Non-linearities in quantum circuits

Serge Florens [Néel Institute - CNRS/UGA Grenoble]

"A gentle journey into not-so-gentle quantum many-body problems"

Plan and purpose of the lecture

Organization of the lectures

Superconducting nanocircuits

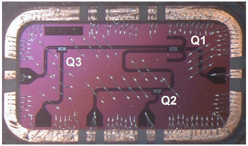
- Lecture 1 : i) General circuitry ; ii) From Josephson junctions chains to spin-boson model
- Lecture 2 : i) Scattering theory with dynamical emitter ; ii) Frequency conversion beyond RWA ; iii) Renormalization from many-body entanglement ; iv) Many-body inelastic scattering theory

Metallic nanocircuits

- Lecture 3 : i) Anderson and Kondo models ; ii) DC Transport ; iii) Non Fermi liquid extensions
- Lecture 4 (closing the loop) : i) Boson-fermion connection in first quantization (using quantum optics concepts) ; ii) Insights into the Kondo wave-function ; iii) Emergent Majorana fermions

Why emphasis on superconducting circuits ?

1) Topic is timely with fast progresses :



Multiple qubit architecture

2) Connects two topics of the school :

Strong link to lectures by Pothier (superconductivity), Splettstoesser (scattering theory), Waintal (finite frequency transport, Luttinger liquids), Alet (many-body wave-functions)

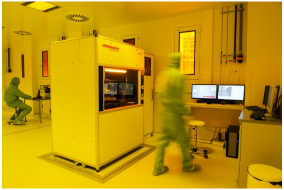
3) Pedagogical : allows to simplify at best tricky concepts

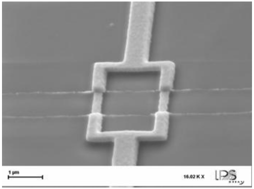
- No quasiparticle excitation in the Fermi sea : life is easy !

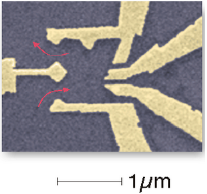
A quick preview on physical systems

<u>Lithographically-defined nano-circuits :</u>

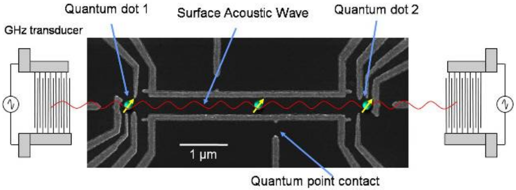
Typical length scale : $\boxed{L = 100nm}$







<u>Complex designs :</u>



Scales at play (mesoscopic domain)

Typical length : $L = 100\text{nm}$

Energies :

- Electrostatic : $E = \frac{e^2}{4\pi\epsilon_0 L} \simeq 10^{-22}$ J $\simeq 10^{-3}$eV
- Confinement : $E = \frac{\hbar^2\pi^2}{m_e L^2} \simeq 10^{-23}$ J $\simeq 10^{-4}$eV

Temperature : $T = \frac{E}{k_B} \simeq 1\text{K}$ for $E = 10^{-4}$ eV

Frequency : $f = \frac{E}{2\pi\hbar} \simeq 20\text{GHz}$ for $E = 10^{-4}$ eV

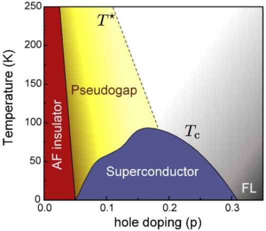
At cryogenic temperatures ($T = 50\text{mK}$) :

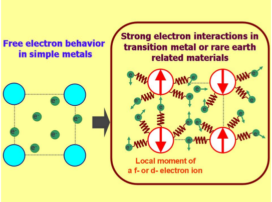
- Single electron charge operations possible
- Quantum confinement takes place (artificial atoms)
- Efficient control by microwave radiation

Why do it (besides for the interesting physics)

To build (or not to build ?) a quantum computer :

To understand correlated matter in simpler settings :





What do we mean by non-linearities ?

Exemples of non-linear phenomena :

- Output not proportional to input
- Transport resonances
- Frequency conversion

In addition, more fundamentally :

- System cannot be described by independent individual entities
  $\Rightarrow$ forget Landauer-Büttiker
- and neither by independent collective entities (on all scales)
  $\Rightarrow$ forget Fermi liquid theory, Luttinger liquids...

A word on methodology in many-body problem

No perfect and unique approach :
Full understanding comes by shining light from various directions

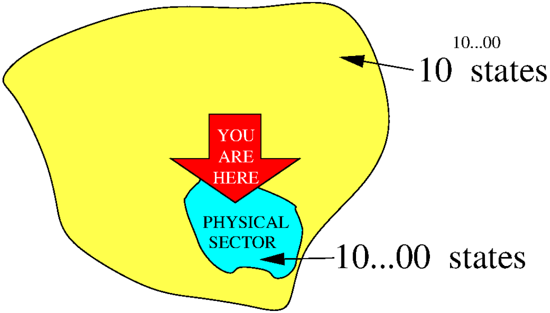
Great variety of methods :

- From simple (but approximate) ones : mean-field, perturbation theory, slave bosons...
- ... To more "exact" (but more technical) ones : many-body Green's functions and Feynman diagrams, Bethe Ansatz, Conformal Field Theory, Bosonization, advanced numerical algorithms (QMC, NRG, DMRG...)

For this lecture : I will use mostly wave-functions, that are more familiar conceptually, more transparent physically

Map of the Hilbert space

Target the physical sector : states accessible upon "reasonable" protocol within decoherence time



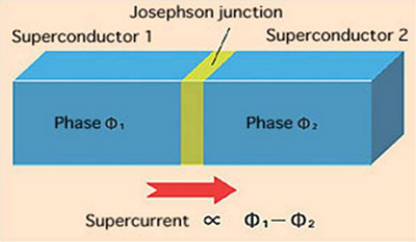
How to span the physical sector ?

- For low entanglement (1d) : Matrix Product States
- Alternative : quantum superposition of "classical" states
- Advantages : input/output friendly, decoherence friendly

Superconducting circuitry : waveguides

Josephson junction = high inductance lossless element

Josephson relations : Φ is phase difference across a junction



$$\boxed{I = I_c \sin \Phi \simeq I_c \Phi} \text{(linearized)}$$

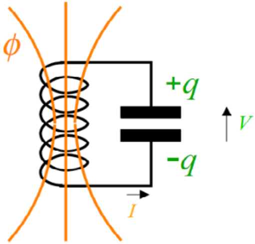
$$\boxed{V = \frac{\hbar}{2e} \frac{\partial \Phi}{\partial t}}$$

$$\Rightarrow V = \frac{\hbar}{2eI_c} \frac{\partial I}{\partial t} = L_J \frac{\partial I}{\partial t} \quad \Rightarrow \quad Z_J(\omega) = iL_J\omega$$

- Josephson inductance density : $\ell_J \simeq 1 \text{ nH}/\mu\text{m} = 10^4 \ell_{\text{geometric}}$
- Geometric self-inductance density :
  $\ell_{\text{geometric}} = \mu_0/(4\pi) = 10^{-7} \text{ H/m} = 10^{-4} \text{ nH}/\mu\text{m}$

On the board : Josephson relations

Crash course on circuit-QED for LC-resonator



$\phi = LI$

$q = CV$

Classical energy : harmonic oscillator

$$H = \frac{Q^2}{2C} + \frac{L}{2} I^2 = \frac{Q^2}{2C} + \frac{L}{2}(\dot{Q})^2$$

Conjugate classical variables : charge/flux

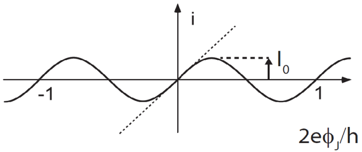
$$\frac{\partial H}{\partial \dot{Q}} = L\dot{Q} = LI = \phi$$

Quantum regime : $\boxed{[\hat{Q}, \hat{\phi}] = i\hbar}$ What does it mean ?

- Tiny electromagnetic signals generated by charge fluctuations of order $\delta Q \simeq 2e$ and flux fluctuations of order $\delta\phi \simeq \hbar/2e$
- Vacuum reached when $k_B T \ll \hbar/\sqrt{LC}$ and low loss

Josephson junction as a circuit element

Linearized limit (non-linearities for later) :



Impedance : $Z(\omega) = \frac{1}{\frac{1}{iL_J\omega}+jC\omega} = \frac{iL_J\omega}{1-L_JC\omega^2}$

Resonance frequency : $f = \frac{1}{2\pi}\sqrt{\frac{1}{L_JC}}$

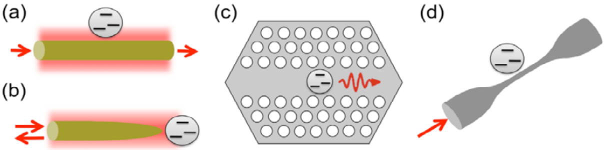
$L_J \simeq 1\text{nH},\ C \simeq 10\text{pF} \Rightarrow f \simeq 1\text{GHz}$ microwave range

Lumped element limit : Maxwell $\rightarrow$ individual circuit elements

- Associated wavelength $\lambda = \frac{c}{f} \simeq 20$ cm
- $\lambda \gg$ jonction size (a few μm)

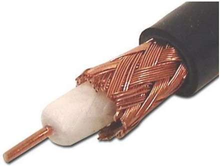
1D Waveguides : optimize coupling to single emitter

Optical domain : requires nanostructuration

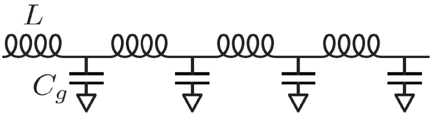


- Surface plasmonic modes in metallic nanowires
- Waveguide in photonic crystal
- Tapered optical fiber

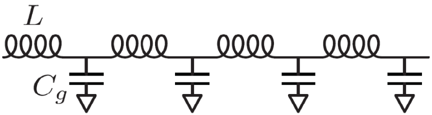
Microwave domain : human scale !



On the board : propagation in waveguide



On the board : propagation in waveguide



Telegraph equation for LC waveguide :

$$I(x,t) = I_1 e^{i\omega[t-\sqrt{\ell c_g}x]} + I_2 e^{i\omega[t+\sqrt{\ell c_g}x]}$$

$$V(x,t) = \sqrt{\frac{\ell}{c_g}}\left[-I_1 e^{i\omega[t-\sqrt{\ell c_g}x]} + I_2 e^{i\omega[t+\sqrt{\ell c_g}x]}\right]$$

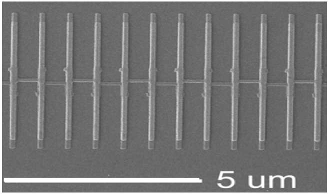
Impedance of waveguide : $Z(\omega) = \sqrt{\ell/c_g} = \sqrt{L/C_g} \simeq 3\text{k}\Omega$

Crucial point : propagation with velocity $v = 1/\sqrt{\ell c_g} \simeq c/50$

High inductance ⇒ slow light ! ⇒ stronger light-matter coupling

Josephson junction arrays

Waveguide : a chain of tunnel-coupled superconducting islands



Generic Hamiltonian : valid for $T \ll T_c^{\mathrm{BCS}}$

$$H = \frac{1}{2}\sum_{i,j}(2e)^2 n_i \left[\hat{C}^{-1}\right]_{ij} n_j - \sum_i E_J \cos(\Phi_i - \Phi_{i+1})$$

$n - \Phi$ are conjugate variables : $[\hat{n}_j, \hat{\Phi}_l] = i\delta_{j,l}$

Non-linearities are controlled by the ratio of charging energy $E_C = (2e)^2/C$ and Josephson $E_J = \hbar^2/[(2e)^2 L_J]$ energy

On the board : classical normal modes (general)

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<u>General result :</u>

ω_k^2 are the eigenvalues of symmetric matrix $\hat{C}^{-1/2}\hat{L}^{-1}\hat{C}^{-1/2}$

valid for any capacitance matrix (even non-uniform waveguides)

On the board : quantum normal modes (waveguide)

On the board : quantum normal modes (waveguide)

Diagonal form :

$$H_{\mathrm{chain}} = \sum_k \hbar\omega_k a_k^\dagger a_k$$

Spectrum : (same as classically)

$$\hbar\omega_k = 2\left|\sin(\frac{k}{2})\right|\sqrt{\frac{E_J(2e)^2}{C_g + 4C_J\sin^2(k/2)}}$$

$k = ap$ dimensionless momentum

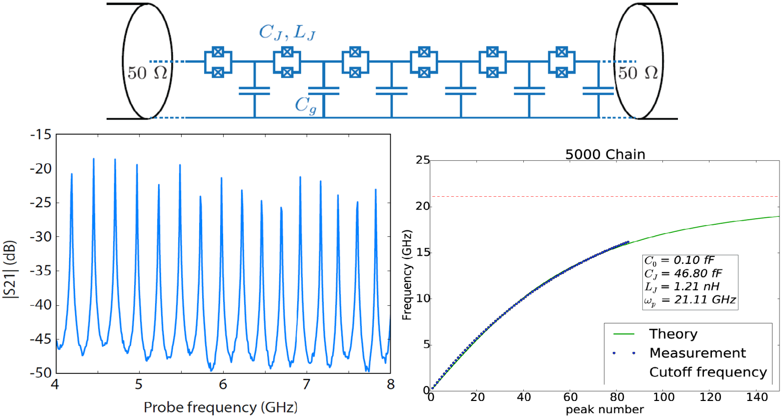
Low frequency limit : $\omega_k \simeq k\sqrt{E_J(2e)^2/C_g} = k/\sqrt{L_J C_g}$

we recover slow light

High frequency limit : ω_k saturates at ω_P

Experimental measurement (@ NEEL)

Finite chain coupled to 50 Ω lines : "giant Fabry-Perot cavity"

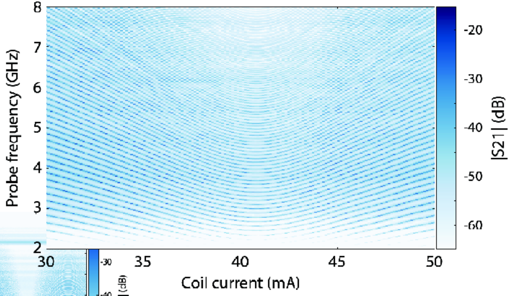


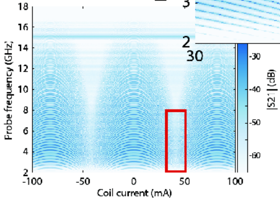
$T = 20$ mK $\Leftrightarrow f_{\text{thermal}} = \frac{k_B T}{h} = 0.4$ GHz $\ll f_{\text{modes}}$: quantum limit

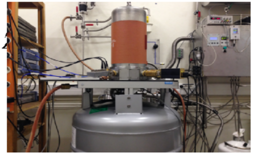
Velocity from dispersion relation : $v \simeq 6.10^6$m/s $= c/50$ (slow light)

Tuning the modes

<u>Chain of SQUIDS :</u> $L_J(\phi)$ is flux dependent







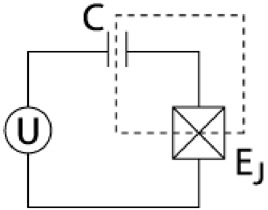
Very clean signal : disorder is quite weak

Superconducting circuitry : emitter

Artificial atom engineering

Cooper pair box : gate-tuned Josephson grain

- For $(2e)^2/C \gg E_J$, charge is locked
  $\Rightarrow$ strong phase fluctuations
- Maximize non-linear effects, but not optimal w.r.t. noise



On the board : charge qubit

On the board : charge qubit

At charge degeneracy point :

$$H = \frac{(2e)^2}{2C}\left[\hat{N} - N_g\right]^2 - E_J \cos\hat{\Phi}$$

$$\Rightarrow H = -E_J \frac{\sigma_x}{2}$$

Two-level system :

$$|g\rangle = \frac{|\uparrow\rangle + |\uparrow\rangle}{\sqrt{2}}$$

$$|e\rangle = \frac{|\uparrow\rangle - |\uparrow\rangle}{\sqrt{2}}$$

On the board : effective Hamiltonian

On the board : effective Hamiltonian

Spin boson model : [Leggett *et al.* RMP (1987)]

$$H = \frac{E_{JQ}}{2}\sigma_x - \sigma_z \sum_k \frac{g_k}{2}(a_k^\dagger + a_k) + \sum_k \omega_k a_k^\dagger a_k$$

Detailed derivation in waveguide QED :

Double dot setup : [LeHur, PRB (2012) ; Goldstein *et al.* PRL (2013)]

Cooper pair box : [Snyman, Florens, PRB (2015)]

Spectral density : charge $\hat{n}_0 \sim \sum_k g_k(a_k^\dagger + a_k)$

$J(\omega) = \mathrm{Im}\langle \hat{n}_0(\omega)\hat{n}_0(-\omega)\rangle = \pi \sum_k g_k^2 \delta(\omega - \omega_k) = 2\pi\alpha\omega e^{-\omega/\omega_P}$

Interaction strength : effective fine structure constant

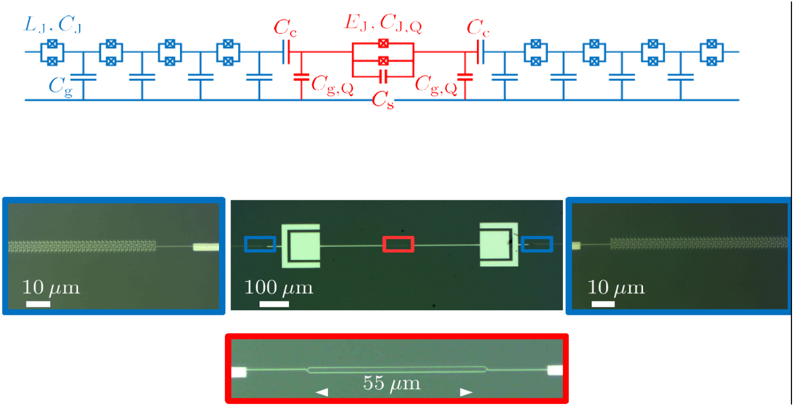
$$\alpha = 2\left(\frac{C_{cQ}}{C_{cQ} + C_{gQ} + 2C_{JQ}}\right)^2 \frac{Z_{\mathrm{chain}}}{R_Q} \;\; \text{with} \;\; R_Q = \frac{(2e)^2}{h} \simeq 6\mathrm{k}\Omega$$

α can reach values of order 1

Preliminary experimental results (NEEL)

Inline qubit coupling :

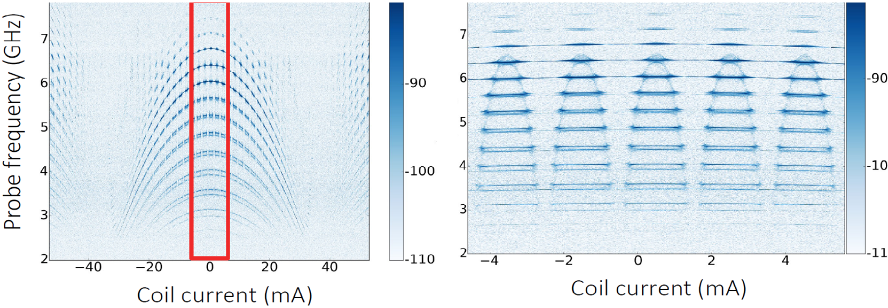
Qubit in regime between CPB (too noisy) and transmon (too linear)



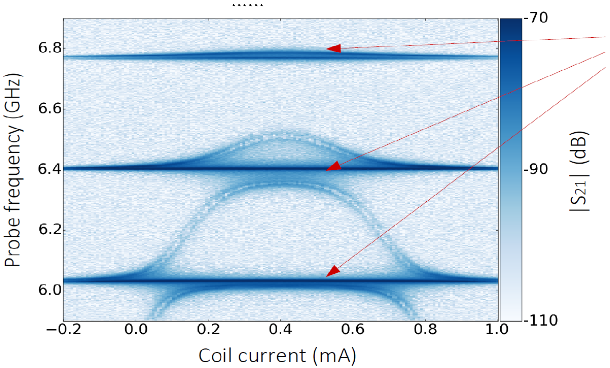
[Nicolas Roch *et al.*, unpublished]

Preliminary experimental results

Inline coupled qubit :



Multimode ultra-strong coupling



Multimode coupling :
At least three modes coupled to the qubit

Here $\alpha < 0.1$ only, but already a premice of physics beyond standard quantum optics

On the menu of the next lecture

- Scattering theory with a dynamical degree of freedom
- How to cut a photon into 3 pieces
- Renormalization from many-body entanglement
- Many-body inelastic scattering theory using coherent states