Non-linearities in quantum circuits

Serge Florens [Néel Institute - CNRS/UGA Grenoble]

“A gentle journey into not-so-gentle quantum many-body problems”

Organization of the lectures

Superconducting nanocircuits

- Lecture 1 : i) General circuitry ; ii) From Josephson junctions chains to spin-boson model
- Lecture 2 : i) Scattering theory with dynamical emitter ; ii) Frequency conversion beyond RWA ; iii) Renormalization from many-body entanglement ; iv) Many-body inelastic scattering theory

Metallic nanocircuits

- Lecture 3 : i) Anderson and Kondo models ; ii) DC Transport ; iii) Non Fermi liquid extensions
- Lecture 4 (closing the loop) : i) Boson-fermion connection in first quantization (using quantum optics concepts) ; ii) Insights into the Kondo wave-function ; iii) Emergent Majorana fermions

Beyond quantum optics in superconducting circuits

Fiat Lux

A bit of philosophy about the fine structure constant :

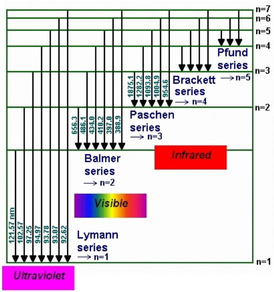
$$\alpha_{\mathrm{QED}} = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137} \quad \text{magic number!}$$

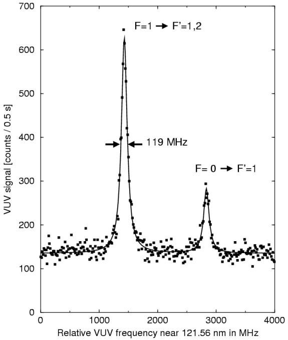
Another fine Feynman quote :

"God's hand wrote α, and we don't know how He pushed his pencil"

What if $\alpha_{\rm QED}$ were much larger ?

- Spectroscopists would hate it :





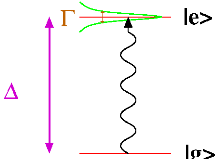
[Eikema, Walz & Hänsch, PRL 2001]

- Natural linewidth Γ (relative to transition frequency Δ) for 3D atomic decay :
$$\frac{\Gamma}{\Delta} \simeq [\alpha_{\rm QED}]^3 \simeq 10^{-7}$$
For 1S-2P transition, $\Gamma = 10^2$ MHz and $\Delta = 10^9$ MHz

Ultra-strong coupling of quantum optics

Linewidth of an atomic transition in vacuum :

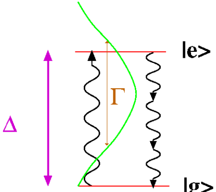
$$\frac{\Gamma}{\Delta} = \left(\frac{P}{e\lambda}\right)^2 \alpha_{\rm QED}$$
$$\simeq \left(\frac{a_{\rm Bohr}}{\lambda}\right)^2 \alpha_{\rm QED} \lll 1$$



- P = transition electric dipole
- λ = wavelength of resonant photon mode

Ultra-strong coupling of QED :

$$\frac{\Gamma}{\Delta} \simeq 1$$



- Higher probability for multi-photon exchange
- Strong non-linearities at small power

Anyway, playing with α_{QED} is not such a good idea

Decrease α_{QED} (with constant α_{strong}) by few % :

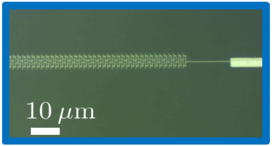
- Fast fusion p+p$\rightarrow$ ^{2}He takes place
- Stars exhaust fuel and quickly collapse to black holes

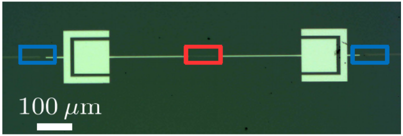
[Barrow,Tipler&Wheeler, *"The anthropic cosmological principle" (Oxford)*]

Increase α_{QED} by few % :

- Blocks nucleosynthesis of heavy elements
- Biology and life are no more possible

Safer approach : go to the lab and emulate this on a chip





Fiat Lux Reloaded

Step 1 : Increase density of states

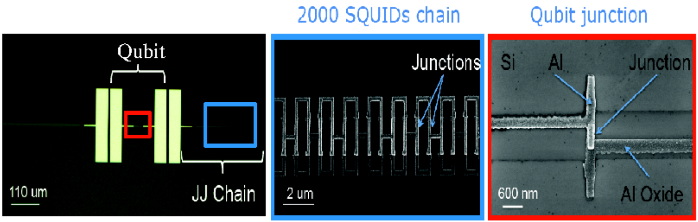
- 1D waveguides $\Rightarrow \Gamma/\Delta \simeq \alpha_{\rm QED}$ instead of $[\alpha_{\rm QED}]^3$

Step 2 : Increase fine structure constant

- Slow down light to enhance interaction with matter
- Use large inductance medium $=$ Josephson arrays

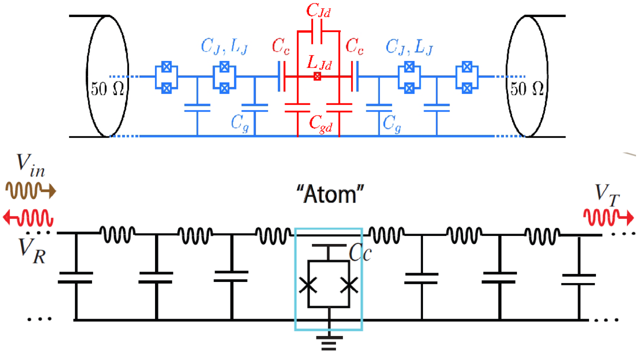
Step 3 : Optimize atomic dipole P

- Use tunable artificial atoms $=$ superconducting qubits



The platform

Device : Cooper pair box capacitively coupled to Josephson chain
$\Rightarrow$ maximizes non-linear effects, but not optimal w.r.t. noise



Effective Hamiltonian : spin-boson model [Leggett *et al.* RMP (1987)]

$$H = \frac{\Delta}{2}\sigma_x - \sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

Reaching the ultra-strong coupling regime

Spectral density : $J(\omega) \equiv \sum_k g_k^2 \delta(\omega - \omega_k)$

- Ohmic spectrum : $J(\omega) = 2\pi\alpha\omega$ for $\omega \ll \omega_P$
- Linewidth from Golden Rule : $\Gamma = \frac{\pi}{2}\alpha\Delta$

Dimensionless coupling strength :

- Effective coupling constant α controlled by geometric capacitances and chain impedance :

$$\alpha = \left(\frac{C_c}{C_c + C_{gd} + 2C_J}\right)^2 \frac{2Z_{\mathrm{chain}}}{R_Q}$$

Enhance light-matter coupling : as expected, we need to

- Maximize the dipole by qubit design
- Maximize the chain impedance

Transport properties : quantum optics regime

[Shen & Fan, PRL (2005)]

[Zheng, Gauthier, Baranger, PRA (2010)]

On the board : scattering theory in RWA ($\alpha \ll 1$)

On the board : scattering theory in RWA ($\alpha \ll 1$)

Transmission :

$$T(k) = \frac{(k-\Delta)^2}{(k-\Delta)^2 + \Gamma^2}$$

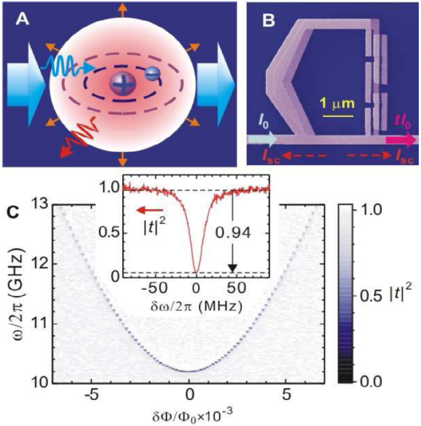
Reflection :

$$R(k) = |1 - t(k)|^2 = \frac{\Gamma^2}{(k-\Delta)^2 + \Gamma^2}$$

- All elastic : $R(k) + T(k) = 1$
- Result for a single photon (Fock) input state

Resonant absorption of a single artificial atom

Experiment with low-Z waveguide : [Astafiev *et al.*, Science 2010]



Resonant transmission vanishes due to perfect interference of incident and emitted waves

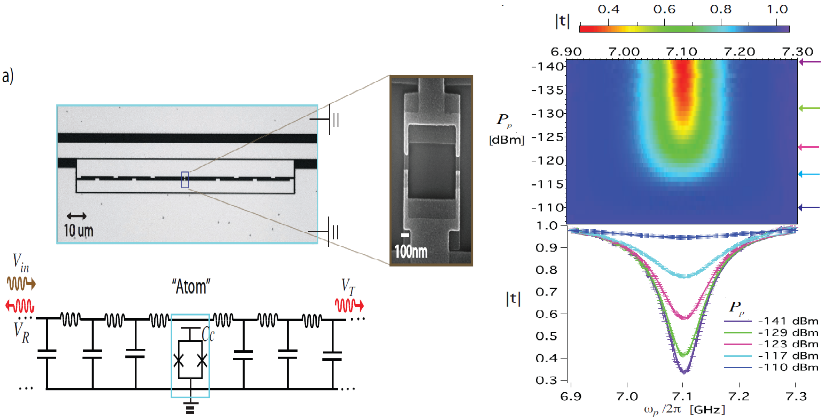
Quantum stuff : antibunching

Coherent state input : (classical signal)

- same transmission as previous at low power
- The reflected signal comes only from emitter
- Emitter has lifetime $1/\Gamma$
  $\Rightarrow$ Photons antibunch at times $t < 1/\Gamma$

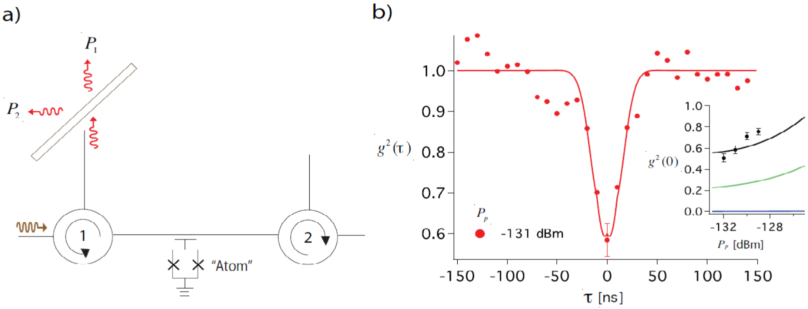
$$g^{(2)}(\tau) = \frac{\langle a^\dagger(t+\tau)a(t+\tau)a^\dagger(t)a(t)\rangle}{\langle a^\dagger(t+\tau)a(t+\tau)\rangle\langle a^\dagger(t)a(t)\rangle}$$

Antibunching measurements in waveguide QED



[Hoi et al., NJP 2013] (Chalmers)

Antibunching measurements in waveguide QED



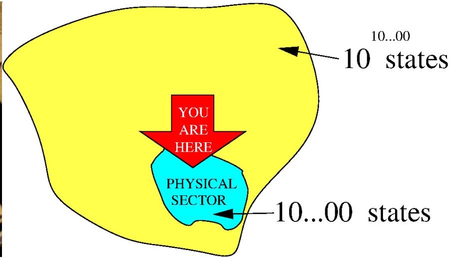
[Hoi et al., NJP 2013] (Chalmers)

Strong inelastic effects in photon scattering

[Goldstein, Devoret, Houzet & Glazman, PRL 2013]
[Bera, Baranger & Florens, PRA 2016]
[Gheeraert, Bera, Baranger, Roch, Florens, in preparation]

Physics beyond RWA

Let's open Pandora's box :



$$H_{\text{coupling}} = \sum_k \frac{g_k}{2} \left[\left(\tau^+ a_k + \tau^- a_k^\dagger \right) + \left(\tau^+ a_k^\dagger + \tau^- a_k \right) \right]$$

Number of excitations is not conserved $\Rightarrow$ full many-body problem

It's not just harder : new phenomena !

Physics beyond RWA

PRL **110,** 017002 (2013) PHYSICAL REVIEW LETTERS week ending 4 JANUARY 2013

Inelastic Microwave Photon Scattering off a Quantum Impurity in a Josephson-Junction Array

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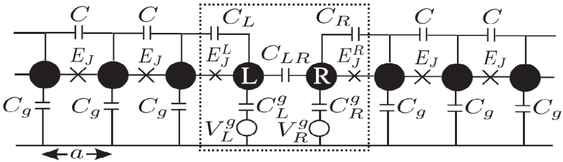
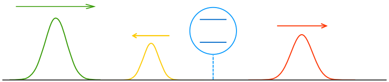
(Received 3 August 2012; published 2 January 2013)

Quantum fluctuations in an anharmonic superconducting circuit enable frequency conversion of individual incoming photons. This effect, linear in the photon beam intensity, leads to ramifications for the standard input-output circuit theory. We consider an extreme case of anharmonicity in which photons scatter off a small set of weak links within a Josephson junction array. We show that this quantum impurity displays Kondo physics and evaluate the elastic and inelastic photon scattering cross sections. These cross sections reveal many-body properties of the Kondo problem that are hard to access in its traditional fermionic version.

DOI: 10.1103/PhysRevLett.110.017002 PACS numbers: 74.81.Fa, 72.10.Fk, 74.25.N−, 85.25.−j



Transport coefficients for infinite lossless waveguide



Transport coefficients : linear response (Kubo)

- Reflection : $R(\omega) = (2\pi\alpha\omega)^2|\chi(\omega)|^2$
- Transmission : $T(\omega) = (2\pi\alpha\omega)^2[\chi'(\omega)]^2 + [1 - 2\pi\alpha\omega\chi''(\omega)]^2$
- Total inelastic losses : $\gamma(\omega) \equiv 1 - R(\omega) - T(\omega)$
- Energy resolved losses : $\gamma(\omega, \omega')$ beyond linear response

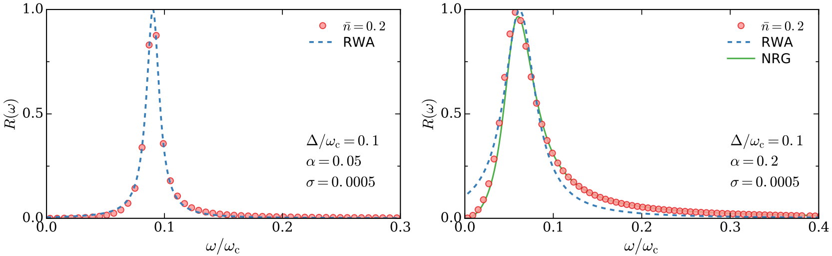
Qubit charge response :

$$\chi(t) = -\frac{i}{4}\theta(t) \langle \mathrm{GS}|[\sigma_z(t), \sigma_z(0)]|\mathrm{GS}\rangle$$

$\gamma(\omega)$ and $\gamma(\omega, \omega')$ computed in perturbation theory [Goldstein et al.]
Only $\gamma(\omega)$ can be computed non-perturbatively by NRG [Bera et al.]

Breakdown of RWA at ultra-strong coupling ?

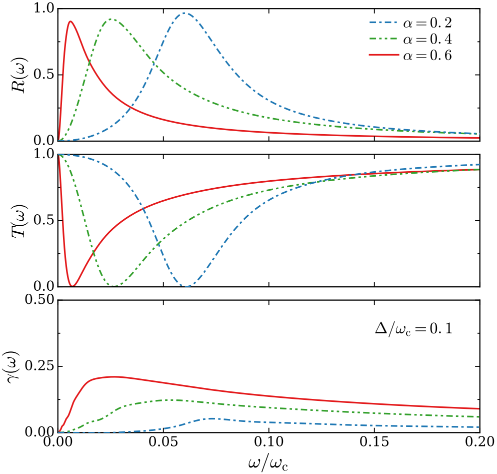
RWA vs. NRG vs. many-body scattering theory :



[Bera, Baranger & Florens, PRA 2016]

- RWA lorentzian lineshape works only if $\alpha \ll 1$
- RWA inaccurate when $\Gamma/\Delta \simeq 1$
- Strong level renormalization $\Delta_R < \Delta$ for increasing α
  About 40% Lamb shift for $\alpha = 0.2$

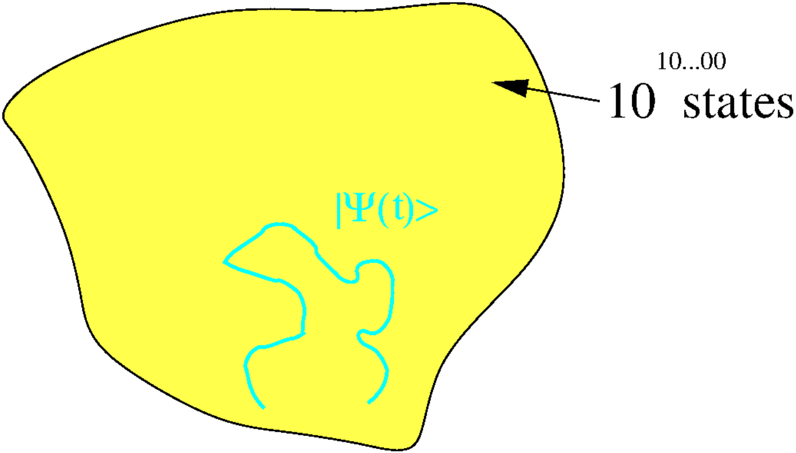
Total inelastic losses at large α (NRG calculation)



- Large inelastic losses : photons scatter at $\omega' \neq \omega$! !
- Computing the distribution of scattered photons is beyond NRG

How do we do many-body scattering simulations ?

- Span physical sector by multicomponent Schrödinger cats
- Crazy quantum states ask an exponential number of cats



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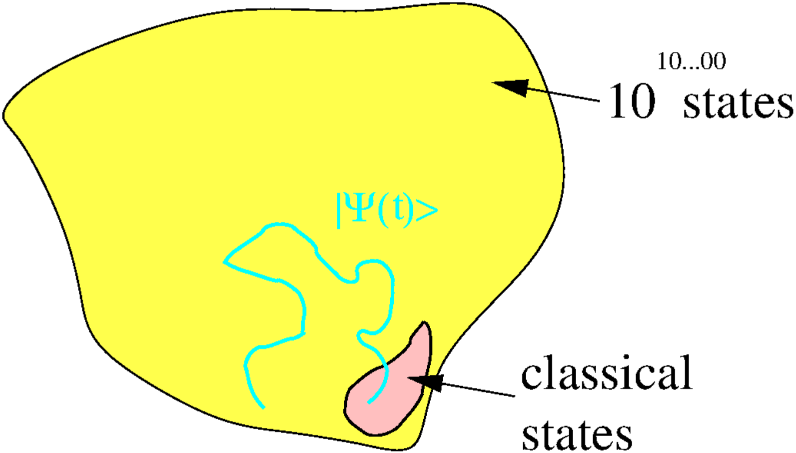

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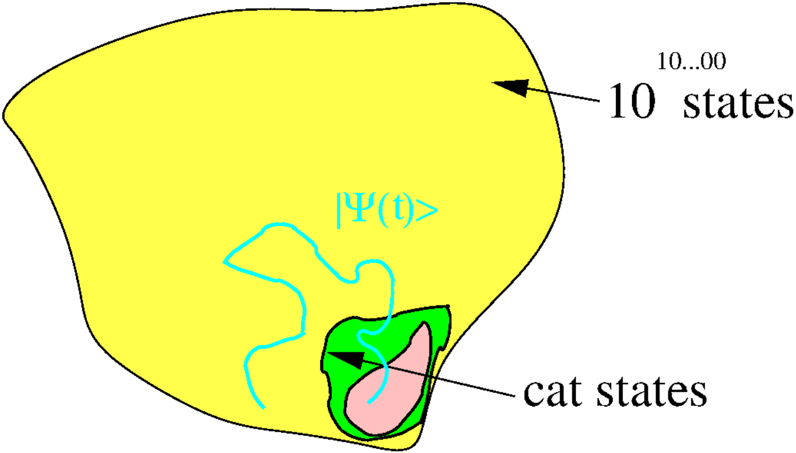

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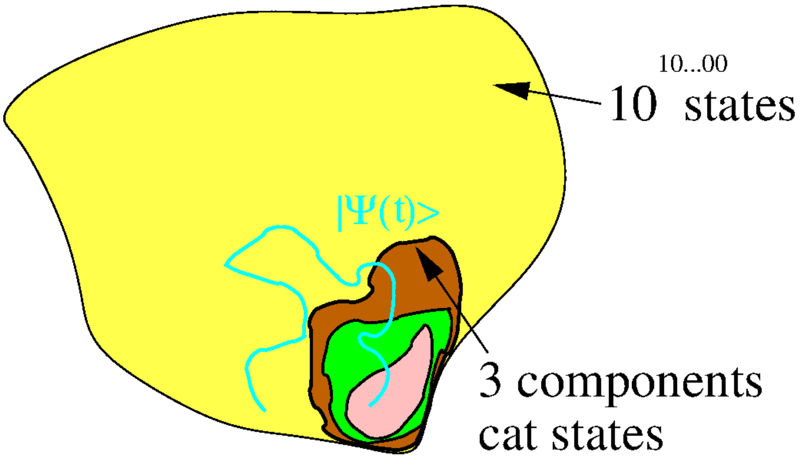


How do we do many-body scattering simulations ?

- Introduce time-dependent multicomponent cats

$$\left|\Psi(t)\right\rangle = \sum_{n=1}^{N_{\mathrm{cs}}} \left[p_n(t) \left| f^{(n)}(t) \right\rangle \otimes \left| \uparrow \right\rangle + q_n(t) \left| h^{(n)}(t) \right\rangle \otimes \left| \downarrow \right\rangle \right]$$

with multimode coherent state $\left|f(t)\right\rangle = e^{\sum_k f_k(t) a_k^\dagger - f_k^*(t) a_k(t)}$

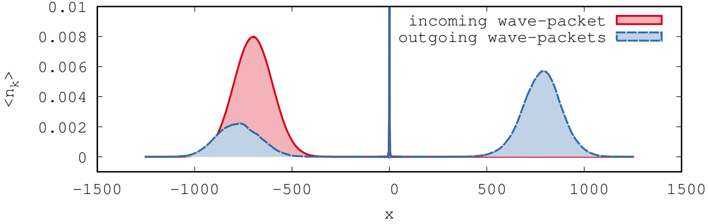
- Construct Lagrangian : $\mathcal{L} = \left\langle \Psi(t) \right| \frac{i}{2} \overleftrightarrow{\partial_t} - \mathcal{H} \left| \Psi(t) \right\rangle$
- Solve Hamilton-Jacobi equations : $\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial \mathcal{L}}{\partial \dot{f}_k^{(n)}} = \frac{\partial \mathcal{L}}{\partial f_k^{(n)}}, \ldots$
- Classical-like equations of motion (after manipulation) :

$$\frac{\mathrm{d}}{\mathrm{d}t} f_k^{(n)} = \sum_{k'=1}^{N_{\mathrm{modes}}} \sum_{n'=1}^{N_{\mathrm{cs}}} C_{k,k'}^{(n,n')}[f, h, p, q]$$

- Numerical cost $\sim \mathcal{O}[(N_{\mathrm{cs}})^3 \times (N_{\mathrm{modes}})^2] \Longrightarrow$ Cheap !
- Norm and energy are conserved by construction
- Non-linear classical circuit theory is included by construction

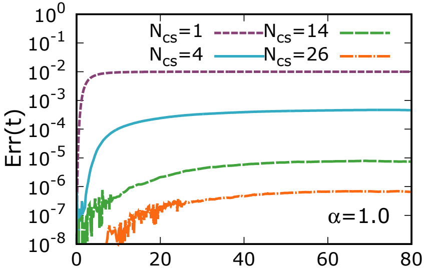
How do we do many-body scattering simulations ?

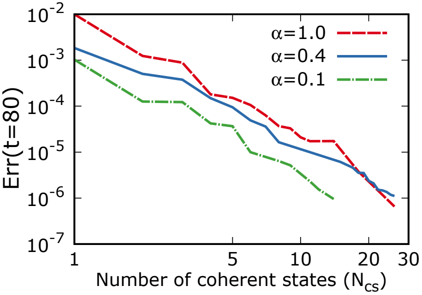
- Classical input : a coherent state distribution with average momentum k_0, width σ, and average photon number $\bar{n}$



- Non-classical input : Fock state = small antisymmetric cat
- Output wavefunction is analyzed spectrally

Control of the error (spontaneous emission)





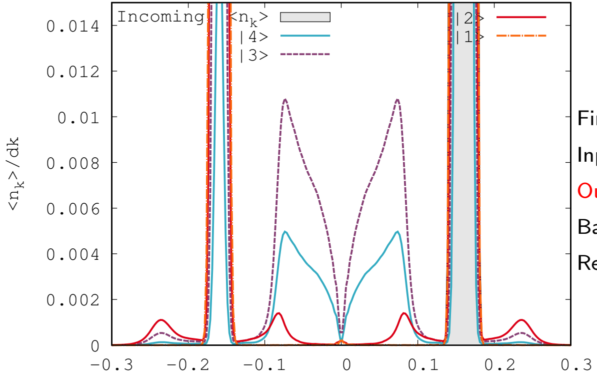
Error in the dynamical simulation :

Define auxiliary state $|\Phi(t)\rangle \equiv (i\partial_t - H)|\Psi(t)\rangle$

$\mathrm{Err}(t) \equiv \langle\Phi(t)|\Phi(t)\rangle$

$\mathrm{Err}(t) = 0$ in the exact Schrödinger dynamics

Inelastic scattering of coherent state with $\langle \hat{n} \rangle = 1$



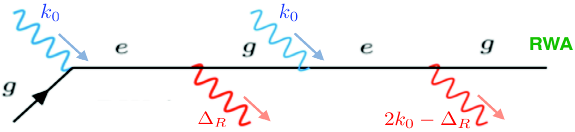
Fine structure constant $\alpha = 0.1$

Input frequency $k_0 = 0.16$

Output frequency k arbitrary

Bare qubit frequency $\Delta = 0.1$

Renormalized qubit $\Delta_R = 0.08$



[Gheeraert, Bera, Baranger, Roch, Florens, in preparation]

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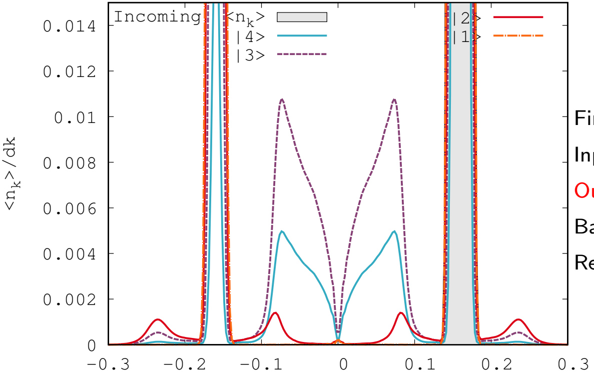

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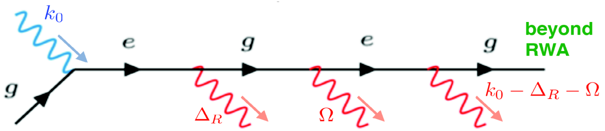

beyond RWA

[Gheeraert, Bera, Baranger, Roch, Florens, in preparation]

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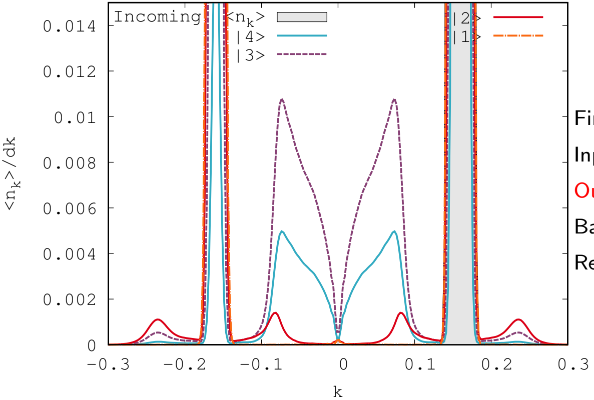

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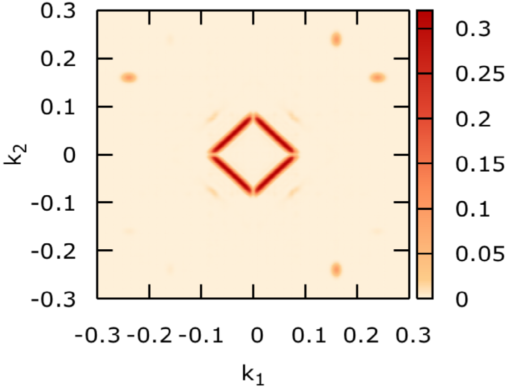
Renormalized qubit $\Delta_R = 0.08$

Size of the full Hilbert space : 1000 modes, Fock states up to $n = 5$ populated $\Rightarrow 1000^5 = 10^{15}$ states in total !

Addressing the physical subspace : $N_{\mathrm{cs}} = 20$ classical states are enough for these calculations

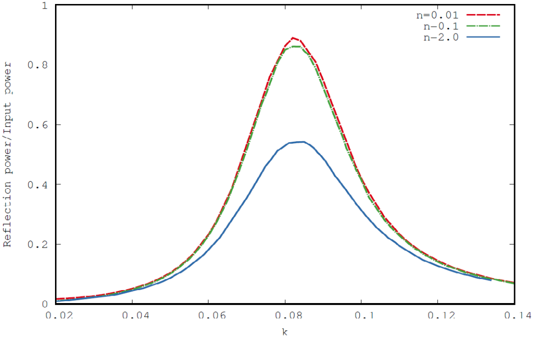
Direct evidence of the 3-photon continuum



- Output wavefunction projected onto Fock state : $\sum_{k_1,k_2,k_3} \Gamma(k_1, k_2, k_3) a^\dagger_{k_1} a^\dagger_{k_2} a^\dagger_{k_3} |0\rangle$
- Amplitude $\Gamma(k_1, k_2, k_3)$ plotted for $k_3 = \Delta_R$
- The two photons accomodate the excess energy $k_0 - \Delta_R$

Saturation effects beyond RWA

Reflection coefficient : for increasing $\bar{n}$



The post-RWA asymmetric lineshape persists at strong power

Open question

True microscopic model : non-uniform Josephson chain

$$H = \frac{1}{2}\sum_{i,j}(2e)^2 n_i \left[\hat{C}^{-1}\right]_{ij} n_j - \sum_i E_i^J \cos(\Phi_i - \Phi_{i+1})$$

$n - \Phi$ are conjugate variables : $[\hat{n}_j, \hat{\Phi}_l] = i\delta_{j,l}$

Experimental perspectives :

- Compromise : decrease qubit non-linearity to avoid charge noise, but not too much
- What is left of the inelastic effects ?

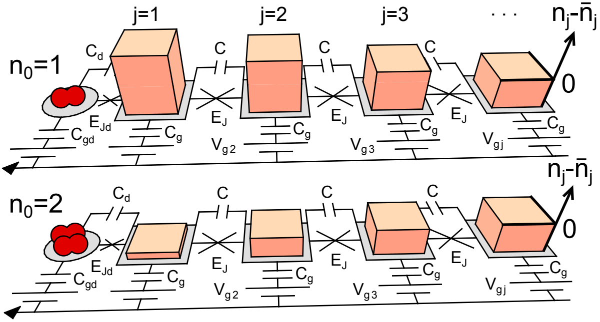
Qubit frequency renormalization from many-body entanglement

[Emery and Luther, PRL (1971)]
[Silbey and Harris, J. Chem. Phys. (1984)]
[Snyman & Florens, PRB 2015]

On the board : qubit renormalization

On the board : qubit renormalization

Physics : each qubit state induces its own charge polarization

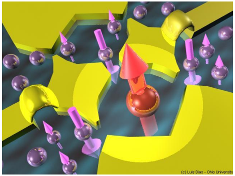


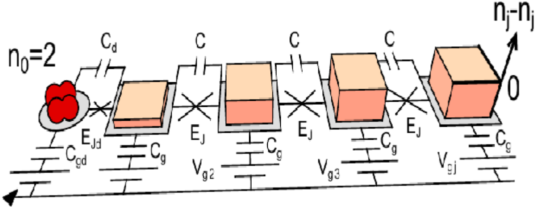
$$|\Psi\rangle \simeq \frac{|\uparrow\rangle \otimes |f\rangle}{\sqrt{2}} - \frac{|\downarrow\rangle \otimes |-f\rangle}{\sqrt{2}}$$

Many-body Bell state between qubit and waveguide

Why the screening cloud has to do with Kondo ?

Metallic vs superconducting quantum dots :





Proper response function : qubit response from non-local gates

$$\chi_j = \langle \sigma_z (n_j - \bar{n}_j) \rangle \neq 0$$

- Charge rather than spin response : easier to detect
- Unidimensional and clean chain : hard to do with 2DEGs
- No quasiparticles : simpler environment (no Friedel oscillations)

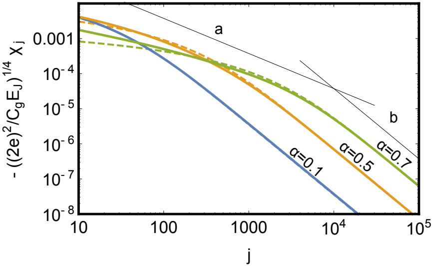
Spatial behavior of the cloud

Variational result : universal function

$$\boxed{\chi_j \propto -L_K \,\mathrm{Re}\left\{ e^{-ij/L_K} \Gamma\left[0, -ij/L_K\right] \right\}}$$

- χ_j decays as $1/j^2$ for $j \gg L_K$
- χ_j drops logarithmically at $1 \ll j \ll L_K$

Same behavior as spin-density of Kondo cloud [Barzyrik&Affleck, PRL 1996]



Dashed line : variational

Full line : exact numerics

On the menu of the next lecture

Metallic quantum circuits :

- Anderson and Kondo models
- Transport properties
- Non Fermi liquid extensions

Extra slides

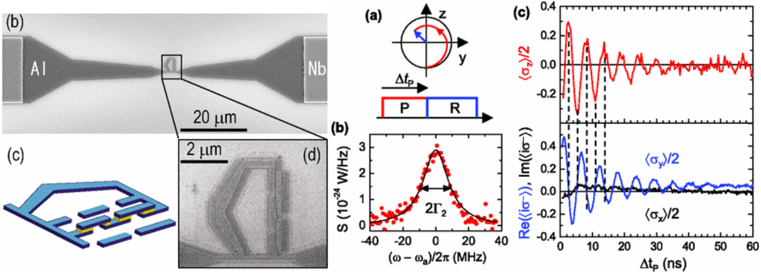
Quantum dynamics : decoupling of relaxation and decoherence

[Gheeraert, Bera & Florens, arXiv :1601.01545]

Dynamics of a single artificial atom

Experiment with low-impedance transmission line :

[Abdumalikov *et al.*, PRL 2011]



- Spin dynamics of single atom is read from collected light
- Coherent radiation reveals atomic superposition :

$$|\Psi\rangle = [\alpha|g\rangle + \beta|e\rangle] \otimes |0\rangle \to |g\rangle \otimes [\alpha|0\rangle + \beta a^\dagger|0\rangle]$$

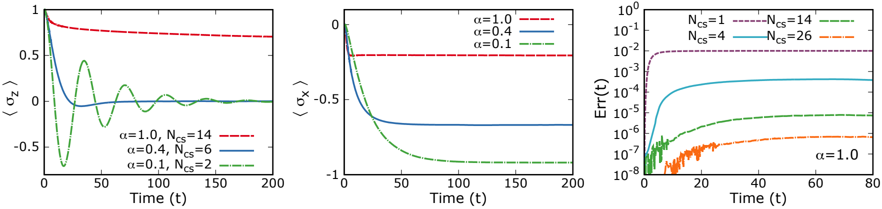
Qubit dynamics at ultra-strong coupling

Protocol : prepare state $\left|\Psi\right\rangle = \left|0\right\rangle \otimes \left|\uparrow\right\rangle = \left|0\right\rangle \otimes [\left|g\right\rangle + \left|e\right\rangle]/\sqrt{2}$

Method : time-dependent variational coherent state expansion

Quantum optics regime $(\alpha \ll 1)$: $T_2 = 2T_1$

- T_2 = time scale for decoherence $\Leftrightarrow$ decay of $\langle\sigma_z(t)\rangle$
- T_1 = time scale for energy relaxation $\Leftrightarrow$ decay of $\langle\sigma_x(t)\rangle$



Ultra-strong coupling regime $(\alpha \simeq 1)$: $T_2 \gg 2T_1$

Relaxation and decoherence time scales decouple !

Decoupling of the dynamics

Fast energy relaxation :

- $|e\rangle$ is very strongly damped at increasing α
- Relaxation time as short as $T_1 \simeq 1/\omega_p$ for $\alpha \simeq 1$

Slow decoherence :

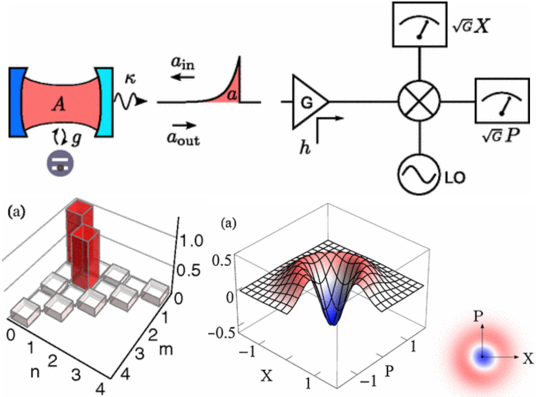
- The system must relax to the many-body ground state
- This can take an exponential time $T_2 \propto L_K \propto 1/\Delta_R$
- Quantum information is transferred slowly from qubit to environment (due to extended size of entanglement cloud)

Spontaneous emission : cat state radiation

[Gheeraert, Bera & Florens, arXiv :1601.01545]

Tomography of itinerant single microwave photon

Wigner distribution of emitted photon : [Eichler *et al.*, PRL 2011]



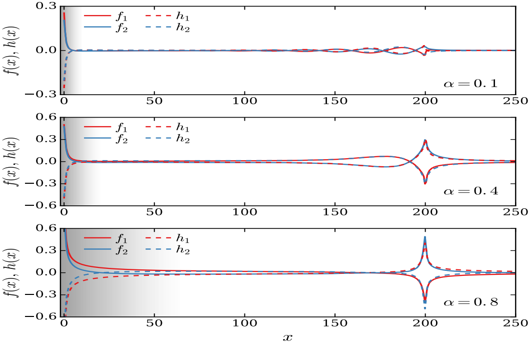
Heterodyne detection : moments $\langle (a^\dagger)^n (a)^m \rangle$ of emitted signal

Power $\propto [E_{\rm signal}\cos(\omega_{\rm signal} t) + E_{\rm LO}\cos(\omega_{\rm LO} t)]^2$

$\rightarrow E_{\rm signal} E_{\rm LO} \cos[(\omega_{\rm signal} - \omega_{\rm LO}) t] + \text{filtered out}$

Spatial profile of electromagnetic modes

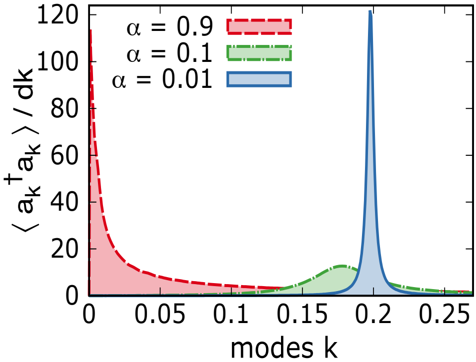
Initial state : $|\Psi\rangle = |0\rangle \otimes |e\rangle = |0\rangle \otimes (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$

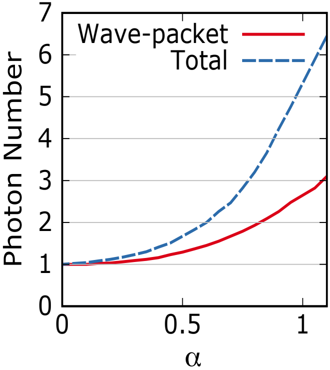


For large α :

- Entanglement spreads within the waveguide (T_2 long)
- Wavepacket is dominated by wavefront (T_1 short)
- Displacements in the wavepacket increase

Distribution of emitted photons at large α





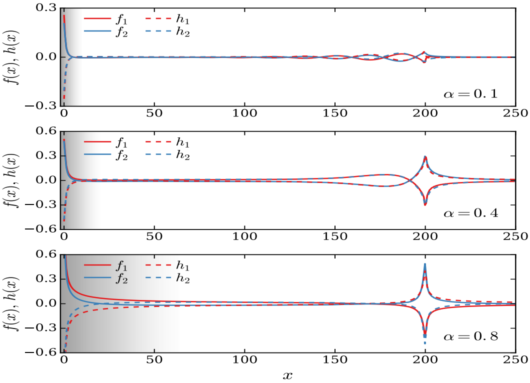
For increasing α :

- Fast energy relaxation $\Rightarrow$ broad spectral distribution
- $\Gamma/\Delta \simeq 1 \Rightarrow$ Photon number $\gg 1$

Emission : from photon to cat

A photon = small Schrödinger kitten

$$|f\rangle - |-f\rangle = \sum_k 2 f_k a_k^\dagger |0\rangle \quad \text{for } f_k \to 0$$

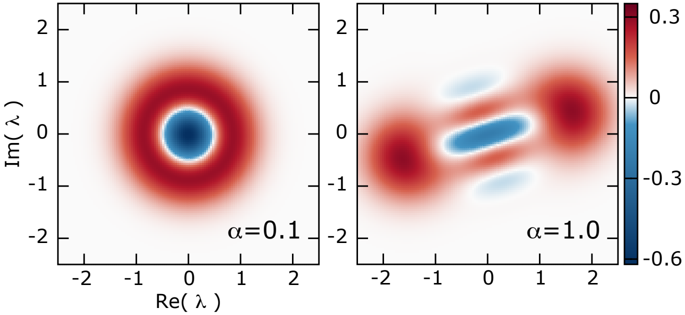


Kitten size grows at increasing α since $n = |f|^2$

Emission : from photon to cat

A photon = small Schrödinger kitten

$$|f\rangle - |-f\rangle = \sum_k 2 f_k a_k^\dagger |0\rangle \quad \text{for } f_k \to 0$$

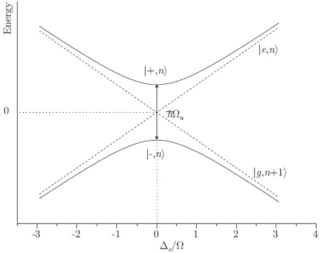


- Cat states with $n = 3$ are radiated for $\alpha \simeq 1$
- Cat coherence is limited by entanglement with the cloud

Standard dressed atom picture

Back to finite chain : near a resonance ($\omega_k \simeq \Delta$) for $\alpha \ll 1$

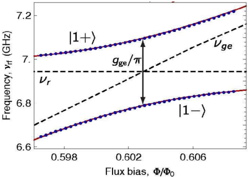
$$\text{Jaynes Cummings : } H \simeq \frac{\Delta}{2}\tau_z - \frac{g_k}{2}(\tau^- a_k^\dagger + \tau^+ a_k) + \omega_k a_k^\dagger a_k$$

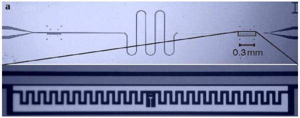


- Only $|g, n+1\rangle$ and $|e, n\rangle$ couple : effective two-level system
- New eigenstates $|\pm\rangle$ mixing $|g, n+1\rangle$ and $|e, n\rangle$
- Anticrossing on resonance : $\Omega_{k,n} = 2g_k\sqrt{n} \propto \sqrt{k}\sqrt{n}$

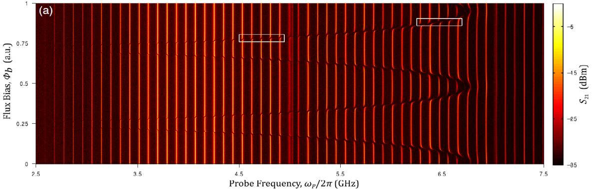
Dressed atom from cavity-QED to multimode resonators

Vacuum Rabi splitting in cavity : [Fink *et al.*, Nature 2008]





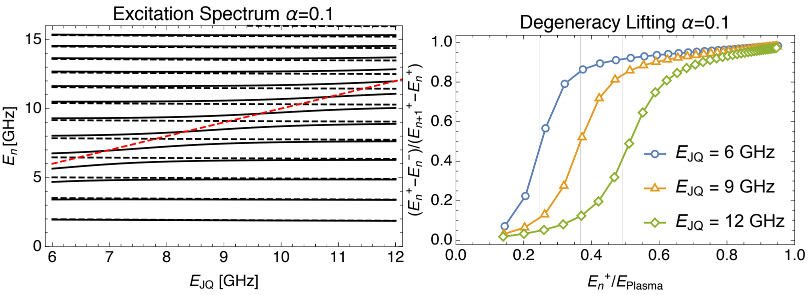
Multimode resonators : [Sundaresan *et al.*, PRX 2015]



- Confirms the $\Omega_{k,n} \propto g_k \propto \sqrt{k}$ scaling
- But $g_0 \simeq 5$ MHz $\ll \omega_{k+1} - \omega_k \simeq 100$MHz : single mode !

Dressed atom in Josephson waveguides

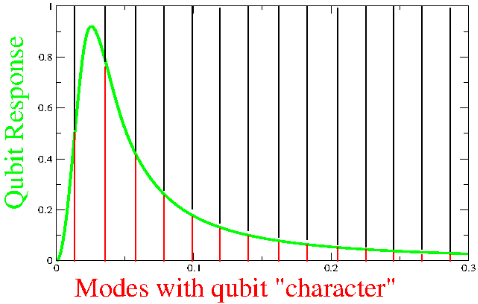
Spectrum from exact diagonalizations (1-photon subspace) :



- Strong dressing even for low-energy modes
- Non-resonant modes are also lifted : many-body dressed state
- Profile of degeneracy lifting is universal (Kondo again)

Re-interpretation of absorption spectra

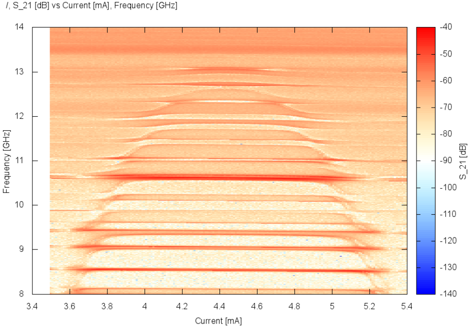
Qubit response χ in infinite system :

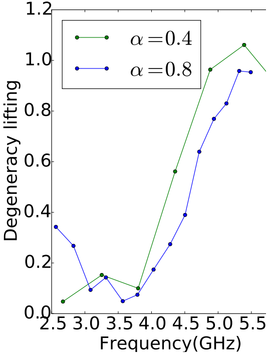


- The broadening and giant Lamb shift come from a broad band of modes with strong qubit "character"

Preliminary experimental results

Two early generations of samples : charge qubit and transmon





- Qualitatively ok with expectations
- Cleaner samples are coming up...

Exact mapping to Kondo model

Spin boson Hamiltonian :

$$H = \frac{\Delta}{2}\sigma_x - \sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

Unitary transformation : $U_\gamma = \exp\{-\gamma\sigma_z \sum_k \frac{g_k}{2\omega_k}(a_k^\dagger - a_k)\}$

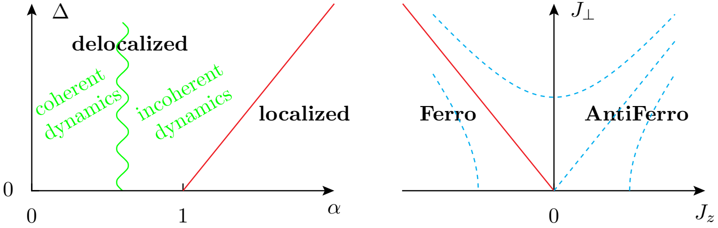
$$U_\gamma H U_\gamma^\dagger = \frac{\Delta}{2}\sigma^+ e^{-\gamma\sum_k \frac{g_k}{\omega_k}(a_k^\dagger - a_k)} + h.c. + (\gamma-1)\sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

Bosonization (ohmic bath only) : [Guinea, Hakim, Muramatsu PRB (1985)]

$$\begin{aligned} U_\gamma H U_\gamma^\dagger &= \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \Delta\sigma^+ \sum_{kk'} c_{k\downarrow}^\dagger c_{k'\uparrow} + h.c. && \rightarrow J_\perp = \Delta \\ &\quad + (1-\sqrt{\alpha})\omega_c \sigma^z \sum_{kk'} [c_{k\uparrow}^\dagger c_{k'\uparrow} - c_{k\downarrow}^\dagger c_{k'\downarrow}] && \rightarrow J_z \propto 1-\sqrt{\alpha} \end{aligned}$$

Relating Kondo to spin-boson

Phase diagram :



Spatial correlations : an exact identity connects bosonic to fermionic Kondo cloud

$$\left\langle \sigma_z [c_\uparrow^\dagger(x) c_\uparrow(x) - c_\downarrow^\dagger(x) c_\downarrow(x)] \right\rangle \sim \chi(x) + \cos(2k_F x)\chi_{2k_F}(x)$$

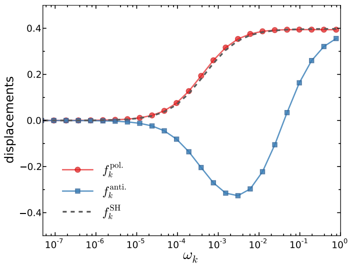
Unveiling the Kondo anti-cloud

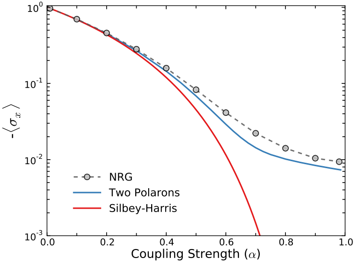
New Ansatz with two coherent states :

$$\big|\Psi\big\rangle = \big|\uparrow\big\rangle \otimes \Big[\big|+f_k^{\mathrm{pol.}}\big\rangle + p\big|+f_k^{\mathrm{anti.}}\big\rangle\Big] - \big|\downarrow\big\rangle \otimes \Big[\big|-f_k^{\mathrm{pol.}}\big\rangle + p\big|-f_k^{\mathrm{anti.}}\big\rangle\Big]$$

$p =$ weight of the anti-cloud relative to the main polarization

Variationally determined displacements and $\langle\sigma_x\rangle$:



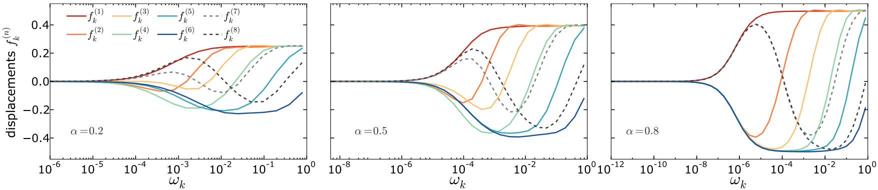


These correlations drastically improve the tunneling amplitude !

General many-body coherent states expansion

Idea : we expand the wavefunction in the coherent state "basis"

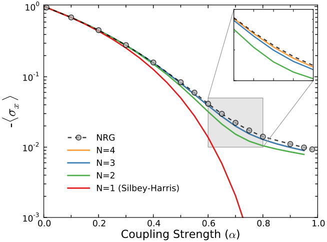
$$\big|\Psi\big\rangle = \sum_{n=1}^{N_{\rm cs}} p_n \Big[\big| + f^{(n)} \big\rangle \otimes \big| \uparrow \big\rangle - \big| - f^{(n)} \big\rangle \otimes \big| \downarrow \big\rangle \Big]$$

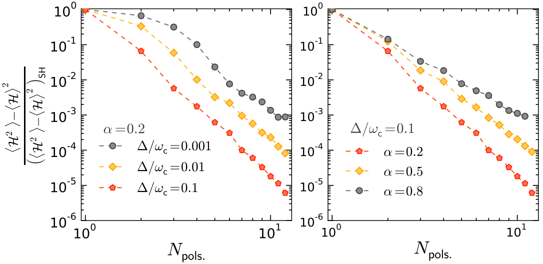


- Development of an efficient numerical algorithm : $f_k^{(n)}$ parametrized by $(N_{\rm cs})^2$ coefficients $\Longrightarrow$ Cheap !

Checking the coherent states expansion

Tunneling amplitude $\langle \sigma_x \rangle$ and energy variance :





- Fast convergent expansion : error vanishes as $N_{cs}^{-2-1/\alpha}$
- Main numerical difficulty : reaching efficiently the global minimum in a very flatish landscape

Kondo scale (extra) renormalization