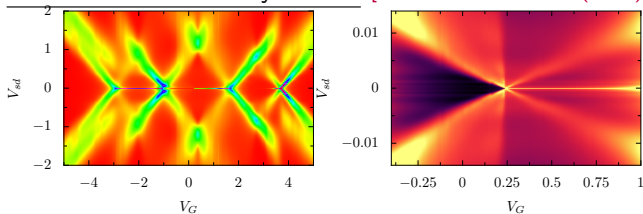
Generalized two-orbital Anderson model :

$$\begin{aligned}
 H = & \sum_{i\sigma} E_i n_{i\sigma} + \sum_i U_i n_{i\uparrow} n_{i\downarrow} + U_{12} n_1 n_2 - J_H \mathbf{s}_1 \mathbf{s}_2 \\
 & + \sum_{\nu k \sigma} \epsilon_{\nu k} c_{\nu k \sigma}^\dagger c_{\nu k \sigma} + \frac{1}{\sqrt{N_k}} \sum_{i \nu k \sigma} (t_{\nu i} c_{\nu k \sigma}^\dagger d_{i\sigma} + \text{h.c.})
 \end{aligned}$$

with $E_2 - E_1 \simeq J_H/2$ (small bare singlet-triplet gap)

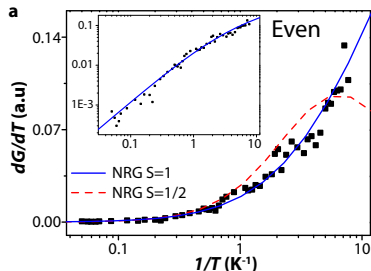
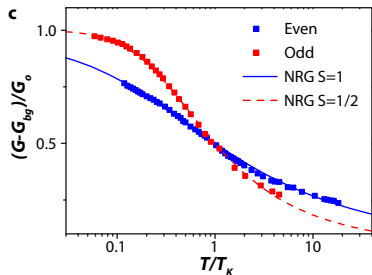
and $t_1 \simeq 2 * t_2$ (moderate hopping asymmetry)

Scenario confirmed by NRG : [Florens *et al.*, JPCM (2011)]



Testing the underscreened Kondo scenario

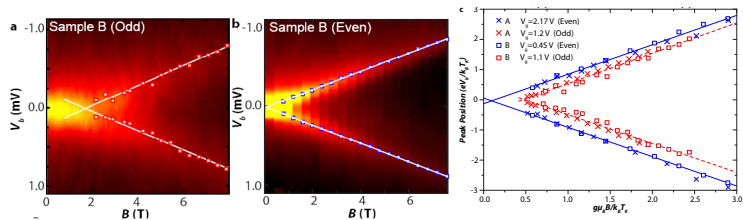
Analysis of the spin $S = 1$ Kondo anomaly : [Roch *et al.*, PRL (2009)]



- ▶ Agreement with $S = 1$ NRG clearly better
... but tough experiment !!
- ▶ $\frac{dG(T)}{dT}$ shows two logarithmic regimes

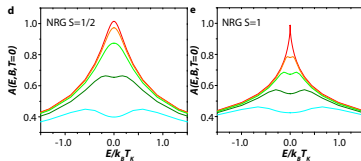
Magnetic field effect : smoking gun !

Comparing $S = 1/2$ and $S = 1$ Kondo anomalies :



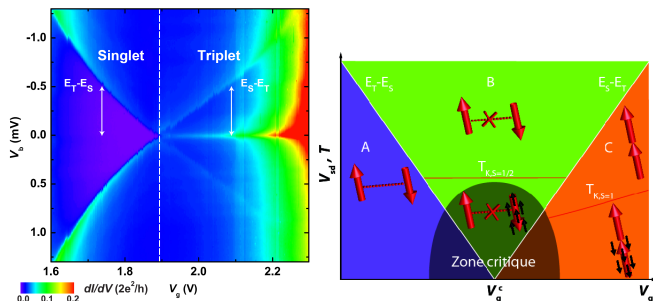
- Splitting occurs at lower magnetic fields for $S = 1$!

NRG calculations : $S = 1$ Kondo resonance very sensitive to field



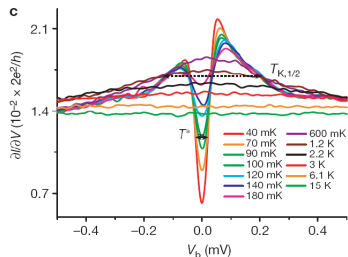
Analysis of the transition

- ▶ Far on the singlet side : spin gap
- ▶ Far on the triplet side : underscreened Kondo
- ▶ Nature of QCP :
one free spin $S = 1/2 \oplus$ one screened spin $S = 1/2$

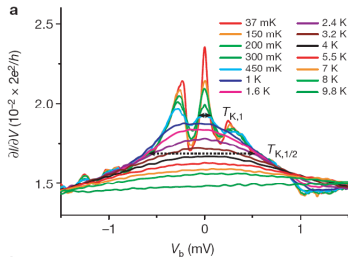


Around the quantum critical point

Singlet side :



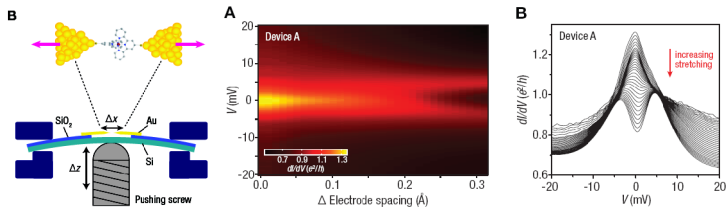
Triplet side :



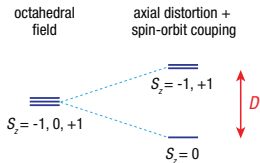
- ▶ For $T \gg |E_T - E_S|$ (on both sides) :
broad spin $S = 1/2$ Kondo anomaly
- ▶ Sharp antiresonance (Friedel!) on the singlet side at low T
 \implies second-stage of Kondo screening

Unexpected effect

Stretching the molecule : splits the Kondo resonance !



Interpretation : strain induced magnetic anisotropy ?

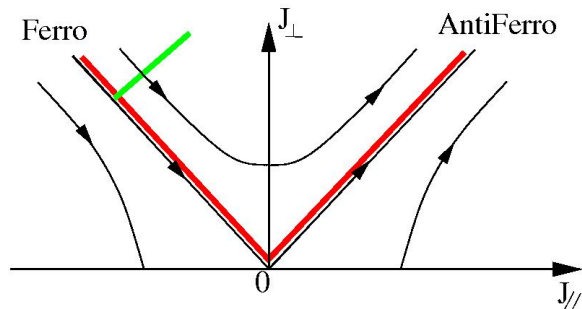


Putting the $|1, 0\rangle$ state down :
kills underscreened Kondo effect

Where are we in parameter space ?

Travelling along Anderson's flow diagram :

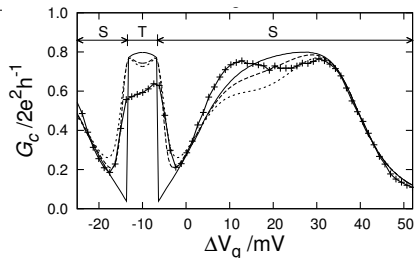
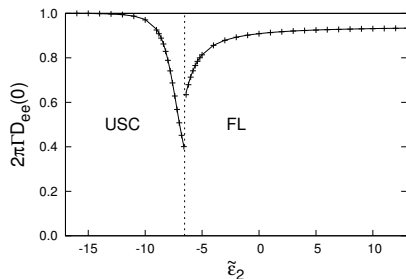
— Cornell
— Grenoble



A third experiment : semiconducting dots

Obvious way to tune QPT : just change the charge!

- ▶ $S = 1/2$: Fermi liquid, sum rule $\sin^2(\pi Q/2)$
- ▶ $S = 1$: non Fermi liquid, sum rule $\cos^2(\pi Q/2)$
 [Pustilnik&Borda PRB 2006 ; Logan et al. PRB 2009 ; Kogan et al. PRB 2003]

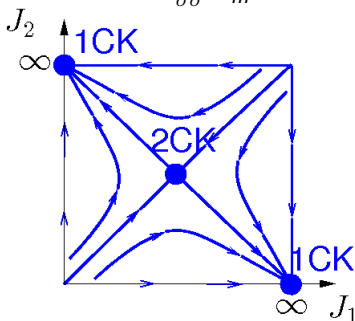


QPT signaled by conductance jump at $T = 0$

Channel anisotropy kills 2CK

Weak coupling RG : with $J_1 \neq J_2$

$$H = H_1 + H_2 + \sum_{\sigma\sigma'} \sum_m J_m c_{\sigma m}^\dagger \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{\sigma'm} \cdot \vec{S}$$

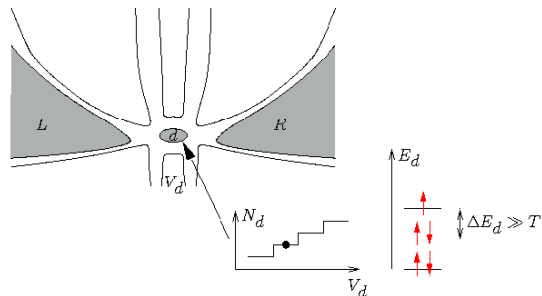


- ▶ \implies fine tuning needed!
- ▶ but can one realize such Hamiltonian with quantum dots?

Why 2 leads experiments give only 1 channel?

Kondo Hamiltonian : $\alpha = L, R$

$$H = H_L + H_R + \sum_{\sigma\sigma'} \sum_{\alpha,\alpha'} J_{\alpha,\alpha'} c_{\sigma\alpha}^\dagger \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{\sigma'\alpha'} \cdot \vec{S}$$

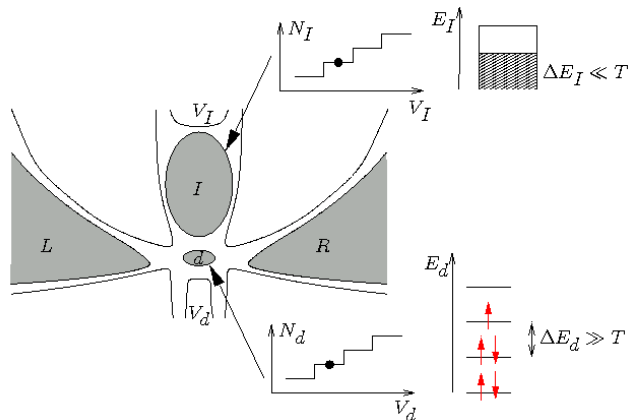


Matrix of couplings : $J_{\alpha,\alpha'} \propto \frac{t_\alpha t_{\alpha'}}{U}$ has only one non-zero eigenvalue

$$\implies H = H_+ + H_- + 8 \frac{t_L^2 + t_R^2}{U} \sum_{\sigma\sigma'} c_{\sigma+}^\dagger \frac{\vec{\tau}_{\sigma\sigma'}}{2} c_{\sigma'+}$$

- ▶ One channel only

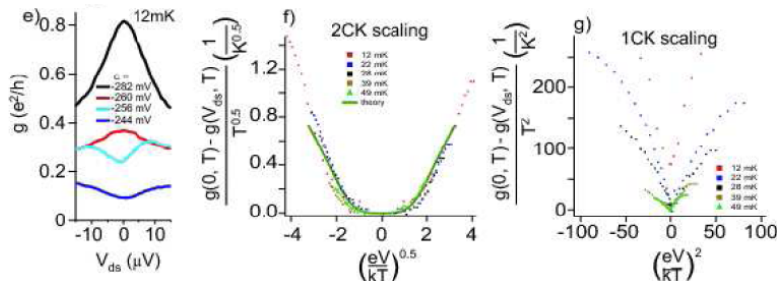
Goldhaber-Gordon/Oreg proposal



- ▶ Charge transfer between leads L/R and island I is suppressed below E_c^I
- ▶ Tough constraint : $\Delta E_I \ll T \ll E_c^I$

Experimental observation

[Potok *et al.* Nature (2007)]



- ▶ some evidence for scaling
- ▶ issues in the absence of complete quantitative comparison
- ▶ is there a better system ?