

Outline of the lectures

- I. Quantum theory of transport (2 hours)
- II. Weak localization (1 hour)
- III. Coherent Back-Scattering (CBS) (1.5 hours)
- IV. Anderson (strong) localization – Scaling theory (2 hours)
- V. Self-consistent theory of localization (1 hour)

Dominique Delande

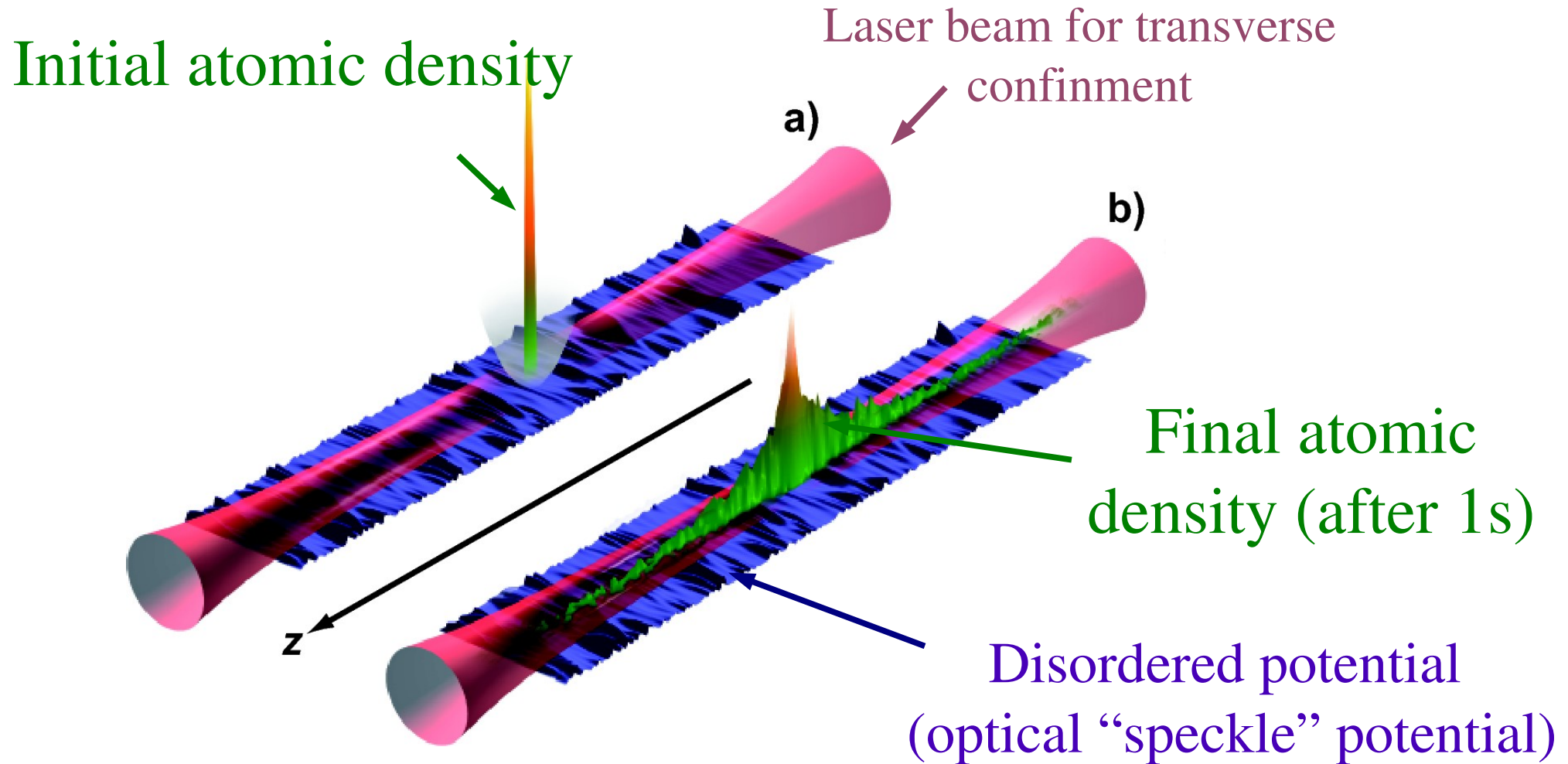
Laboratoire Kastler-Brossel

Ecole Normale Supérieure, Université Pierre et Marie Curie,
Collège de France (Paris)

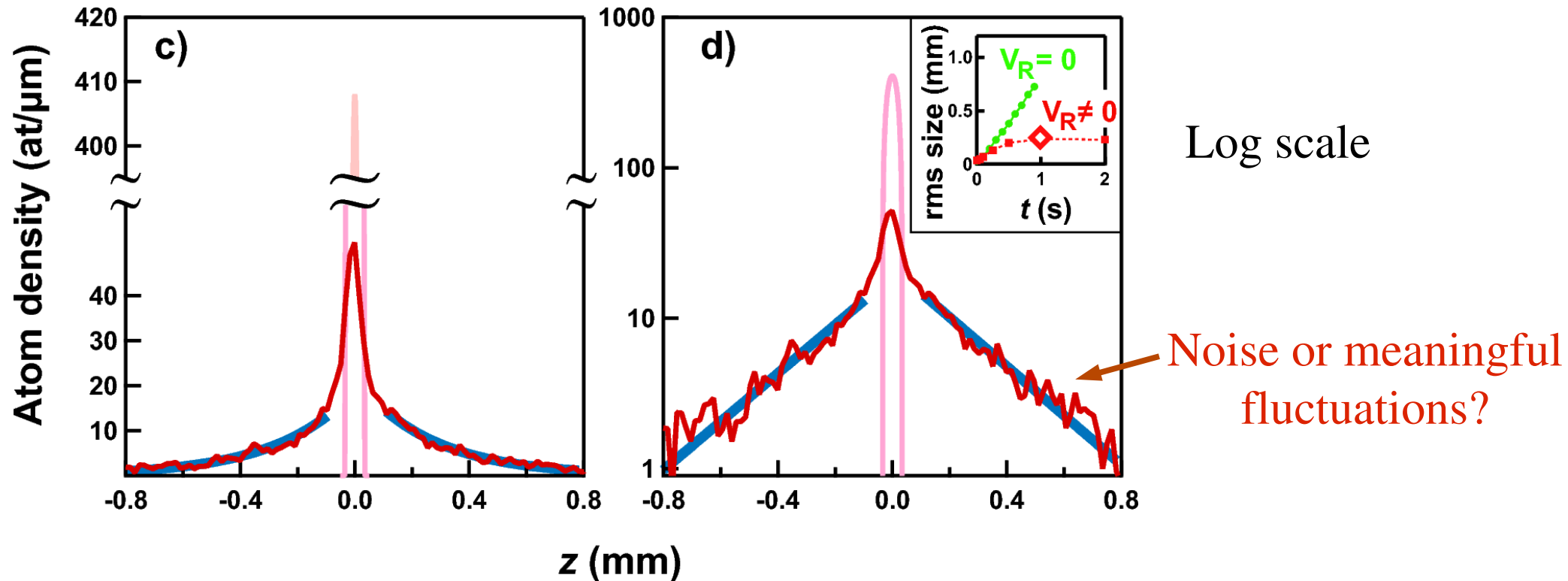
<http://www.lkb.upmc.fr/complexquantumsystems/>

Dominique.Delande@lkb.upmc.fr

Experiment on localization of atomic matter waves

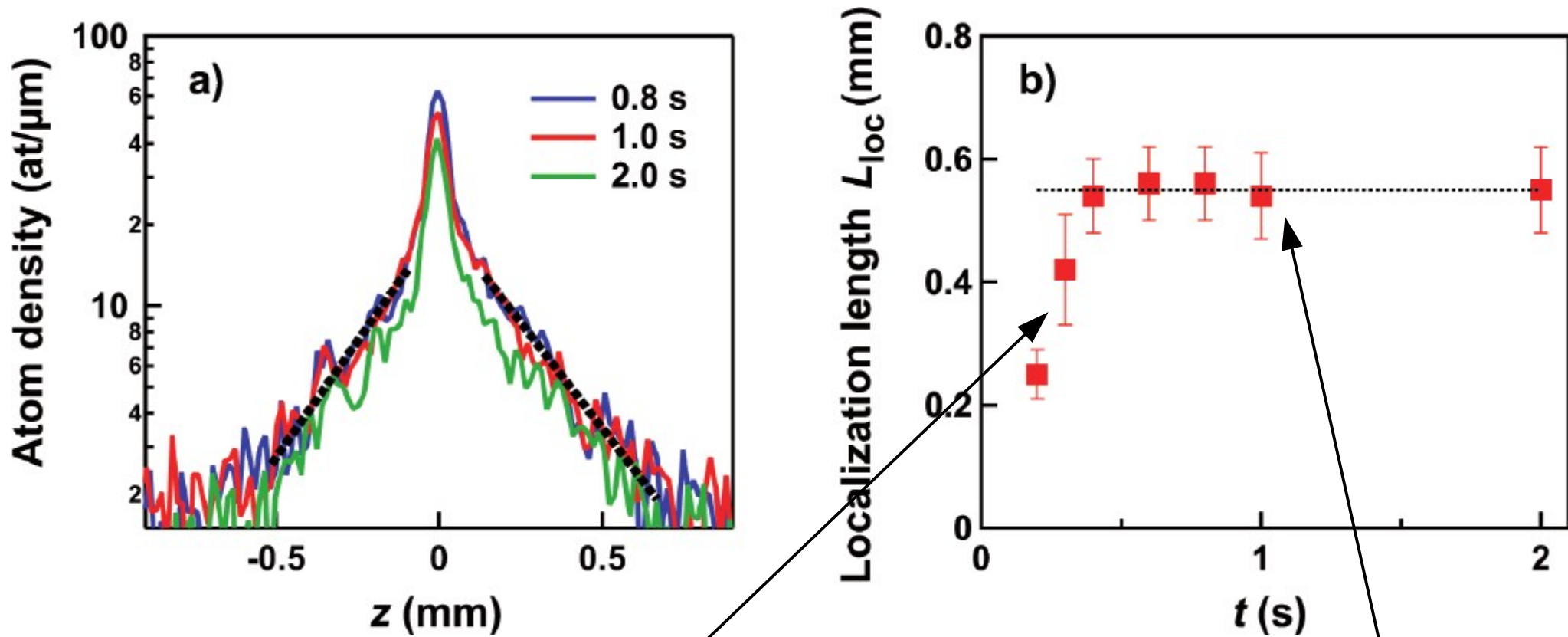


Experiment on localization of atomic matter waves



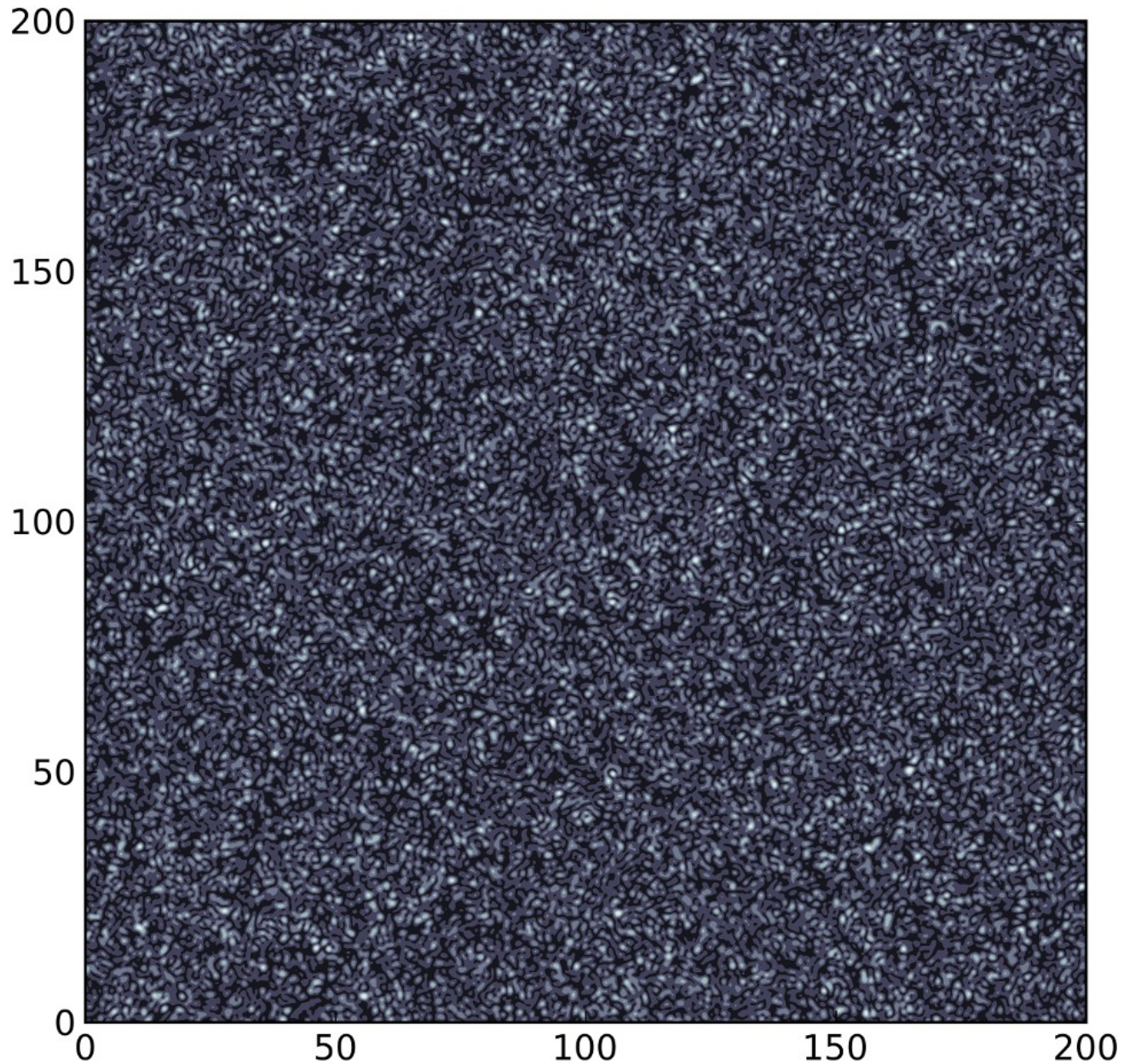
The final atomic density shows exponential localization with localization length of few $100\mu\text{m}$!

Experiment on localization of atomic matter waves



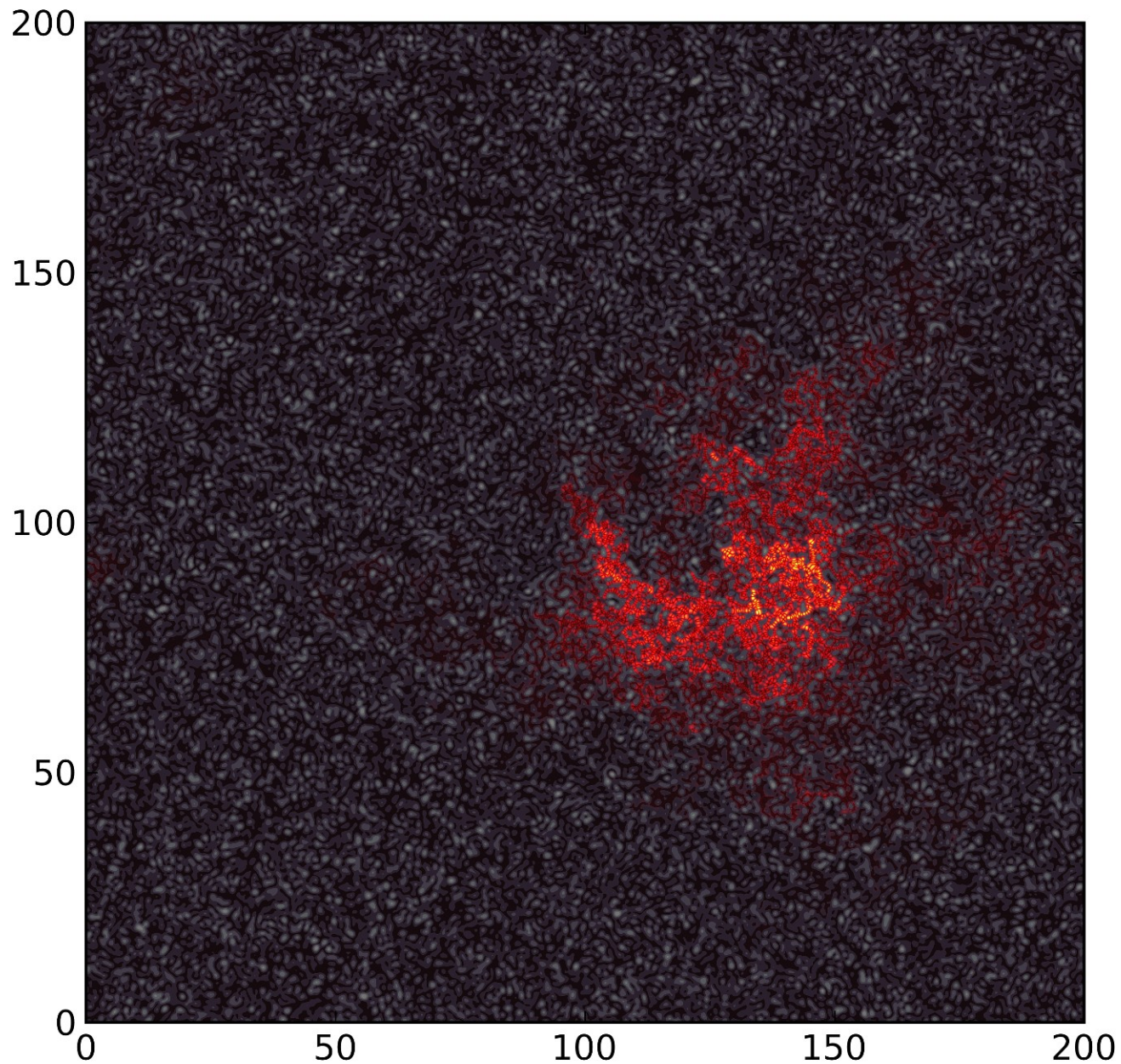
Short time expansion (diffusion?) followed by localization

Disordered potential in 2D



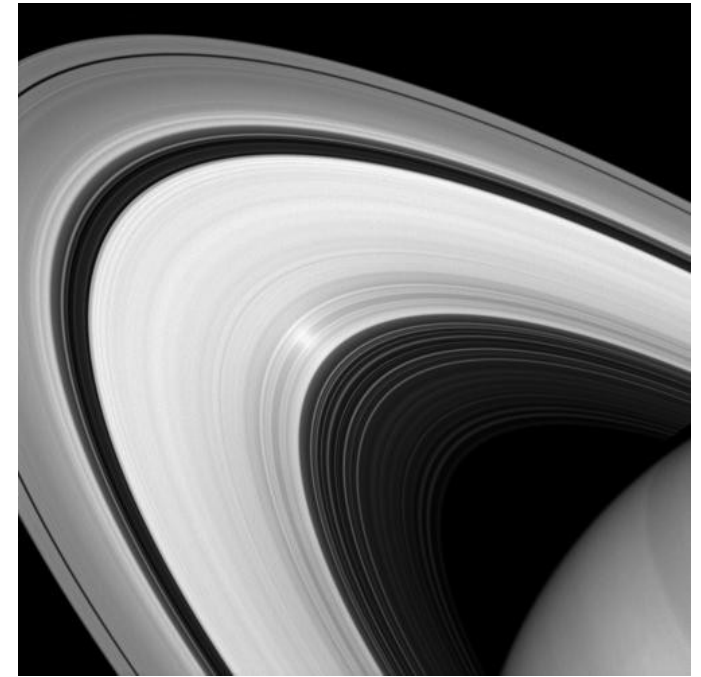
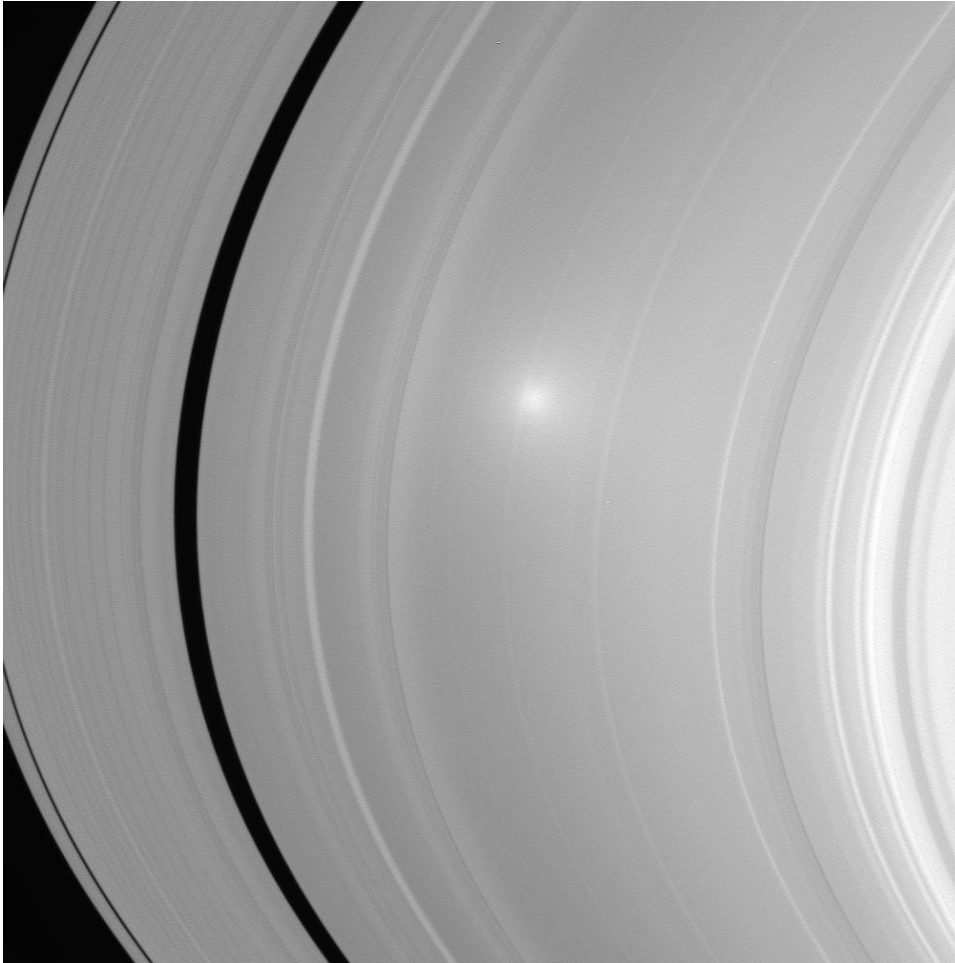
How do eigenstates look like?

Disordered potential in 2D



A typical localized eigenstate

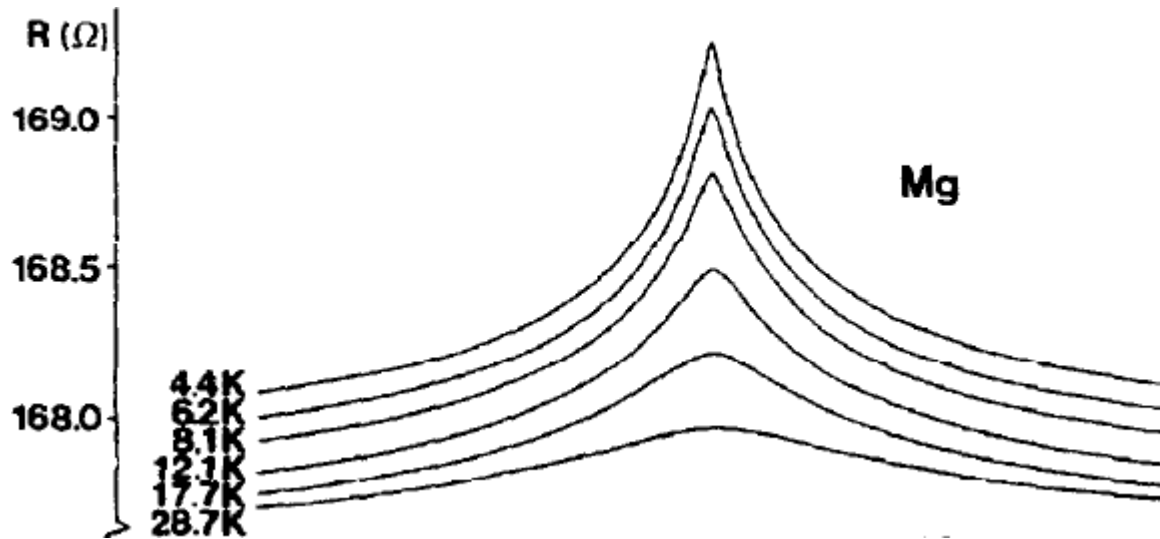
Opposition Effect on the Rings of Saturn



January, 20, 2014

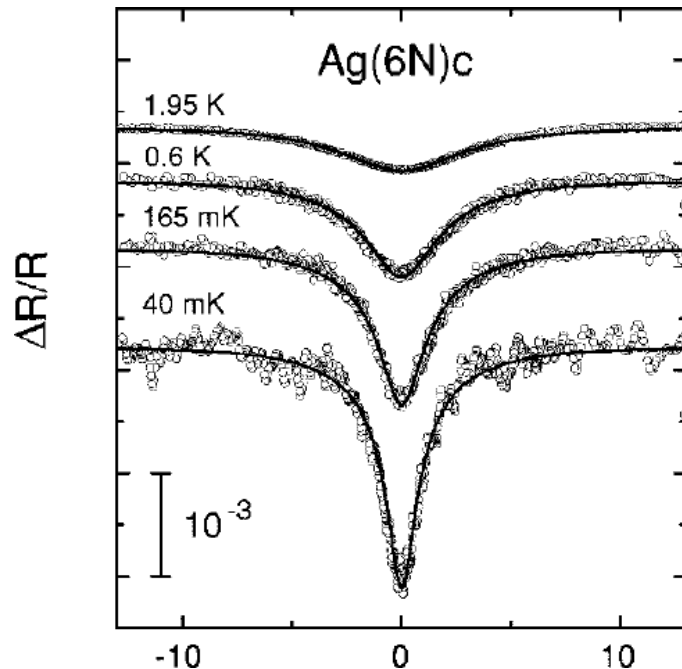
Observations by the Cassini mission, see <http://saturn.jpl.nasa.gov/>

Magneto-resistance of thin metallic films



M. Janssen, Phys. Rep. 295, 1 (1998)

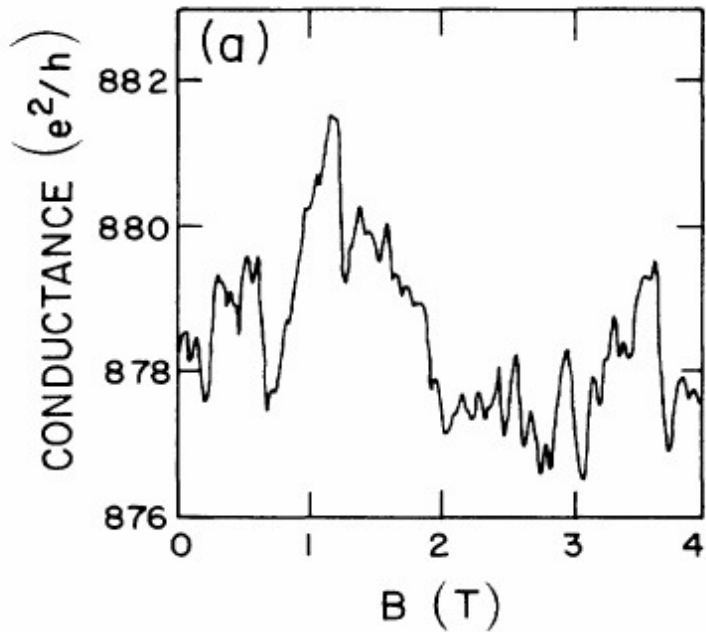
Magnetic field (few Teslas)



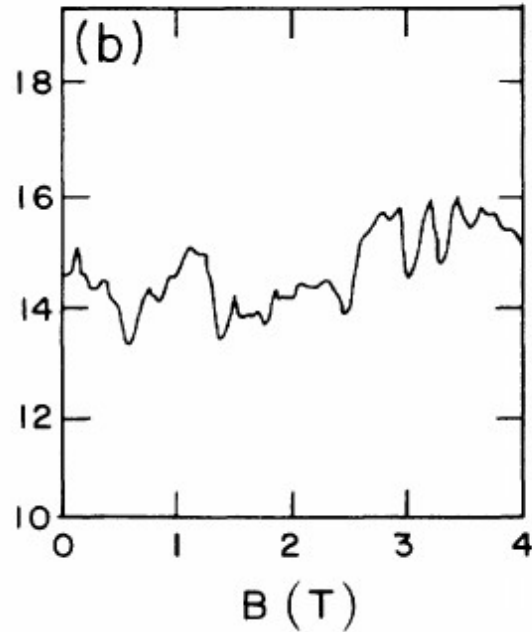
F. Pierre et al, PRB, 68, 085413 (2003)

Magnetic field (few Teslas)

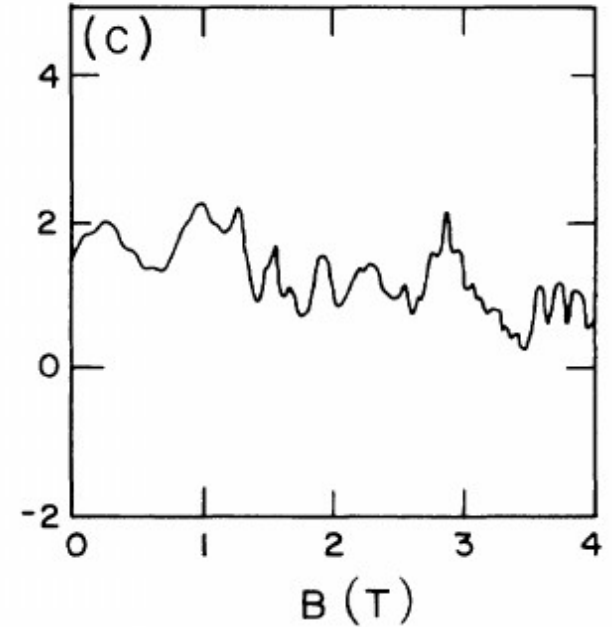
Universal conductance fluctuations



Gold ring



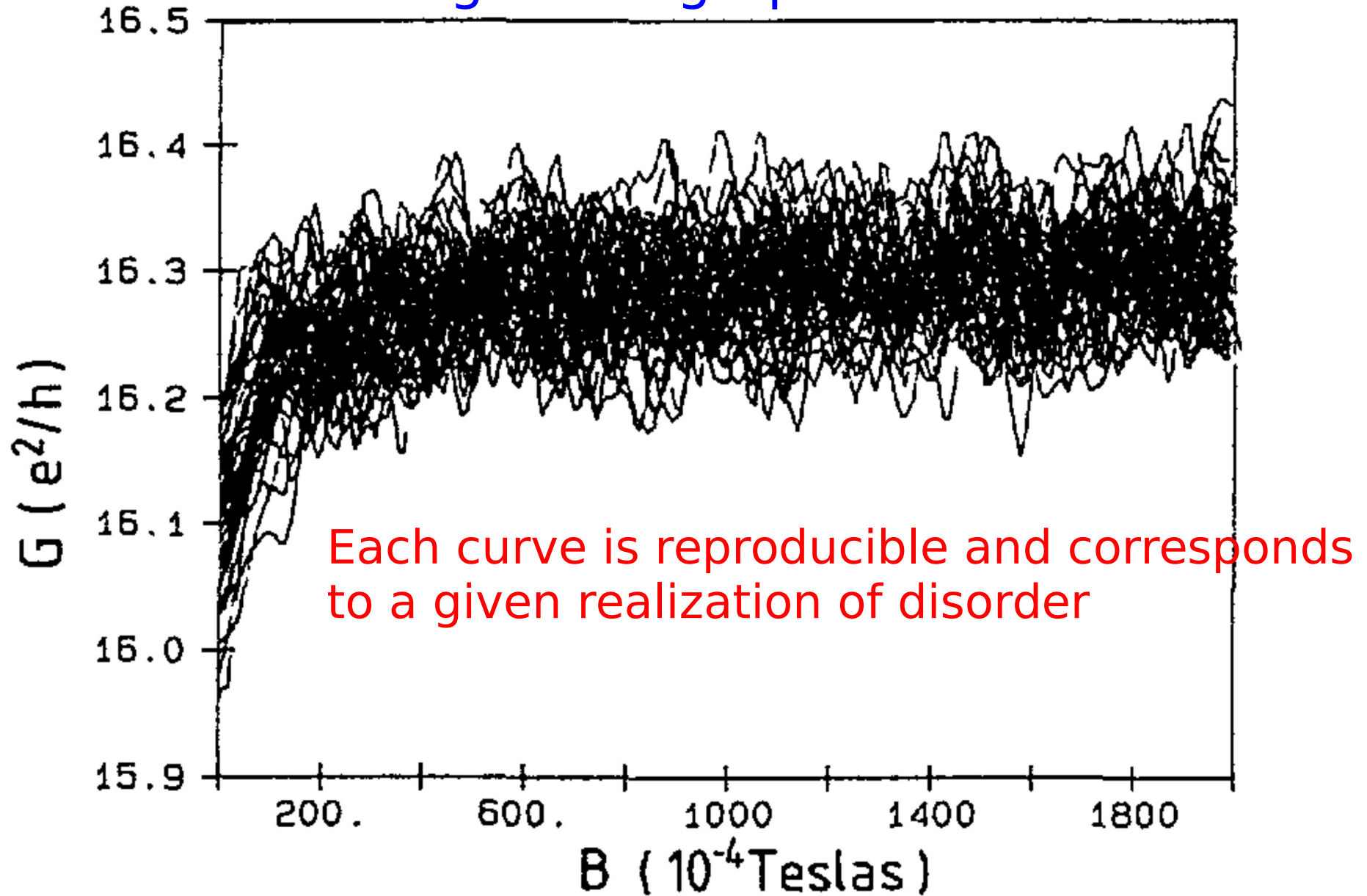
Quasi-1D
Silicon MOSFET



Numerics of the
Anderson model

Lee et al, PRB 35, 1039 (1987)

Magnetofingerprint



D. Mailly and M. Sanquer, J. Phys. I France 2, 357 (1992)

Some orders of magnitude

Electrons (in Gold)

$$k_F^{-1} = 0.085 \text{ nm}$$

$$m^* = 1.1 m_e$$

$$v_F = 1.4 \cdot 10^6 \text{ ms}^{-1}$$

$$\ell_e = 30 \text{ nm up to } 1 \mu\text{m}$$

$$k_F \ell_e = 360$$

$$\tau = 20 \text{ fs}$$

meV-eV, THz, 1000 K

Atoms (Rubidium 87)

$$\lambda_L = 780 \text{ nm}$$

$$k_L = \frac{2\pi}{\lambda_L} = 8 \cdot 10^6 \text{ m}^{-1}$$

$$v_R = \frac{\hbar k_L}{m} = 5.9 \text{ mm/s}$$

$$E_R = \frac{\hbar^2 k_L^2}{2m} = 15 \text{ peV}$$

$$\nu_R = \frac{E_R}{h} = 3.8 \text{ kHz}$$

$$T_R = \frac{2E_R}{k_B} = 360 \text{ nK}$$

$$\Gamma/2\pi = 6 \text{ MHz}$$

$$\text{Lifetime} = 27 \text{ ns}$$

peV-neV, kHz, nK- μ K

Conventions

$$f(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$$

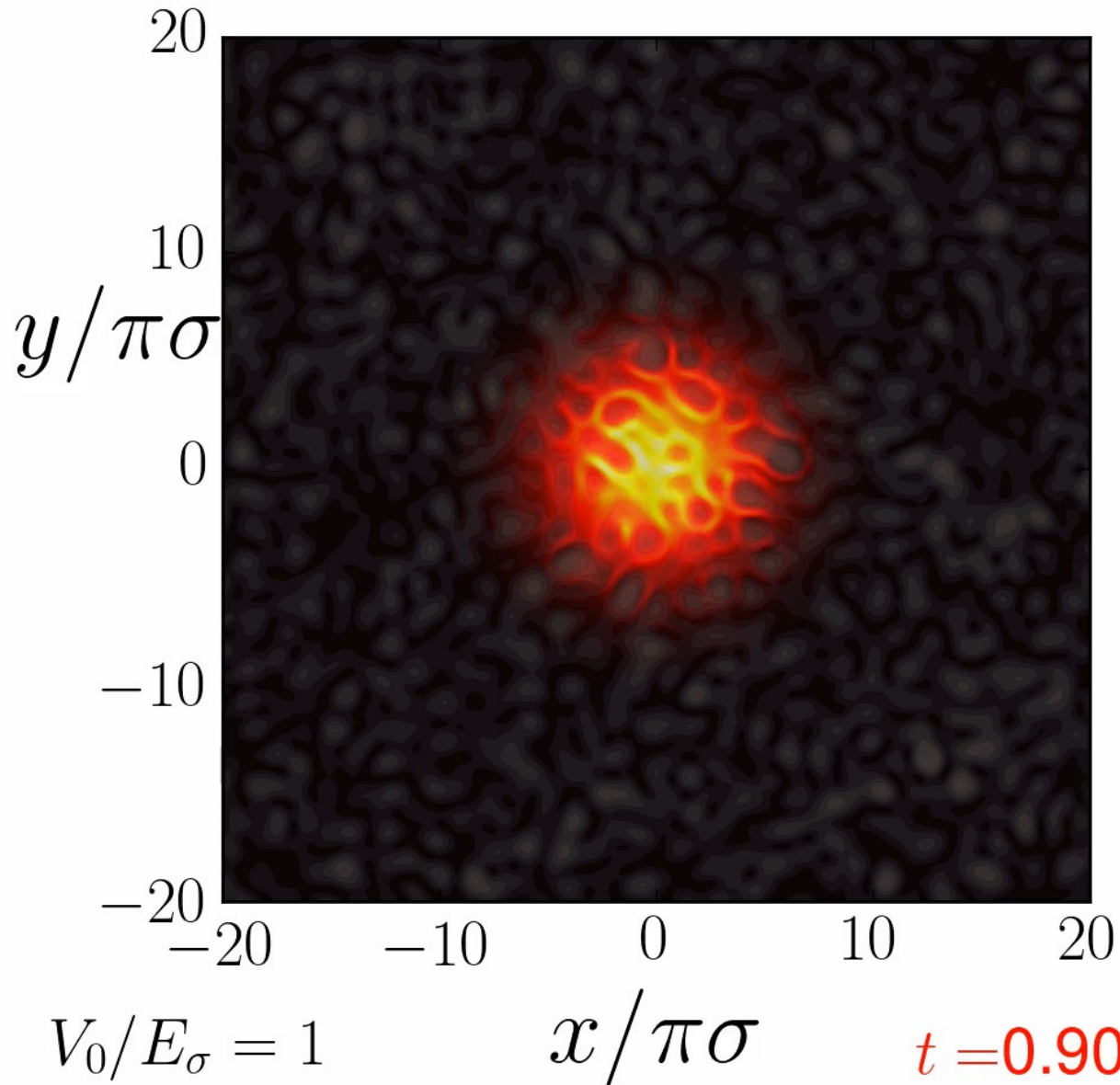
$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\omega) e^{-i\omega t} d\omega$$

$$f(\mathbf{q}) = \int f(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}} d^d \mathbf{r}$$

$$f(\mathbf{r}) = \int f(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}} \frac{d^d \mathbf{q}}{(2\pi)^d}$$

Short-time dynamics of a Gaussian wavepacket

- In the presence of a moderate speckle disorder

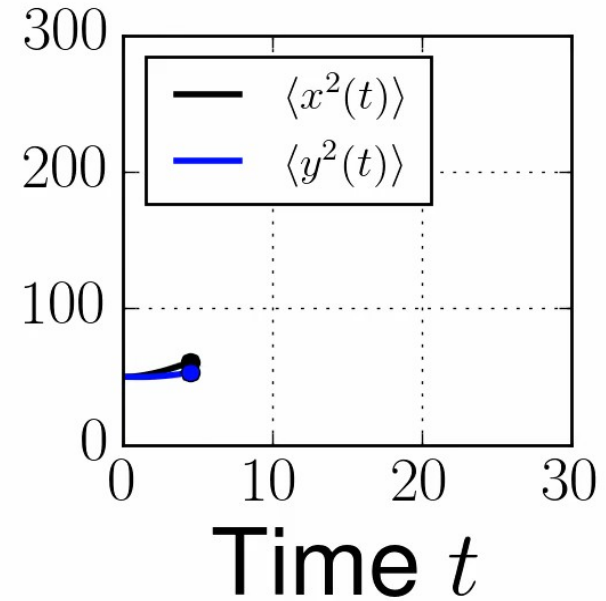
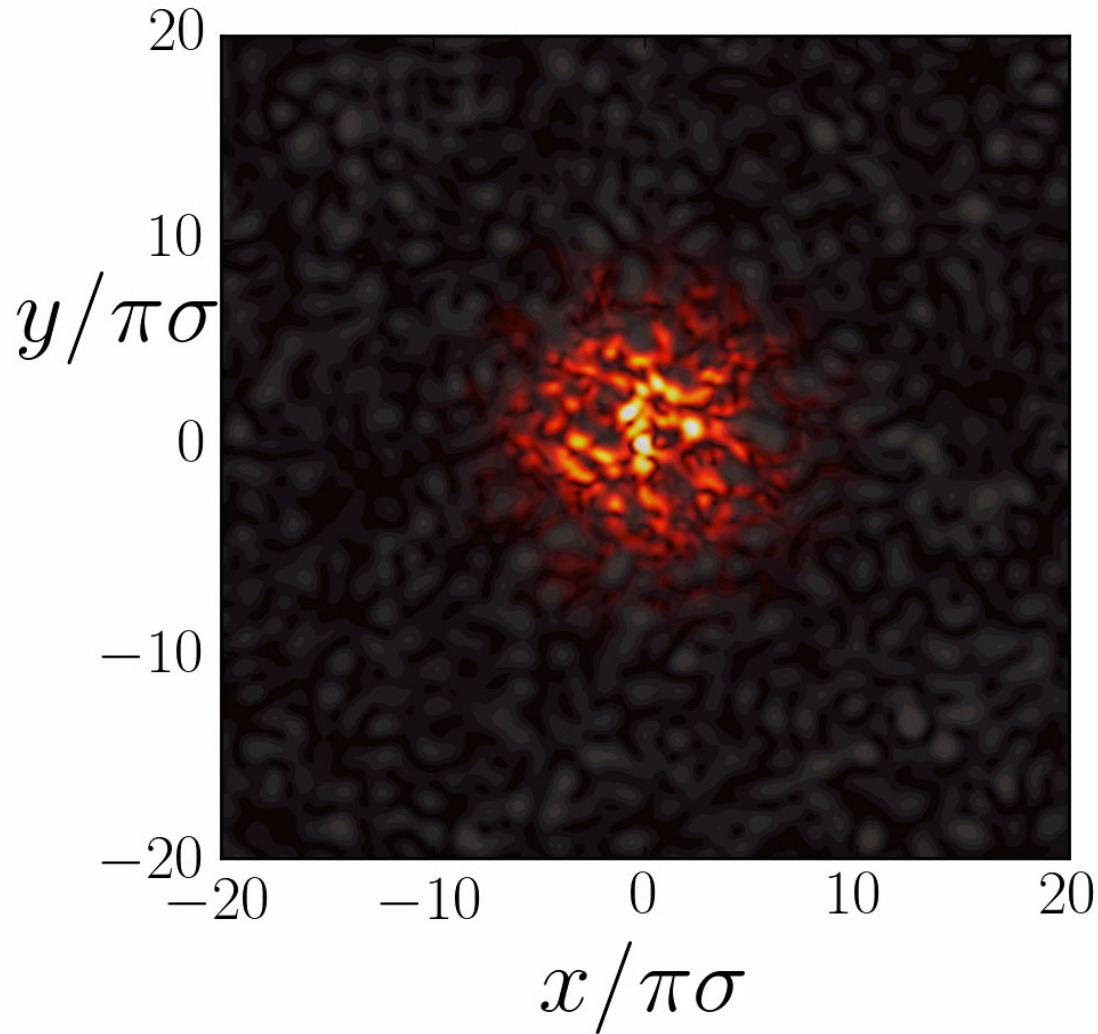


1. Very short time: The atoms fall into the potential minima and convert potential energy into kinetic energy

2. The atomic matter wave is later scattered by the potential hills

Movie in file [wavepacket_propagation_short_time.mp4](#)

Beyond the single scattering time



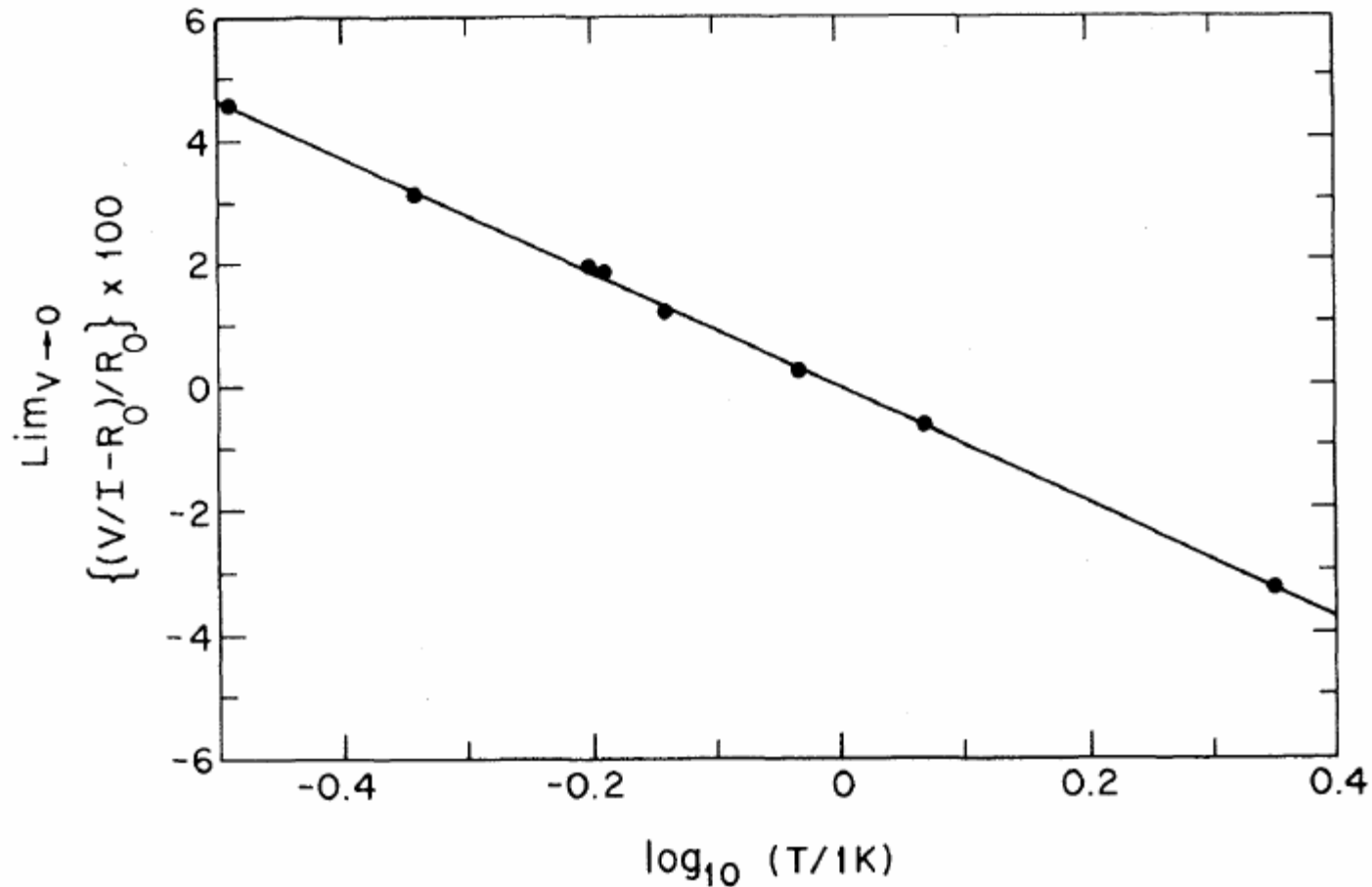
$$V_0/E_\sigma = 1$$

$$t = 4.5$$

Diffusive motion: $\langle r^2(t) \rangle \propto Dt$

Movie in file [wavepacket_propagation_diffusive.mp4](#)

Resistance of a PdAu film vs. temperature

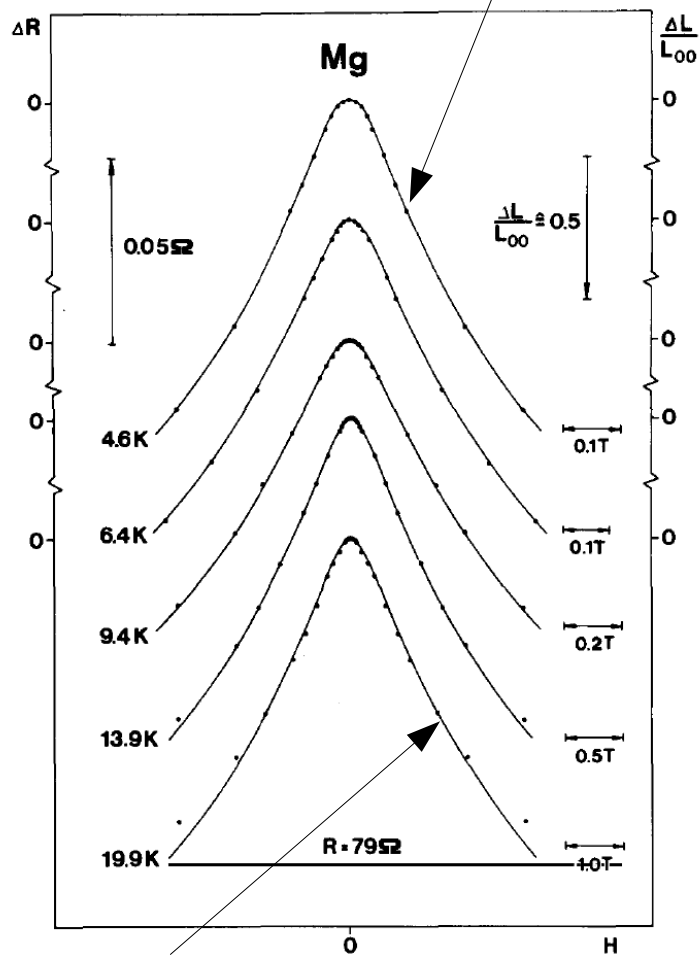


Logarithmic dependance vs. temperature
(weak localization limited by dephasing)

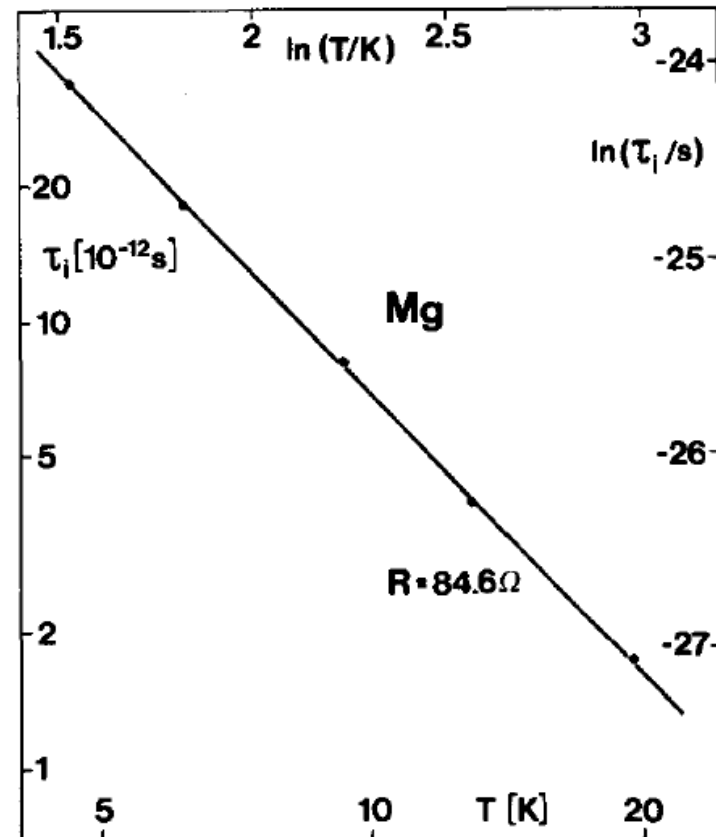
G. Dolan et al. PRL 43, 721 (1979)

Magnetoresistance of a Mg film

Experimental data



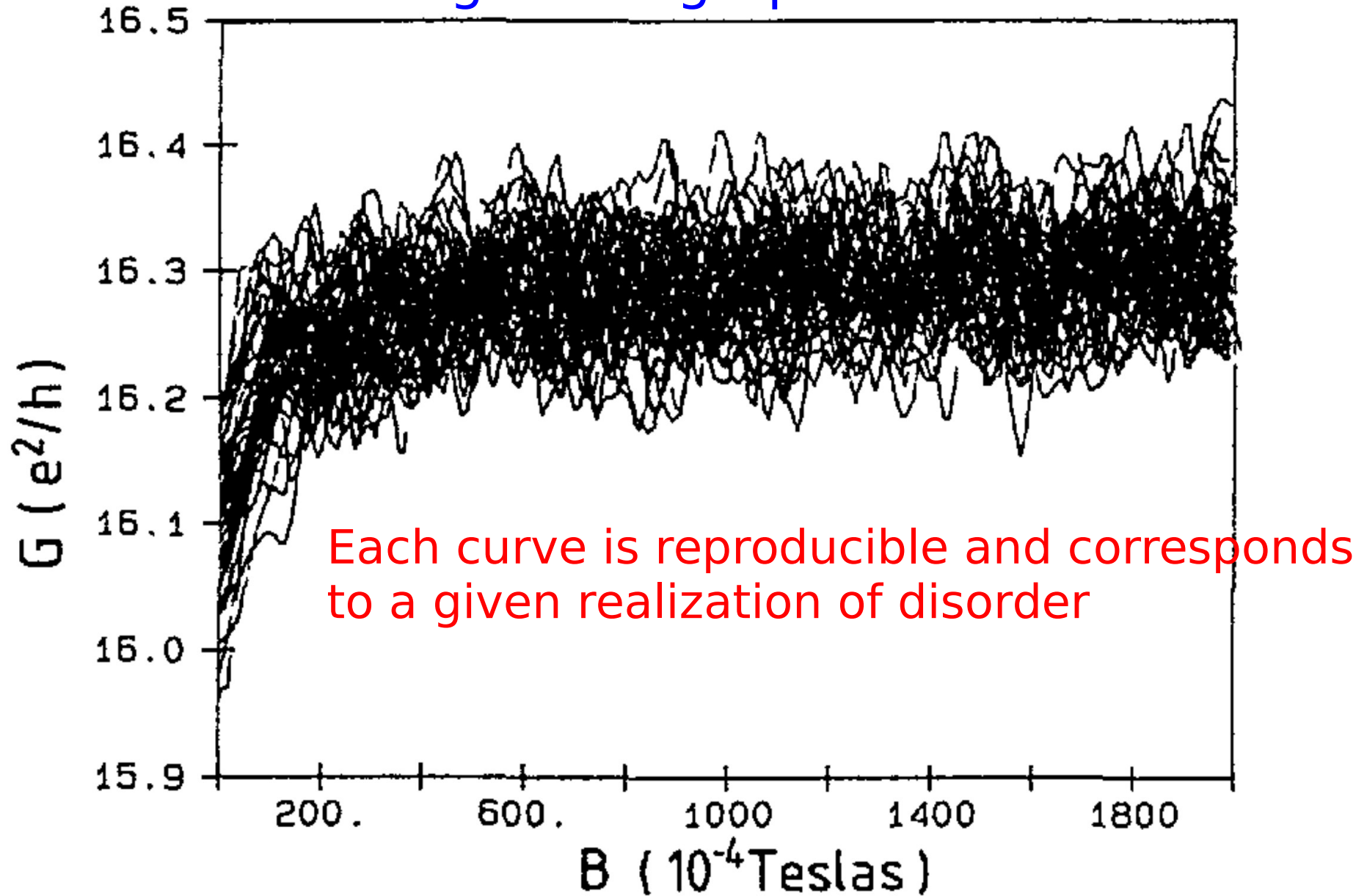
Fit



Dependence of phase coherence time with temperature $1/T^2$

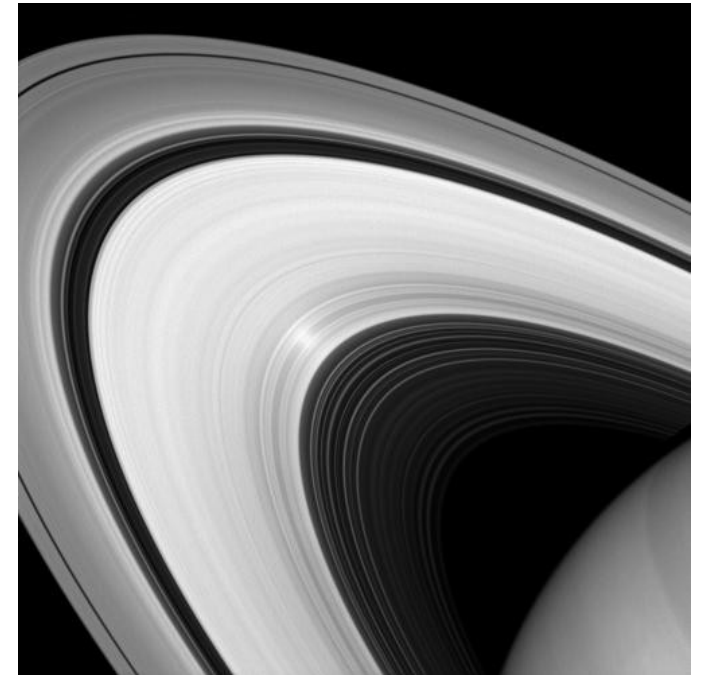
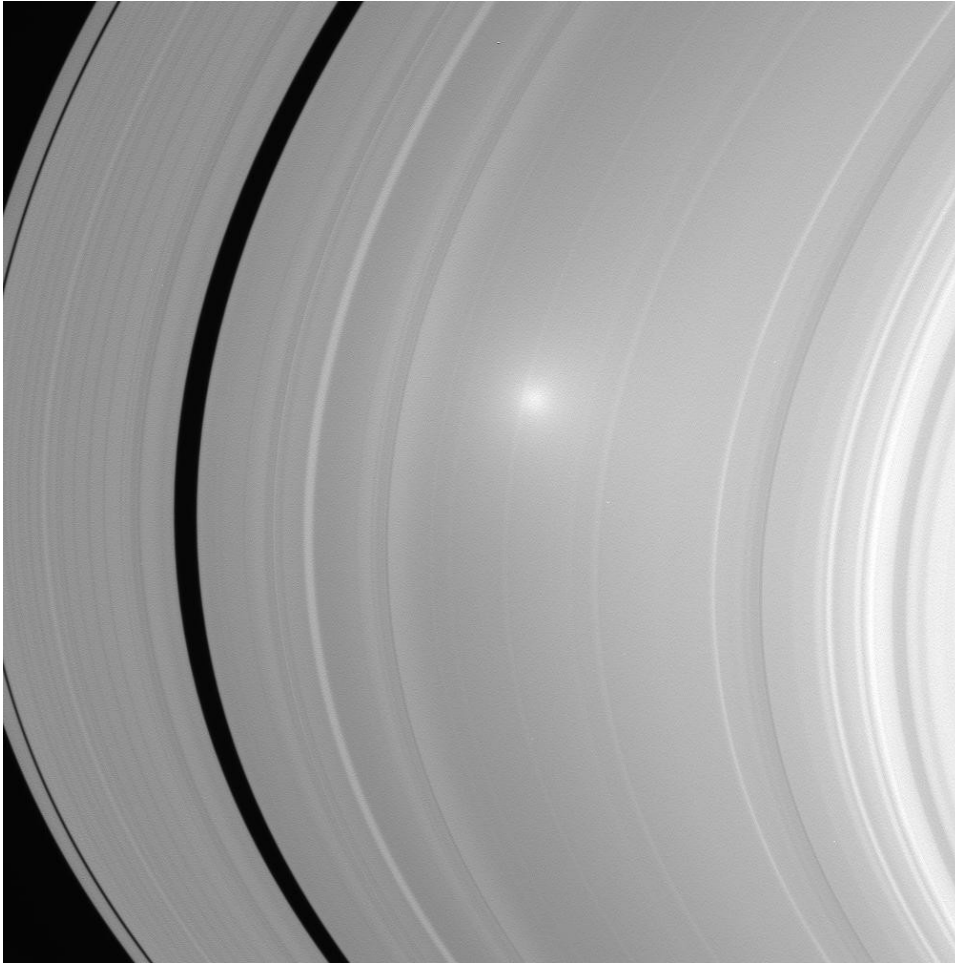
G. Bergmann, Phys. Rep. 107, 1 (1984)

Magnetofingerprint



D. Mailly and M. Sanquer, J. Phys. I France 2, 357 (1992)

Opposition Effect on the Rings of Saturn

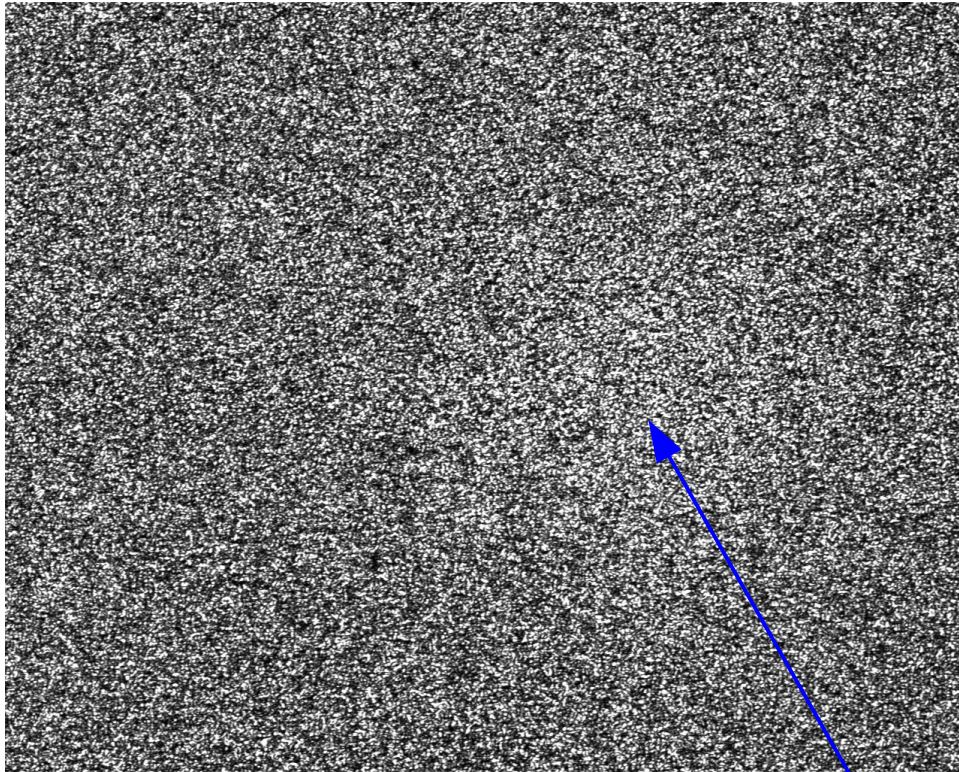


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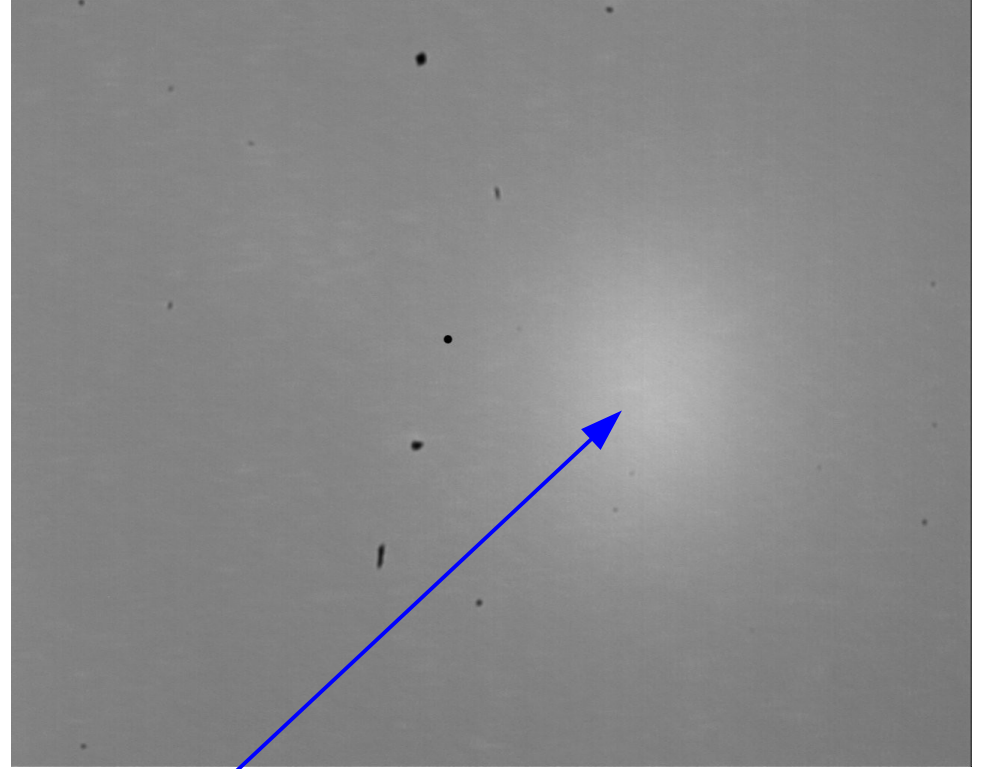
Observations by the Cassini mission, see <http://saturn.jpl.nasa.gov/>

Simple experimental observation of CBS

Red laser sent on a piece of paper, intensity scattered close to the back-scattering direction (± 20 mrad)



Single configuration: speckle
created by random interference



Averaged over random
configurations

Back-scattering direction

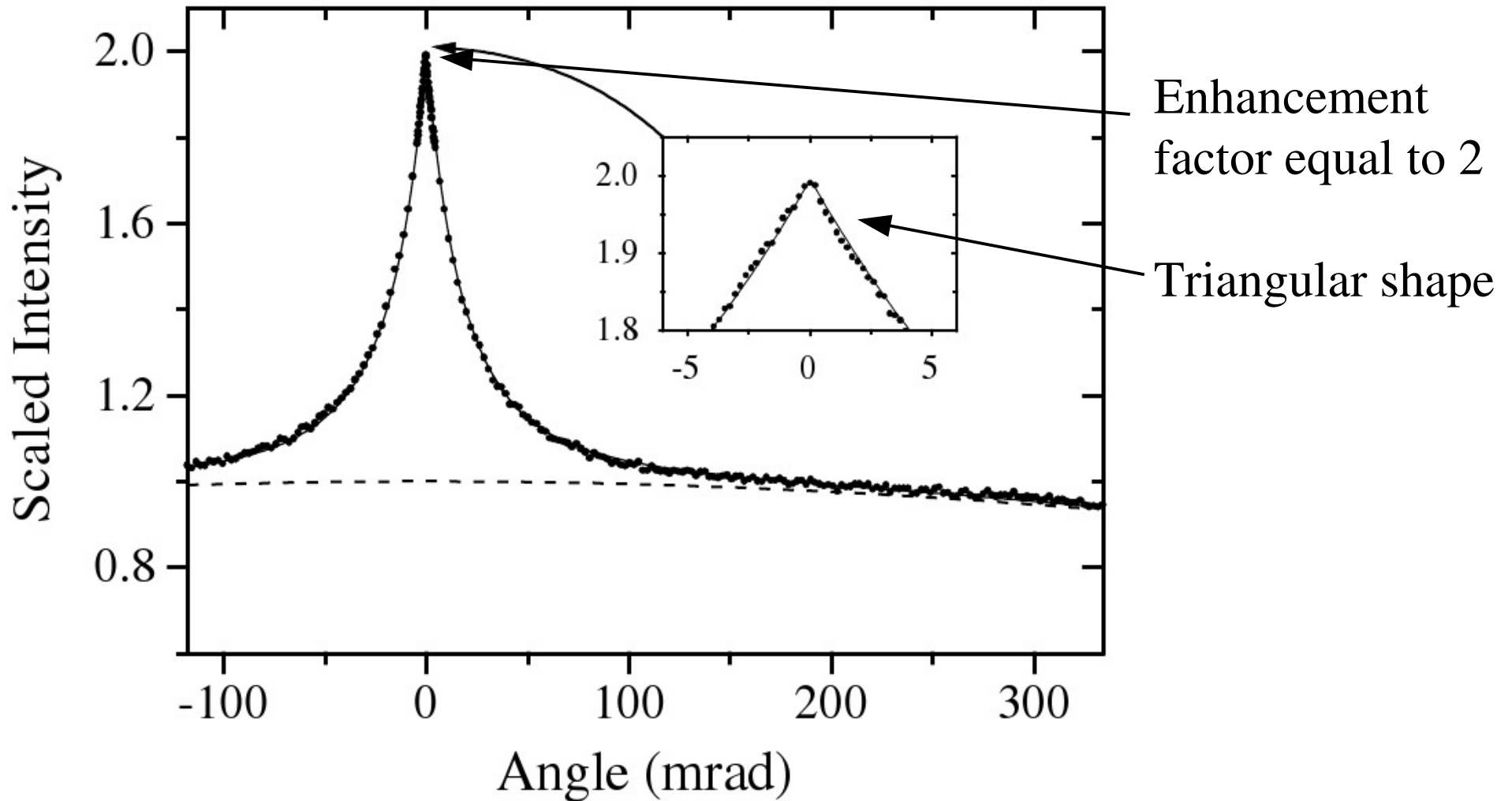
Live experiment on CBS

Experiment built by V. Josse and V. Volchkov
(Institut d'Optique, Palaiseau)

See video on

<https://www.youtube.com/watch?v=Uh-bLRkXL40>

CBS of light (more sophisticated setup)



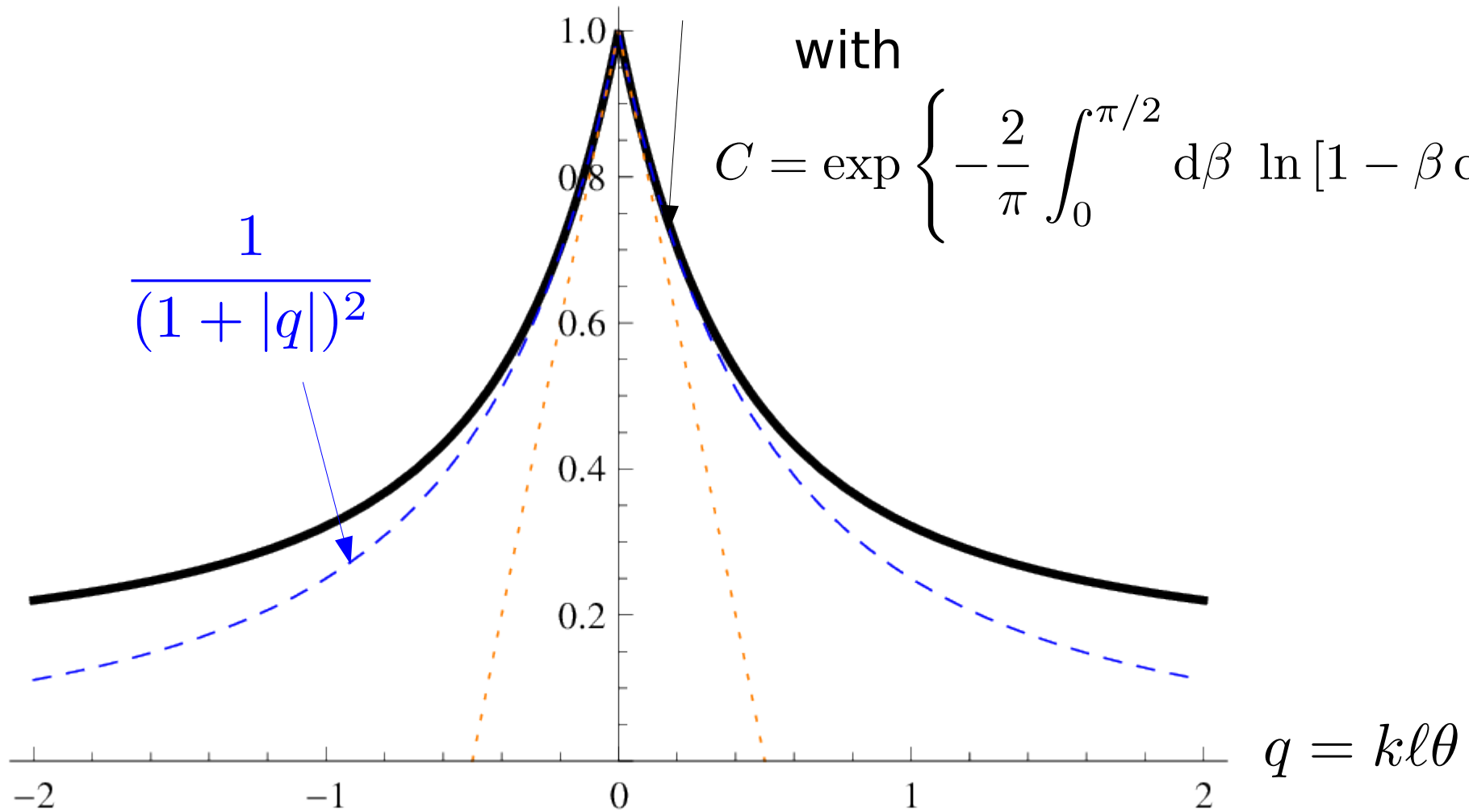
D. S. Wiersma et al, PRL 74, 4193 (1995)

Shape of the CBS peak: $\frac{I_c(\theta)}{I_L}$

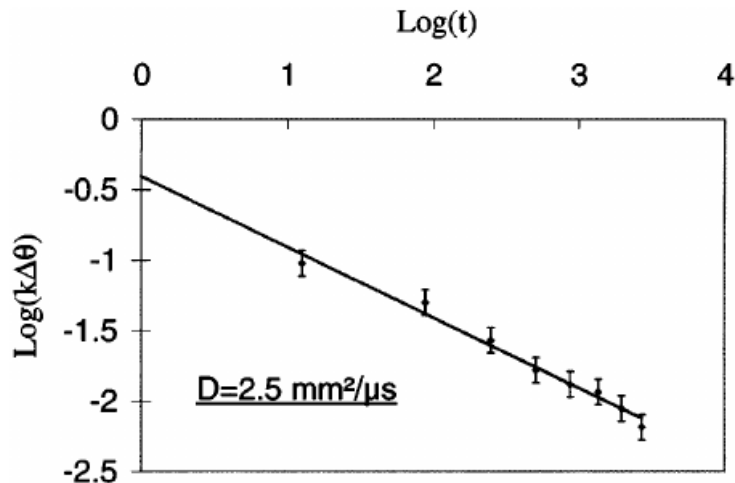
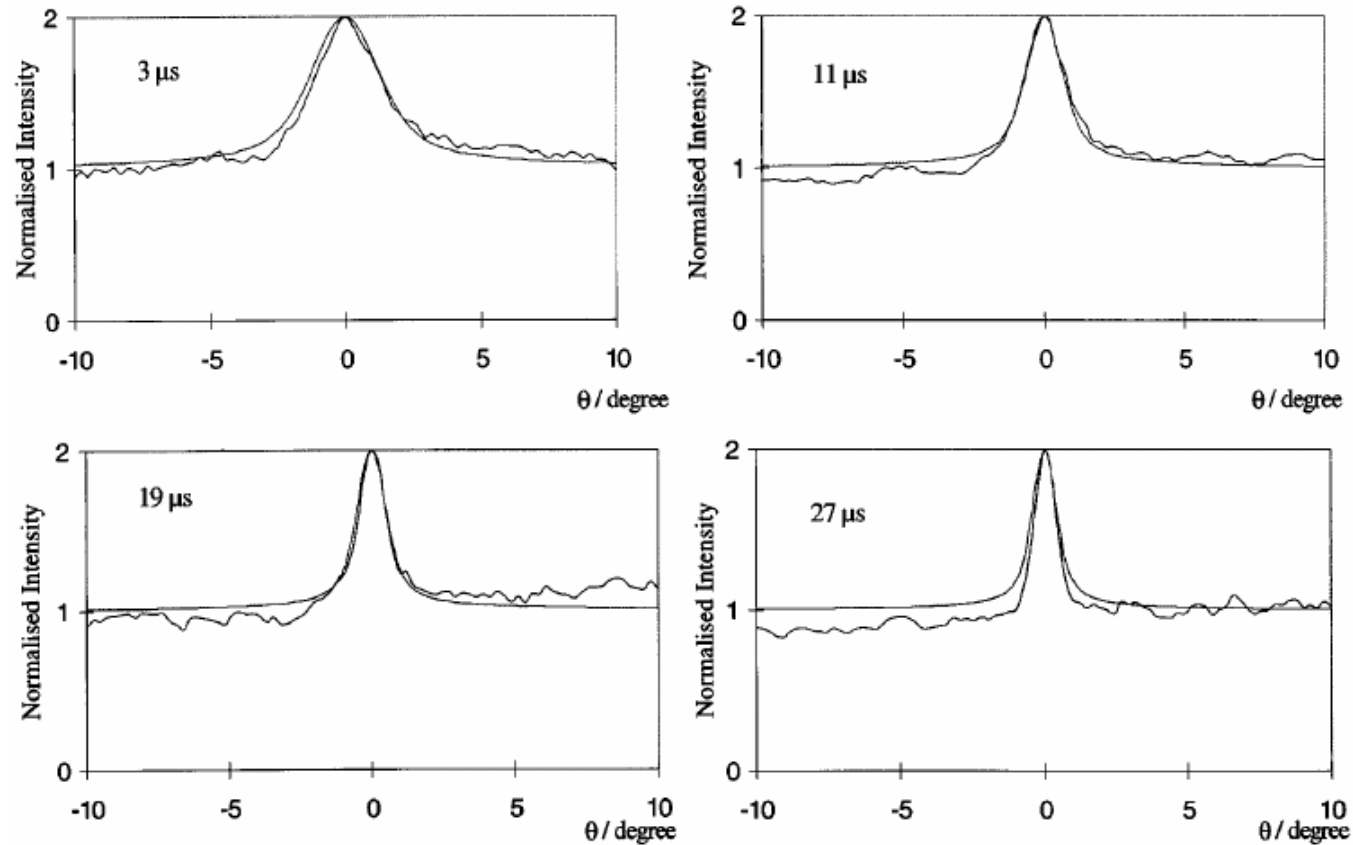
Exact result: $\frac{1}{C} \exp \left\{ -\frac{2}{\pi} \int_0^{\pi/2} d\beta \ln \left[1 - \frac{\arctan \sqrt{q^2 + \tan^2 \beta}}{\sqrt{q^2 + \tan^2 \beta}} \right] \right\}$

with

$$C = \exp \left\{ -\frac{2}{\pi} \int_0^{\pi/2} d\beta \ln [1 - \beta \cot \beta] \right\}$$



CBS of acoustic waves



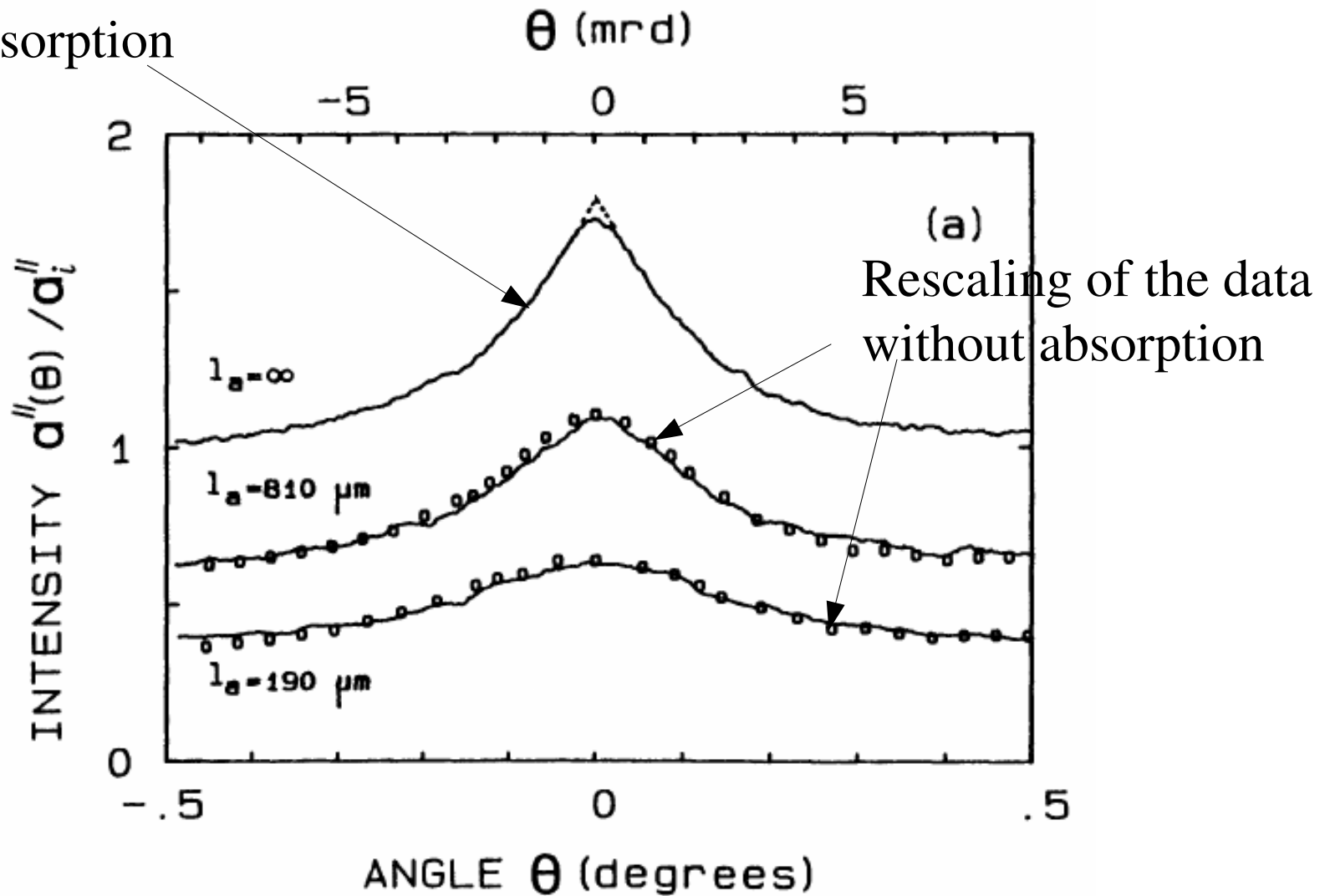
Narrowing of the CBS peak with time

A. Tourin et al, PRL 79, 3637 (1997)

FIG. 4. Cone width versus time. Slope of the fit: 0.49.

Effect of absorption on CBS

Without absorption



P. Wolf et al, J. Phys. France, 49, 63 (1998)

Practical use of CBS

SCIENTIFIC
REPORTS



OPEN

SUBJECT AREAS:
IMAGING AND SENSING
BIOPHYSICS

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4 September 2014

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5 November 2014

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Correspondence and
requests for materials

Ultrasensitive and fast detection of denaturation of milk by Coherent backscattering of light

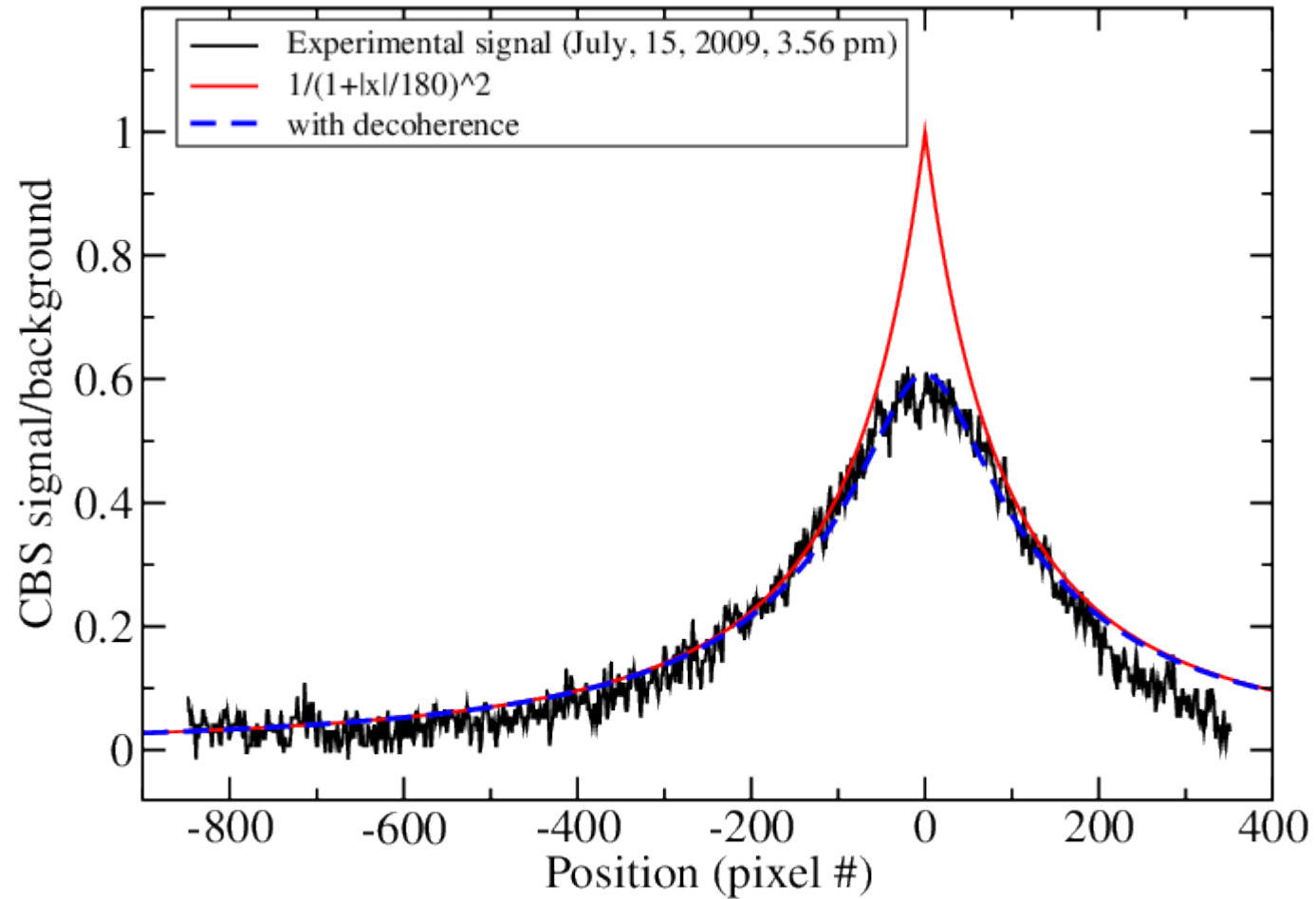
Manish Verma^{1,2}, Dilip K. Singh¹, P. Senthilkumaran², Joby Joseph² & H. C. Kandpal¹

¹Quantum Optics and Photon Physics, CSIR-National Physical Laboratory, New Delhi-110012, ²Department of Physics, Indian Institute of Technology Delhi, New Delhi-110016.

In this work, Coherence backscattering (CBS) of light has been used to detect the onset of denaturation of milk. The CBS cone shape and its enhancement factor are found to be highly sensitive to the physical state of the milk particles. The onset of denaturing of milk not visible to the naked eye, can be easily detected from changes in the CBS cone shape. The onset of denaturation is confirmed by spectral changes in Raman spectra from these milk samples. Further, the possibility to estimate the dilution of milk by water as an adulterant is demonstrated. The method reported has a broad scope in industry for making an inline ultrafast cost effective sensor for milk quality monitoring during production and before consumption.

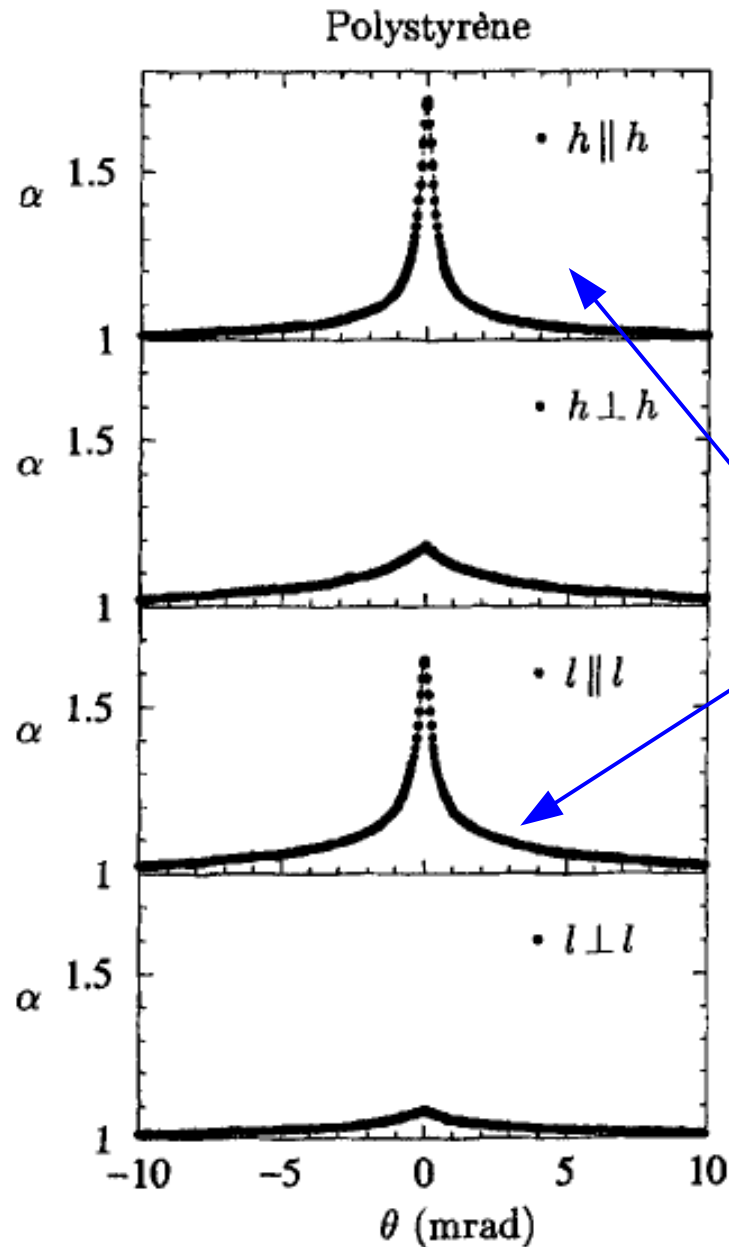
Scientific Reports 4, 7257 (2014)

Simple experimental observation of CBS



$$\tau_\phi \approx 38\tau$$

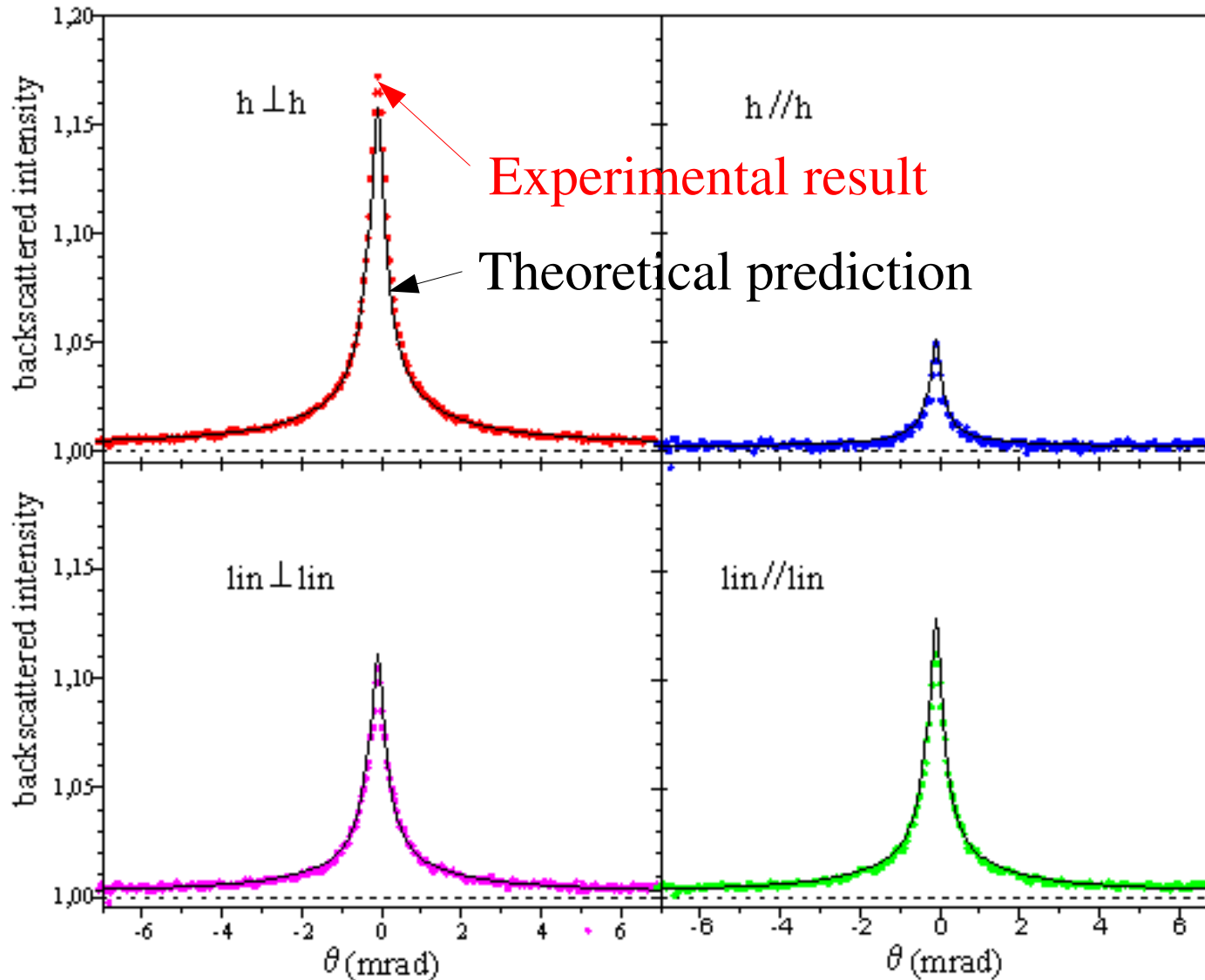
Influence of the polarization on CBS



Enhancement factor varies between 1.1 and 2, depending on the polarization channel

Large enhancement factor in the “parallel” channels

CBS on a cold Rubidium gas

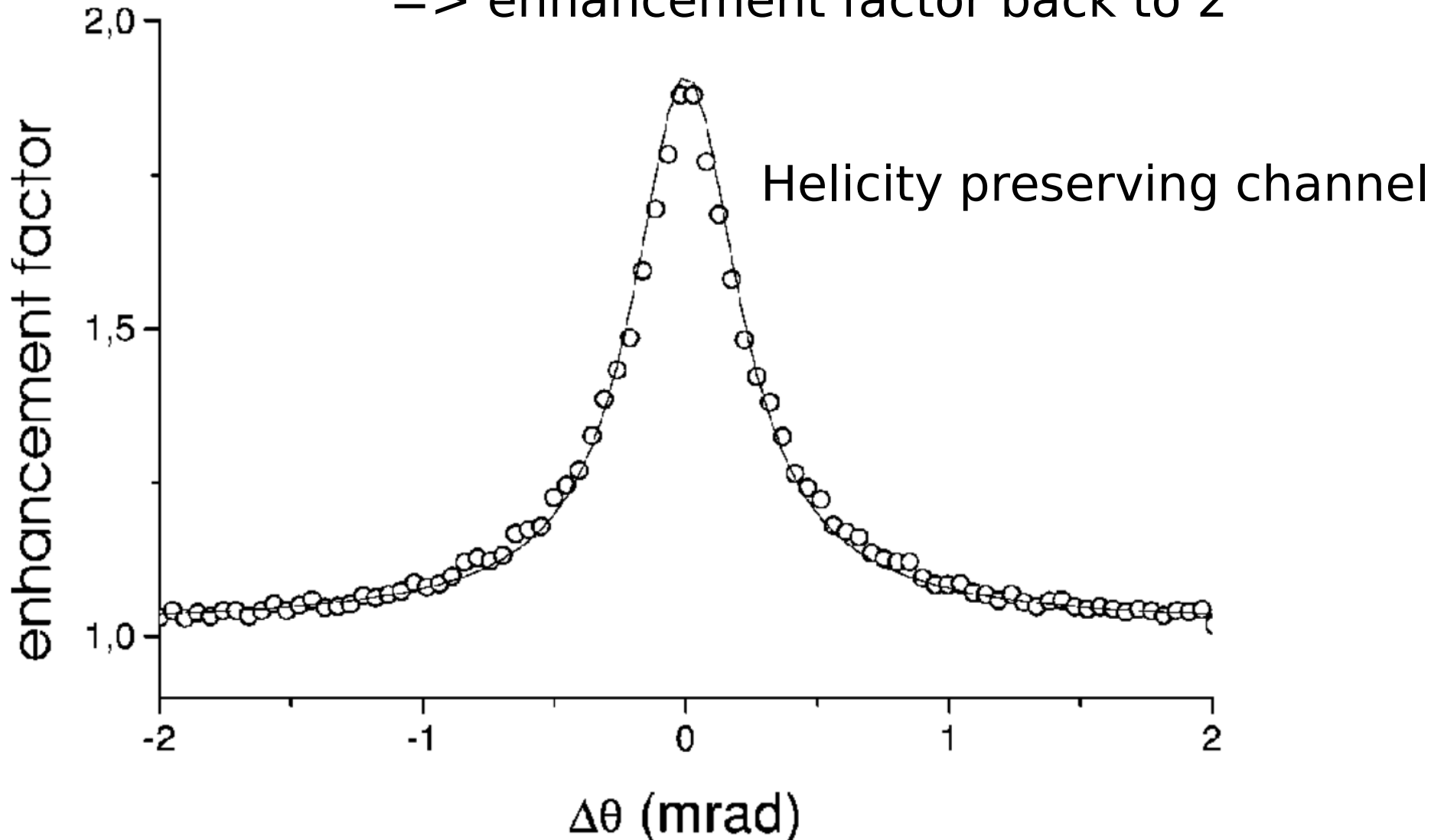


Small enhancement factor, because of “internal” decoherence (degeneracy of the atomic ground state $J=3$)

G. Labeyrie et al, Europ. Lett. 61, 327 (2003)

CBS on a cold Strontium gas

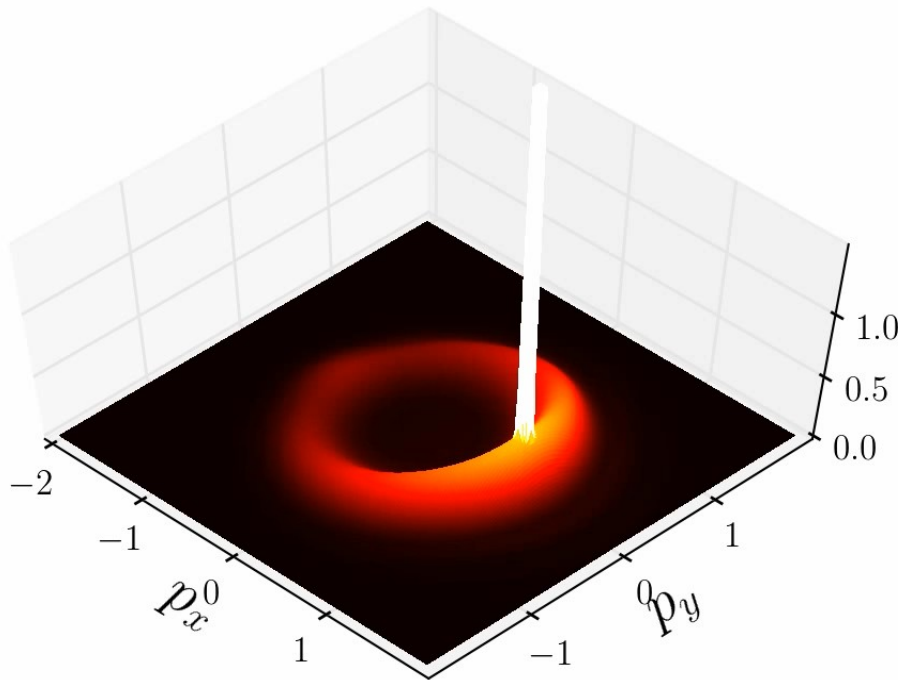
No degeneracy in the ground state
=> enhancement factor back to 2



Y. Bidel et al, PRL 88, 203902 (2002)

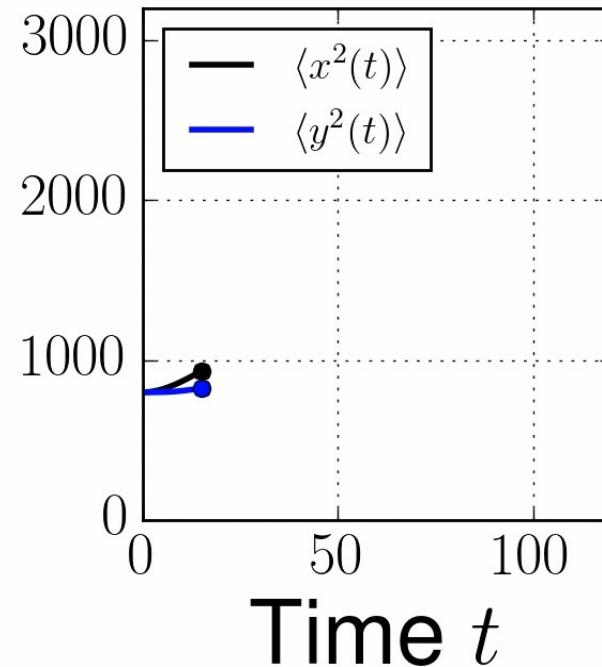
Temporal dynamics in momentum space

- Start from a wavepacket with non-zero initial velocity.
- Weak disorder: scattering by disorder to different direction, but with roughly the same velocity => **isotropization of the momentum distribution**



$t = 15.0$

Initial momentum $p_x = 1.0$, $p_y = 0.0$

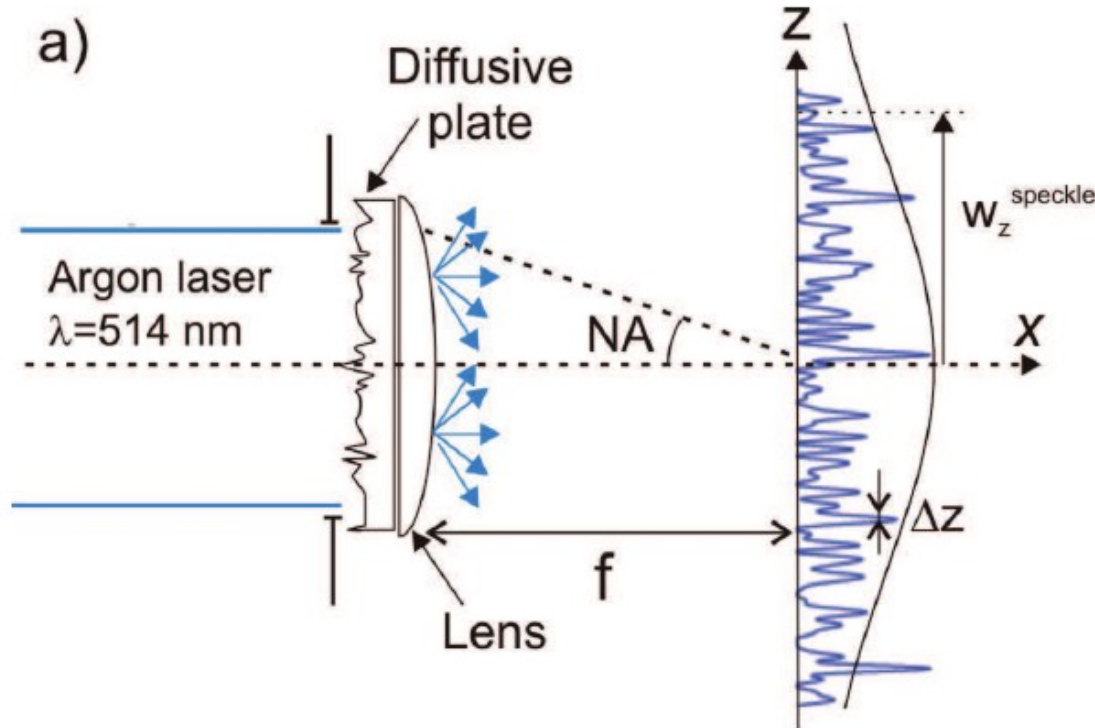


$V_0/E_\sigma = 0.2$

Movie in file [CBS_in_momentum_space.mp4](#)

Speckle optical potential (2D version)

- Speckle created by shining a laser on a diffusive plate:



Courtesy of V. Josse,
Institut d'Optique
(Palaiseau)

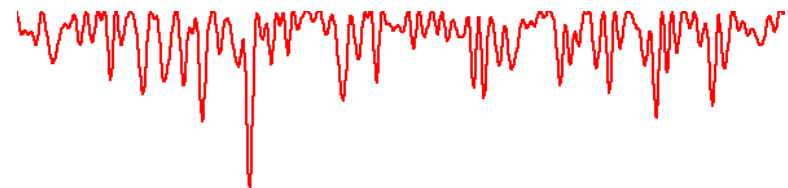
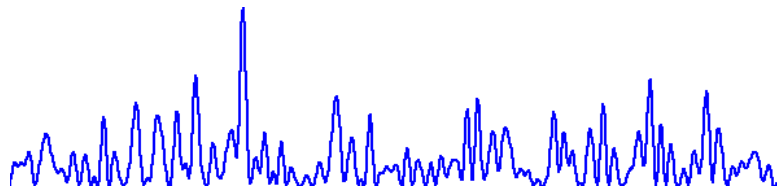
Speckle spot size

$$\sigma \approx \frac{\lambda}{NA}$$

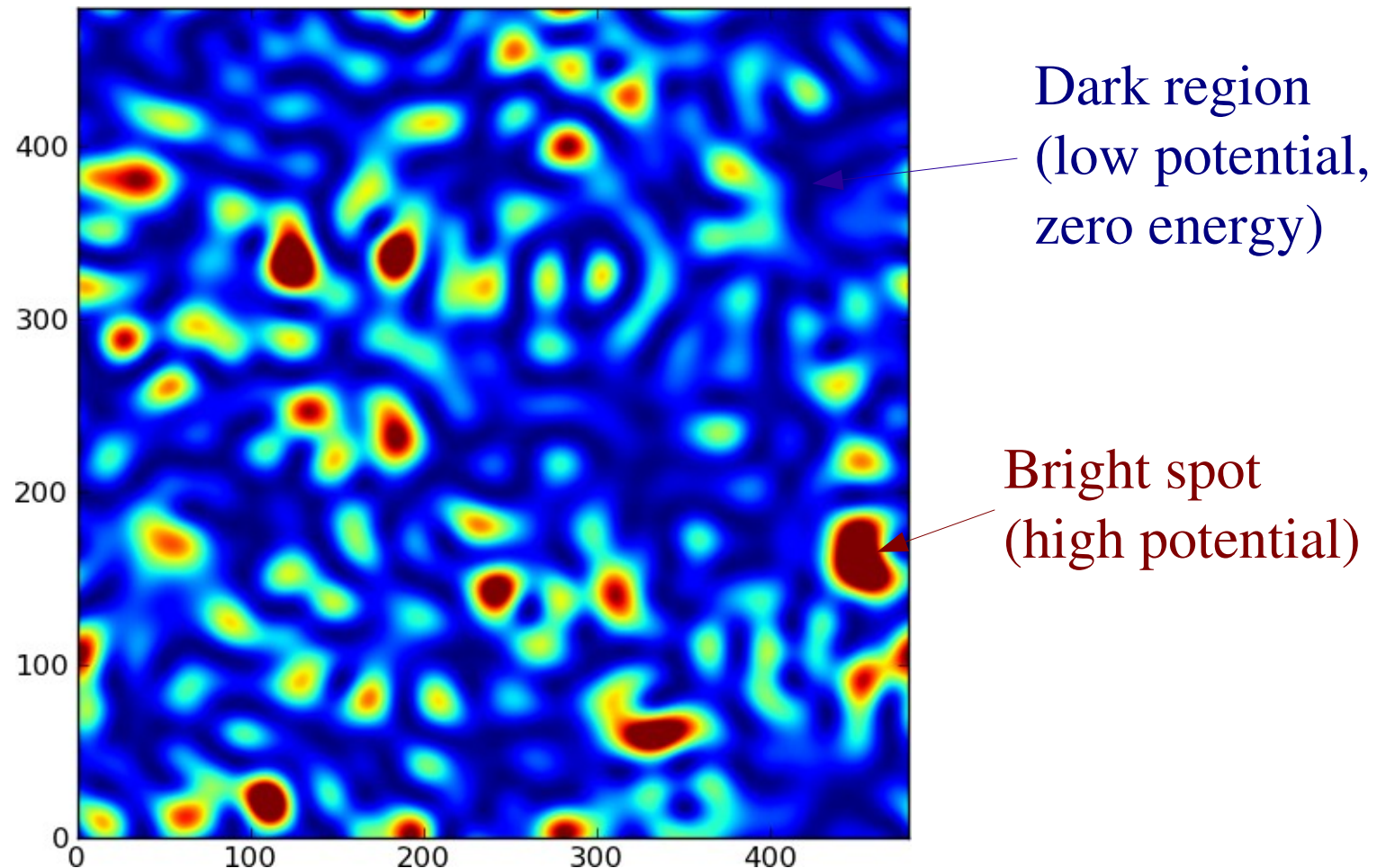
λ : laser wavelength

NA : Numerical Aperture

- The speckle electric field is a (complex) random variable with Gaussian statistics. All correlation functions can be computed.
- Depending on the sign of the detuning, the optical potential is bounded either from above or from below



A typical realization of a 2D blue-detuned speckle potential

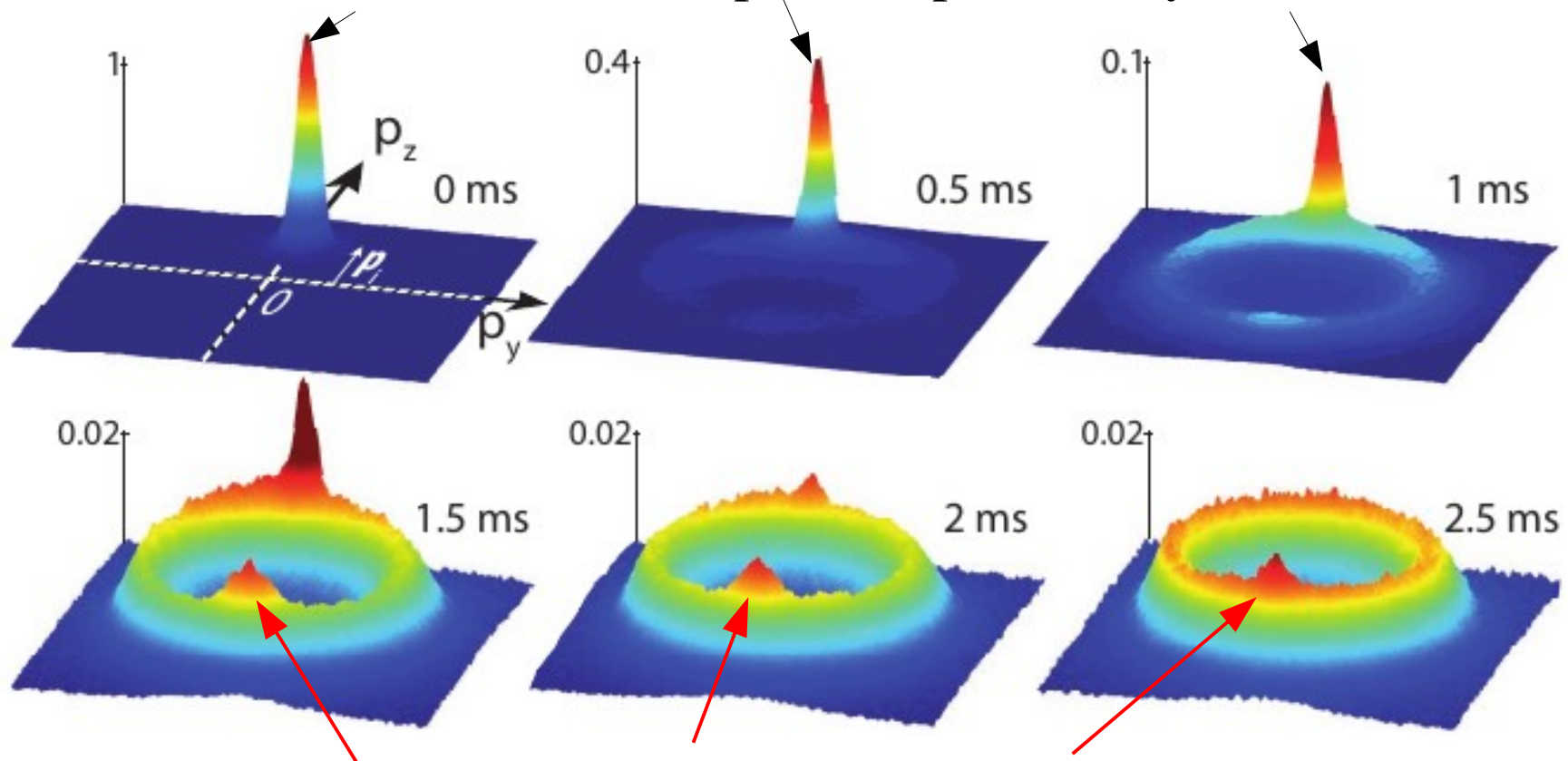


Distribution of potential value $P(V) = \frac{\exp(-V/V_0)}{V_0} \Theta(V)$

Rigorous low energy bound, no high energy bound

Experimental result on CBS of cold atoms

Forward coherent peak (exponentially attenuated)

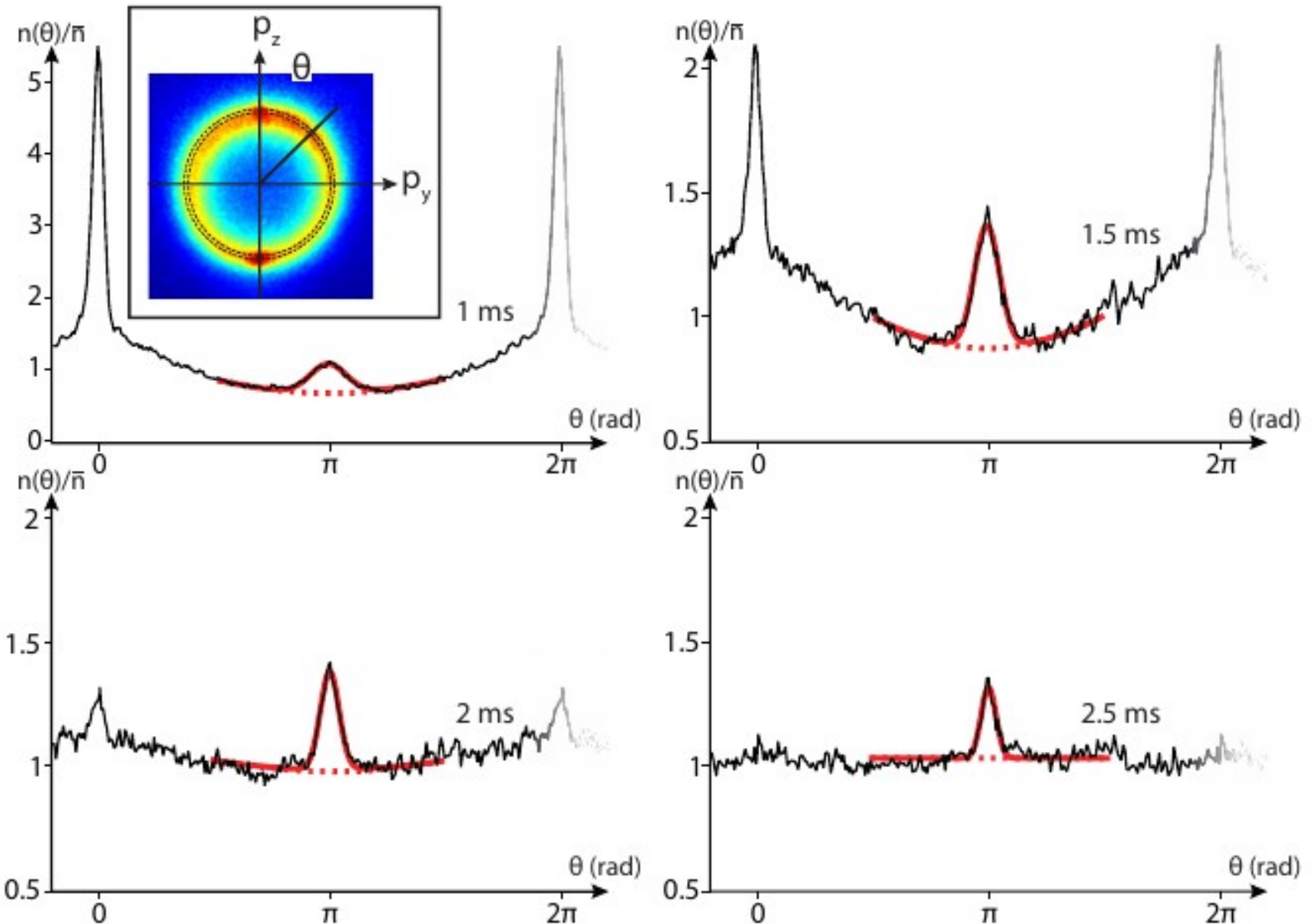


Coherent back-scattering peak

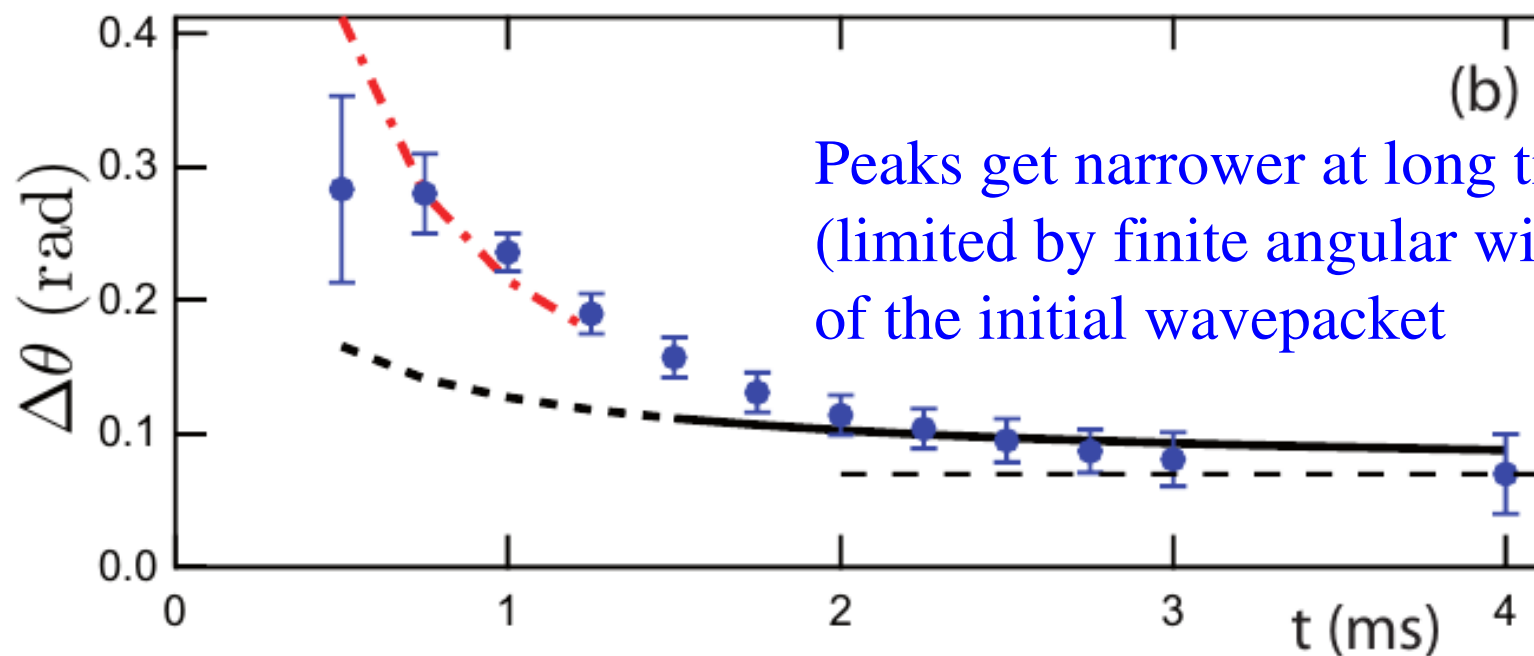
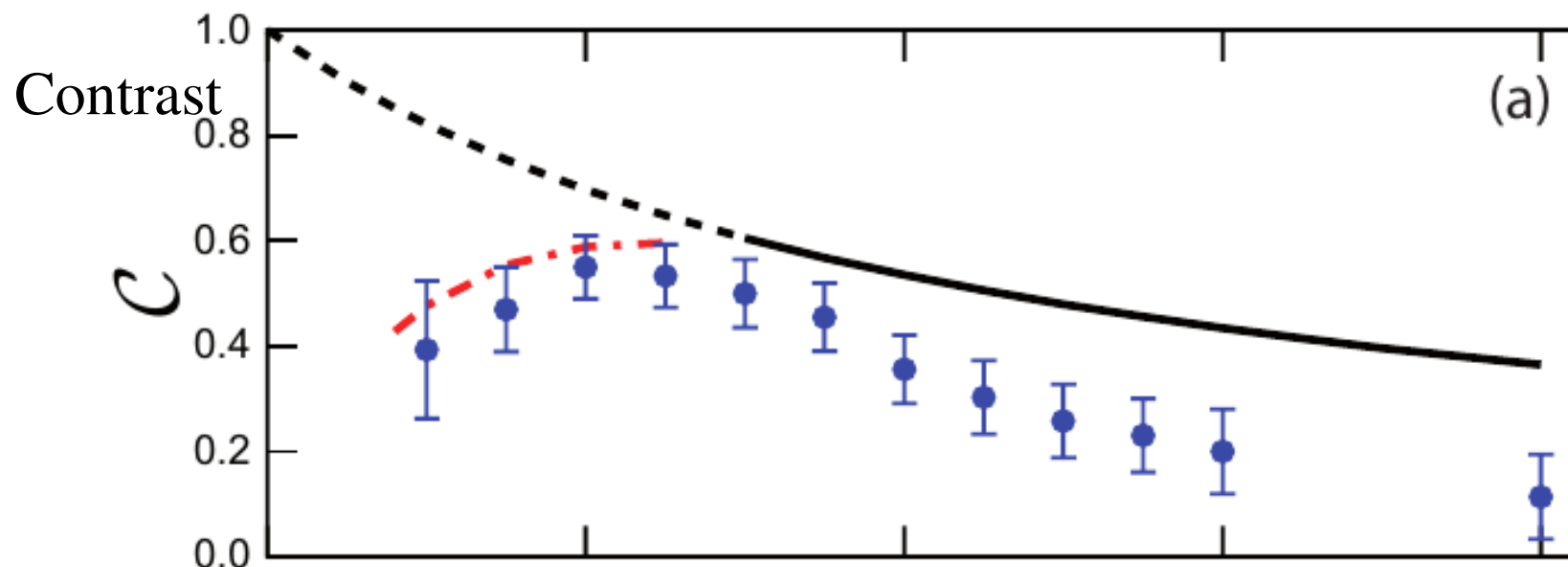
Velocity distribution of ultracold Rb atoms interacting with a disordered optical potential

F. Jendrzejewski et al, Phys. Rev. Lett. 109, 195302 (2012), arxiv:1207.4775

Temporal evolution of the the CBS peak

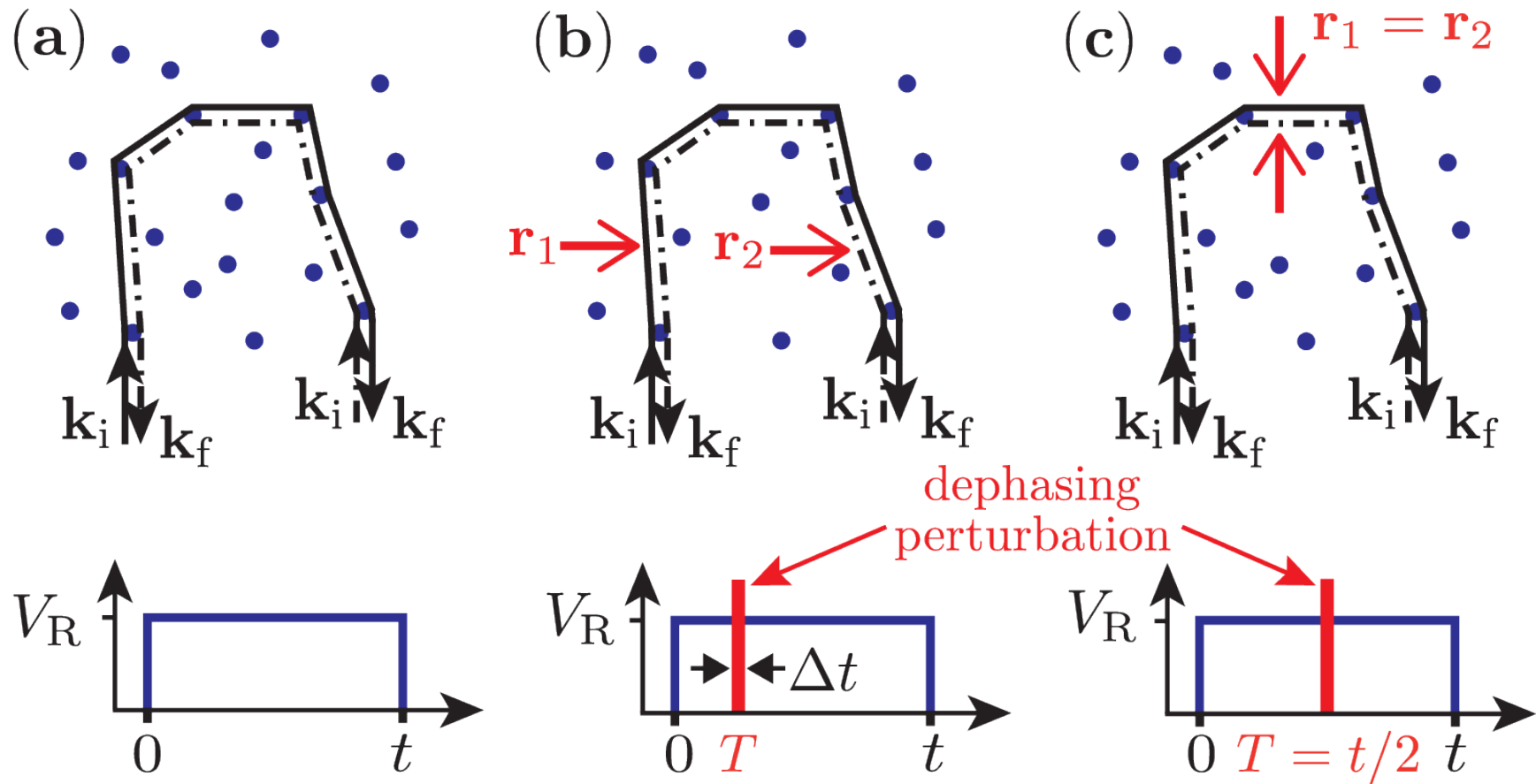


Temporal evolution of the the CBS peak

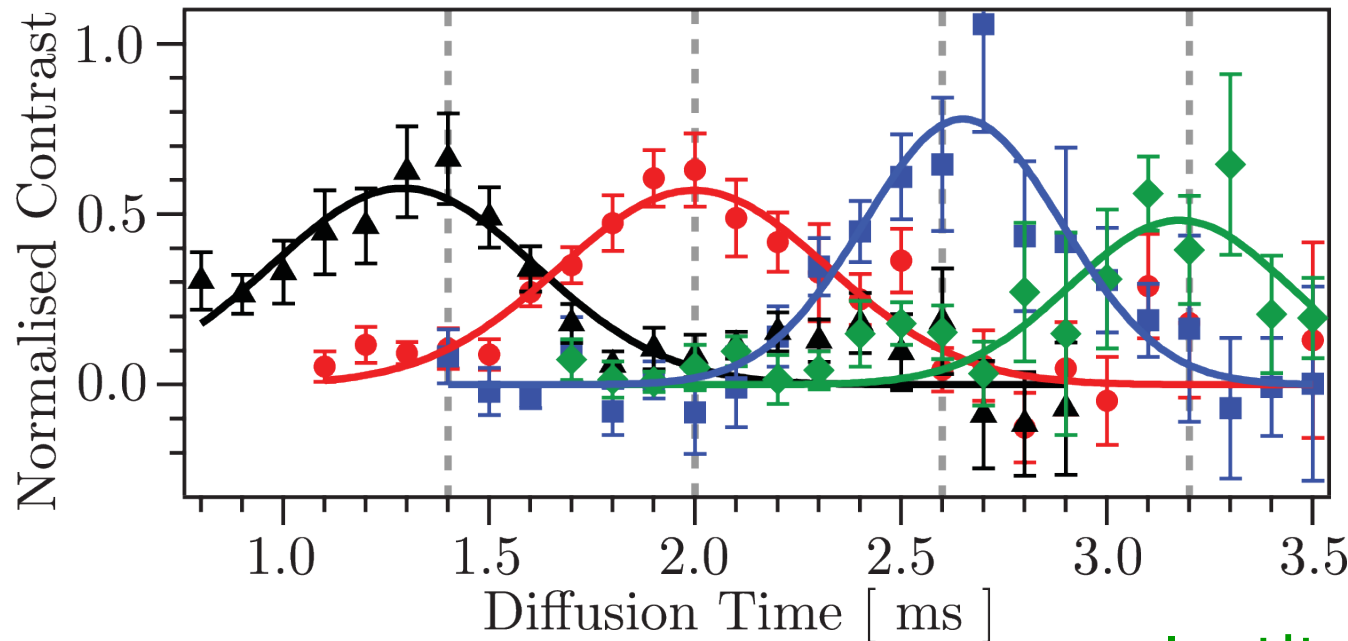
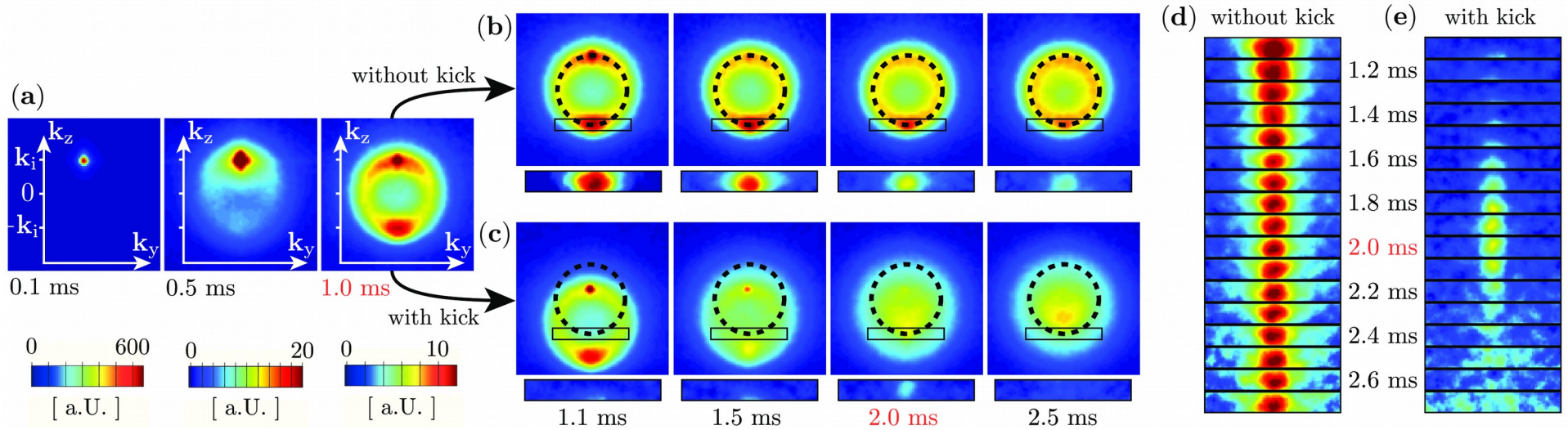


Breaking and restoring CBS

- Idea: apply a “kick” on the systemt after the wave has propagated for some time T in the medium.
- CBS disappears after T , but revives after $2T$.



Breaking and restoring CBS: experimental results



K. Müller et al,
PRL 114, 205301 (2015)
Following an idea of
T. Micklitz, C. Müller
and A. Altland,
PRB 91, 064203 (2015)

Institut d'Optique (Palaiseau)