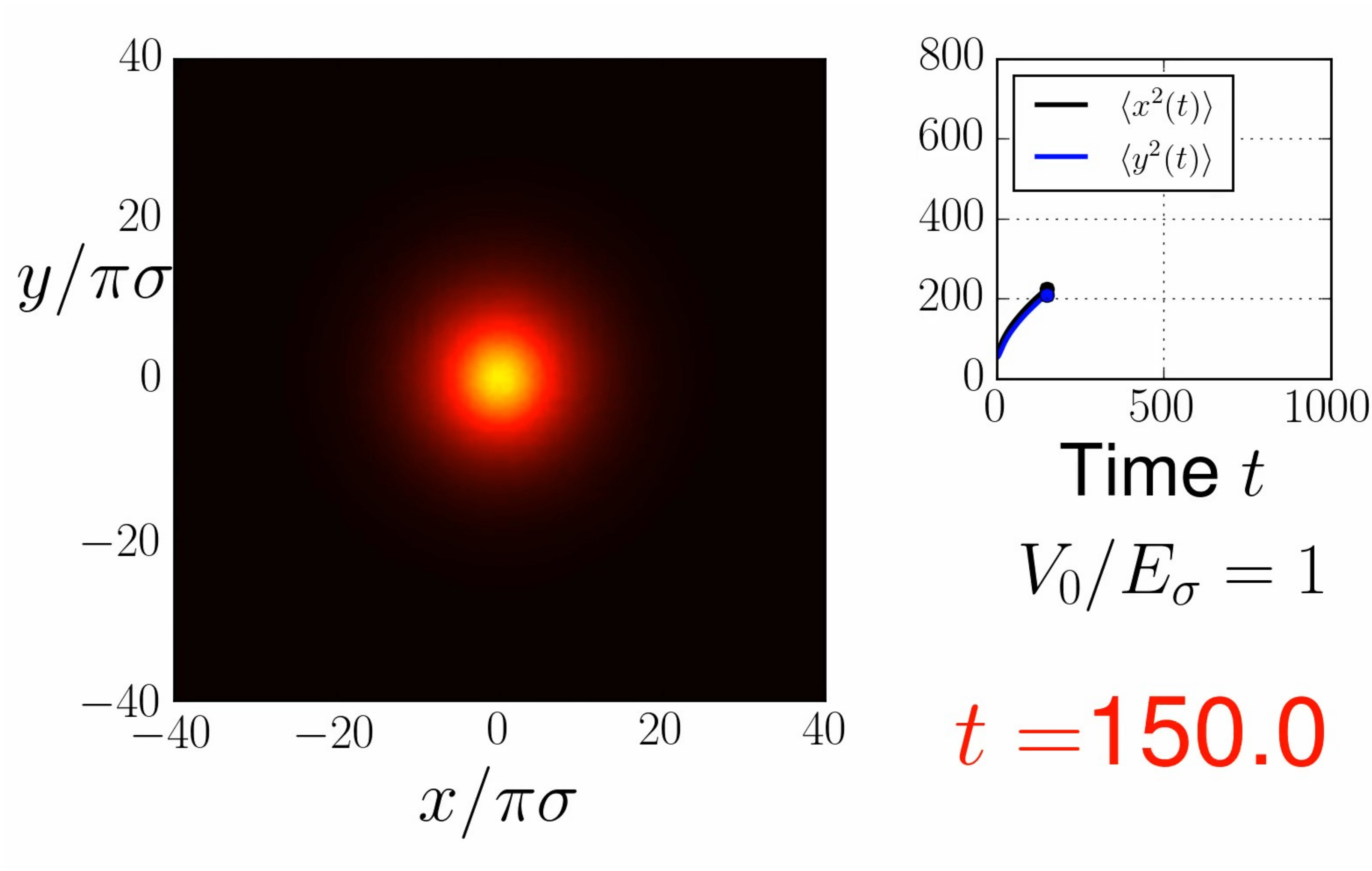
Outline of the lectures

- I. Quantum theory of transport (2 hours)
- II. Weak localization (1 hour)
- III. Coherent Back-Scattering (CBS) (1.5 hours)
- IV. Anderson (strong) localization – Scaling theory (2 hours)
- V. Self-consistent theory of localization (1 hour)

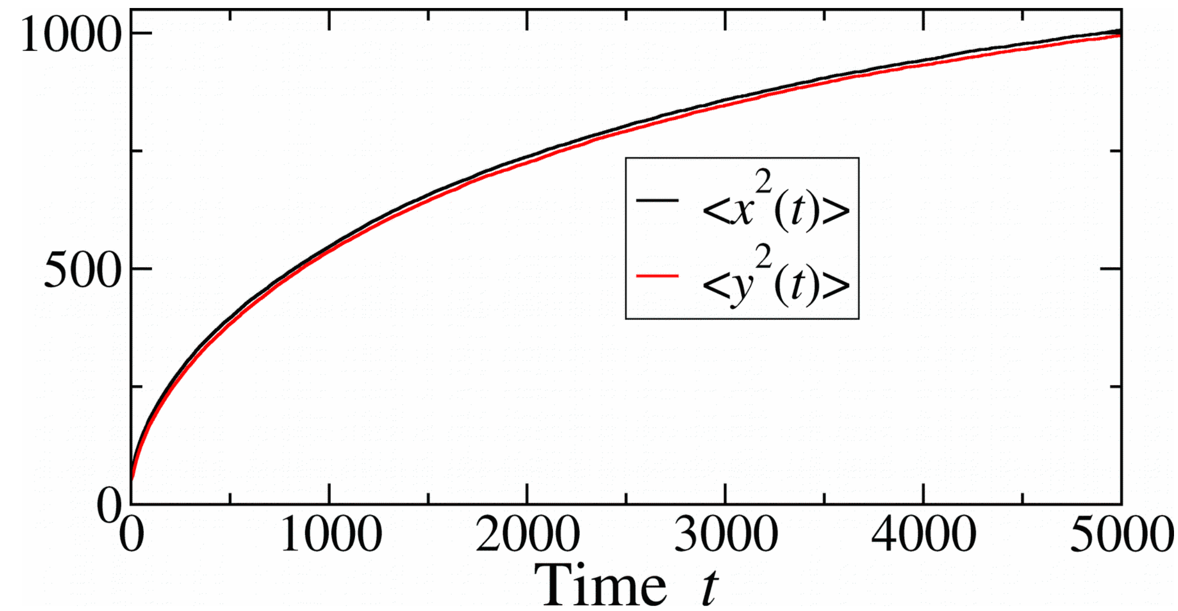
Dominique Delande
Laboratoire Kastler-Brossel
Ecole Normale Supérieure, Université Pierre et Marie Curie, Collège de France (Paris)
http://www.lkb.upmc.fr/complexquantumsystems/
Dominique.Delande@lkb.upmc.fr

Long time dynamics: towards Anderson localization



Anderson localization takes a VERY long time!

- On a longer time scale:

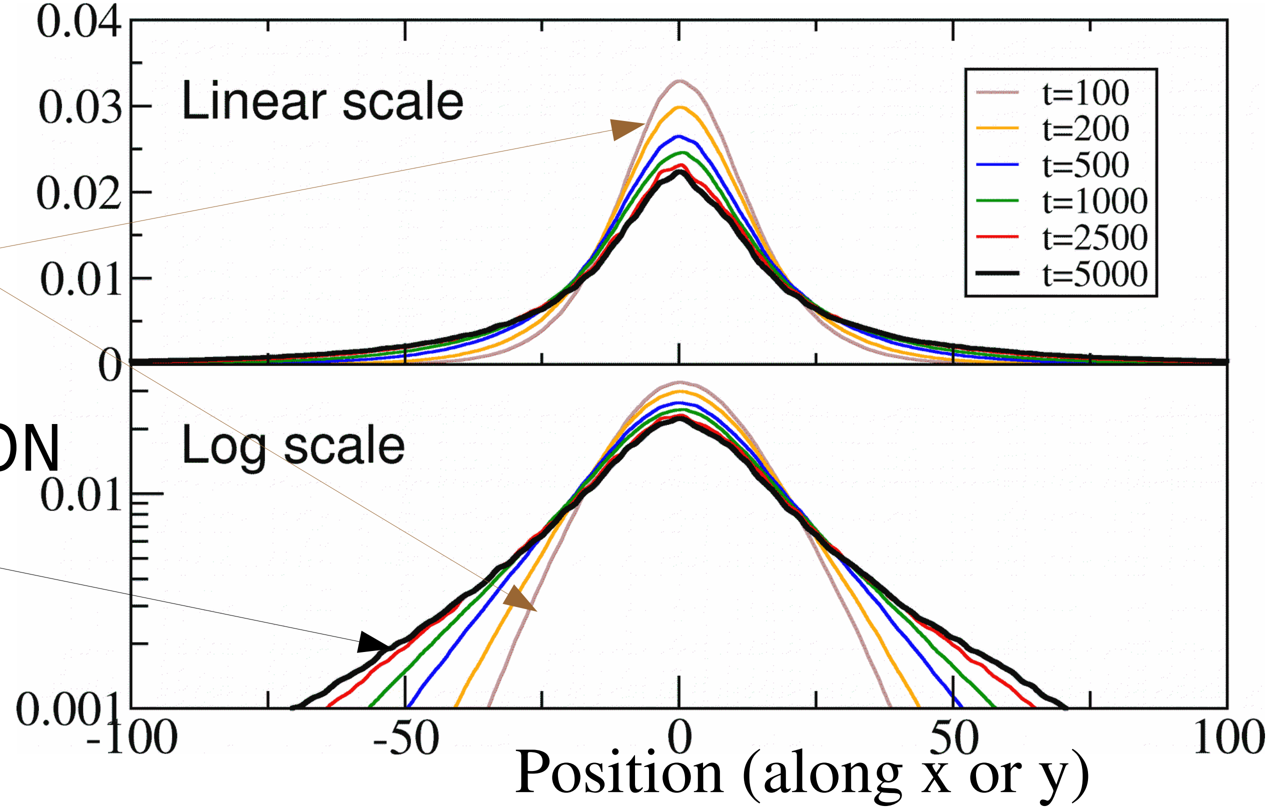


- Average spatial density:

Gaussian profile at short time

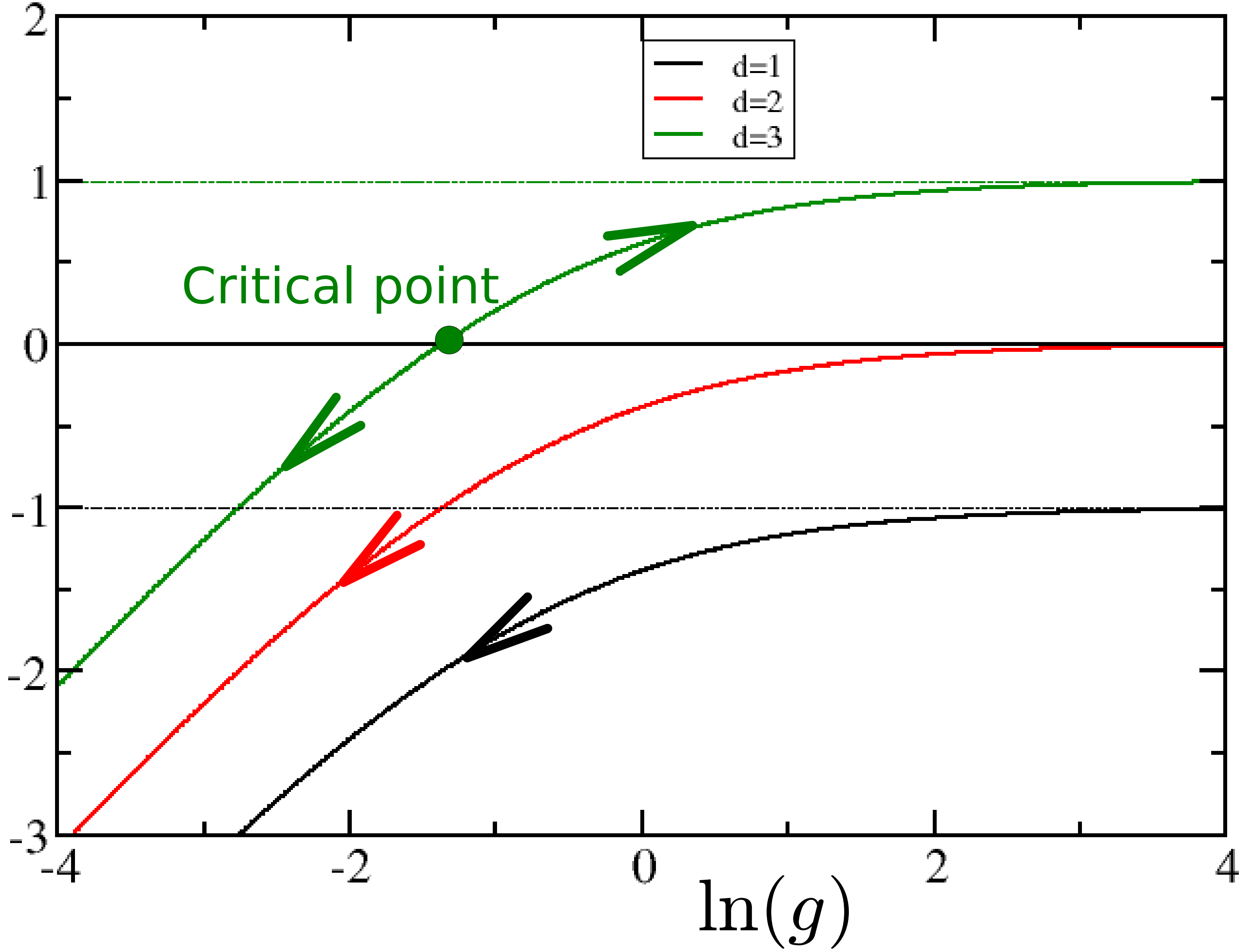
ANDERSON LOCALIZATION

Approximately exponential profile at long time

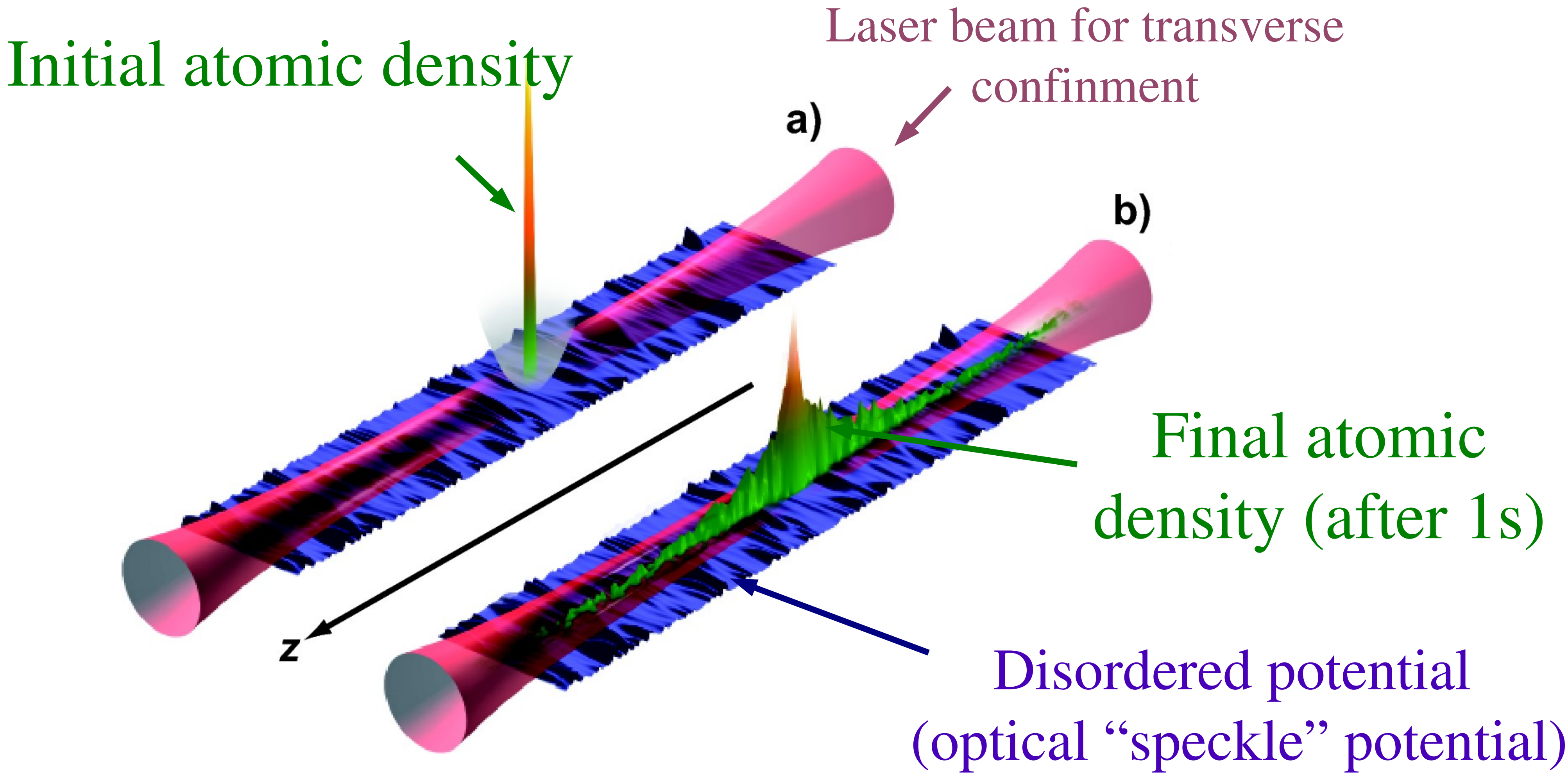


Scaling function in dimension 1, 2 and 3

$$\beta(\ln(g)) = d - 1 - (1+g)\ln(1+g^{-1})$$

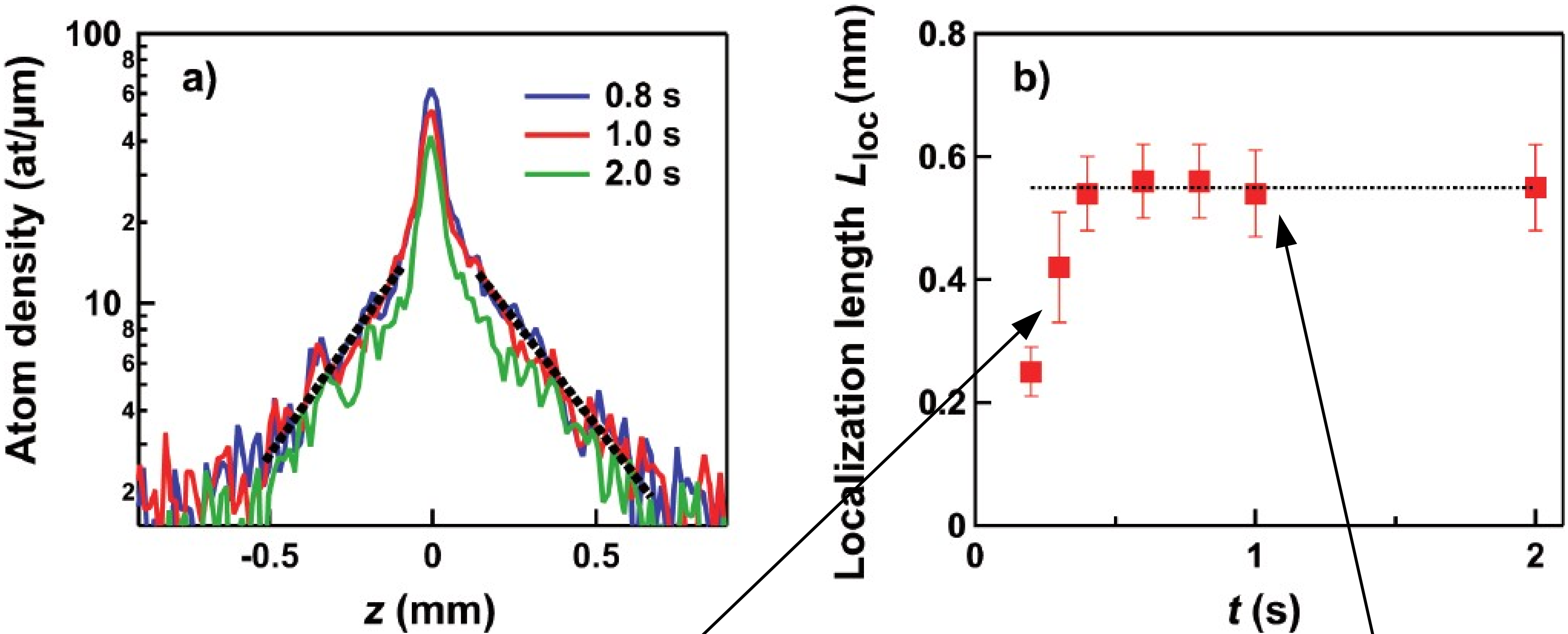


Experiment on localization of atomic matter waves



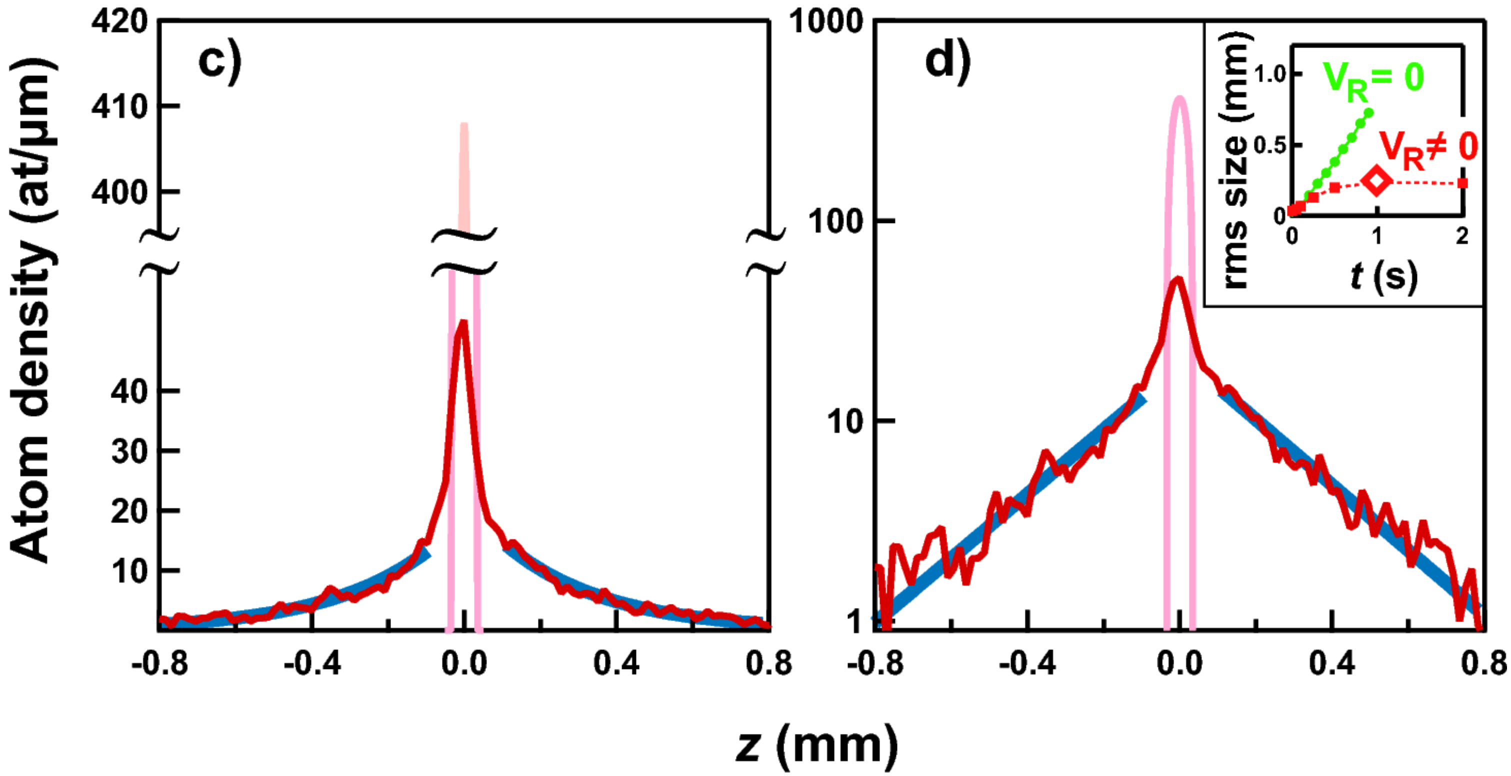
J. Billy et al, Institut d'Optique (Palaiseau, France), Nature, 453, 891 (2008)

Experiment on localization of atomic matter waves



Short time expansion (diffusion?) followed by localization

1d Anderson localization of atomic matter waves

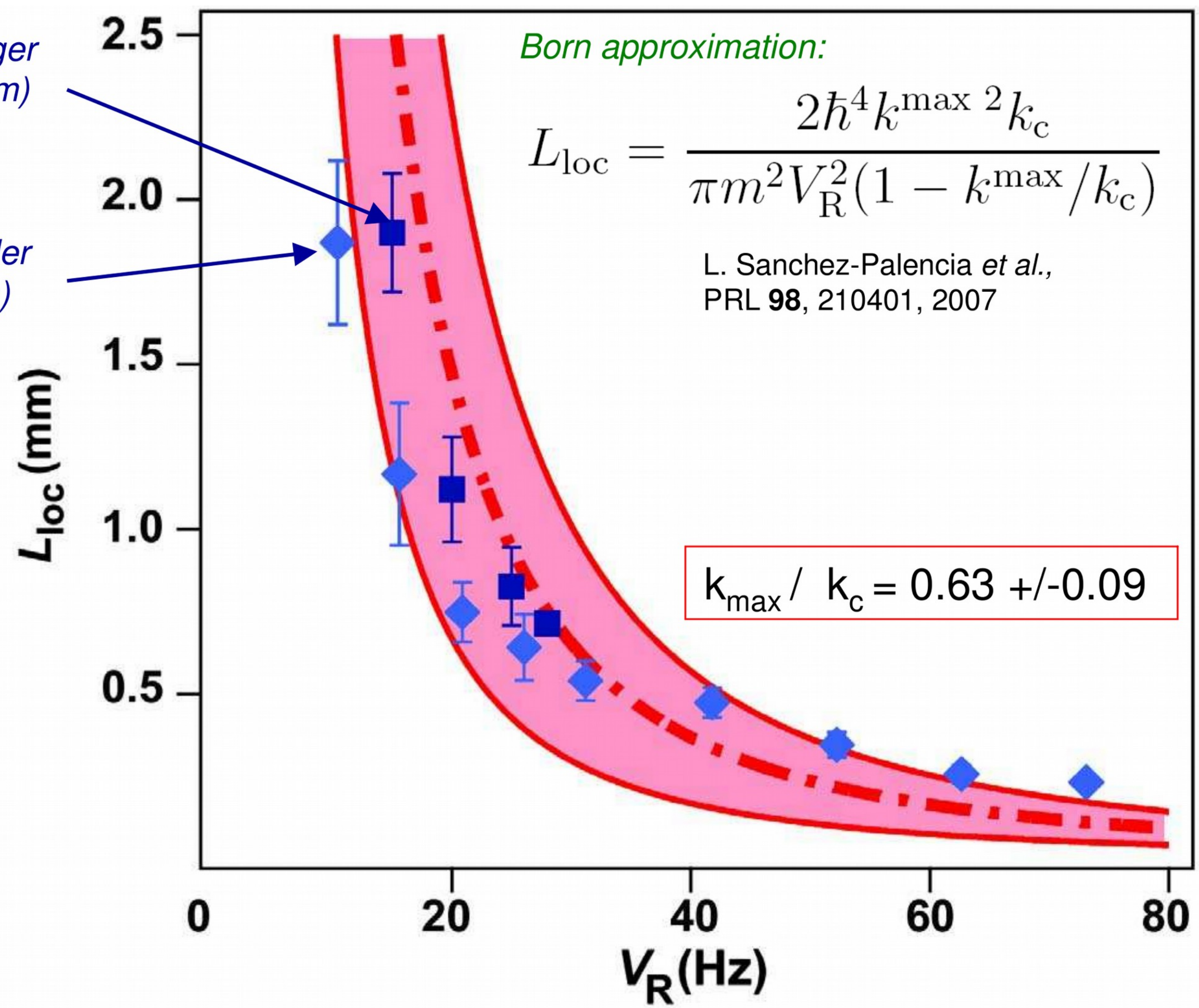


Log scale

The final atomic density shows exponential localization with localization length of few 100μm!

J. Billy et al, Institut d'Optique (Palaiseau, France), Nature, 453, 891 (2008)

Comparison between the measured and calculated localization lengths



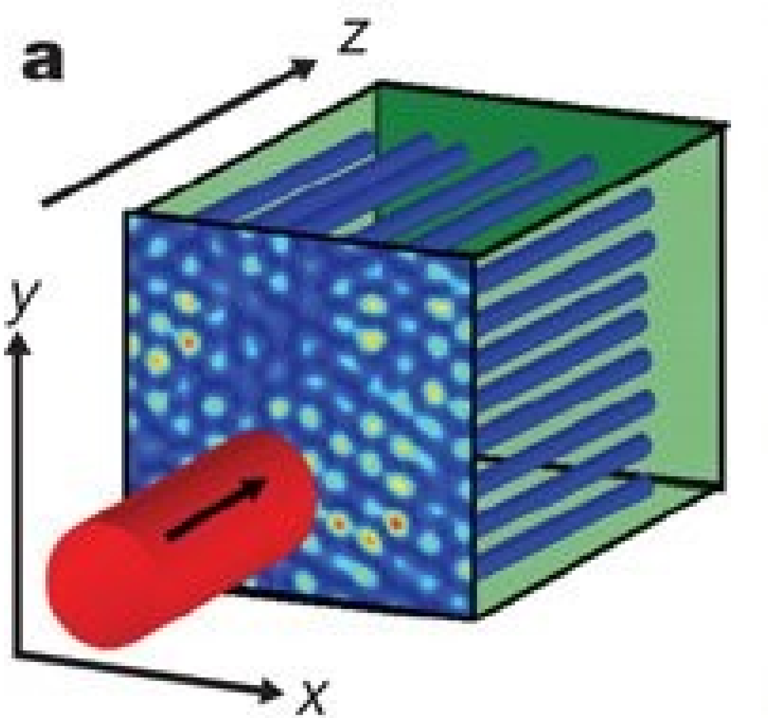
Typical localization length >> atomic de Broglie wavelength

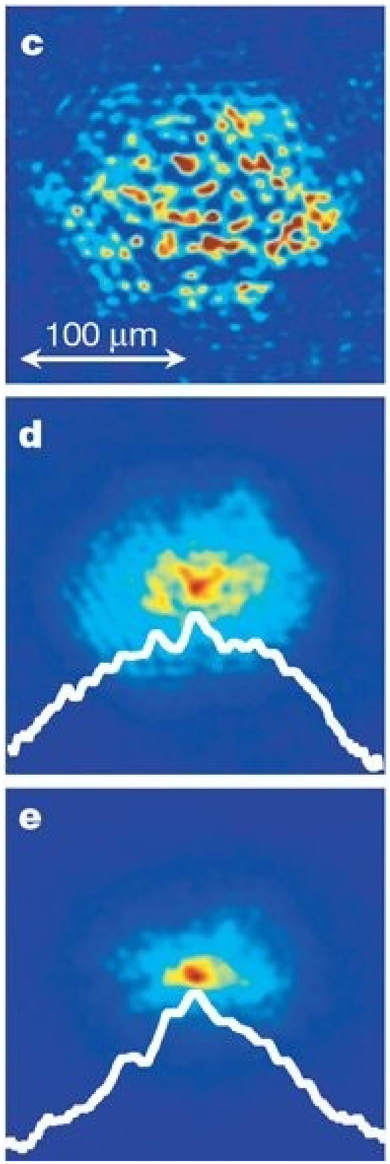
$$k\ell \gg 1$$

Disordered potential strength

J. Billy et al, Institut d'Optique (Palaiseau, France), Nature, 453, 891 (2008)

Anderson localization in 2d photonic lattices





Ballistic regime
(no disorder)

Diffusive regime
(small disorder)

$\ln(|\psi(x)|^2)$

Gaussian shape

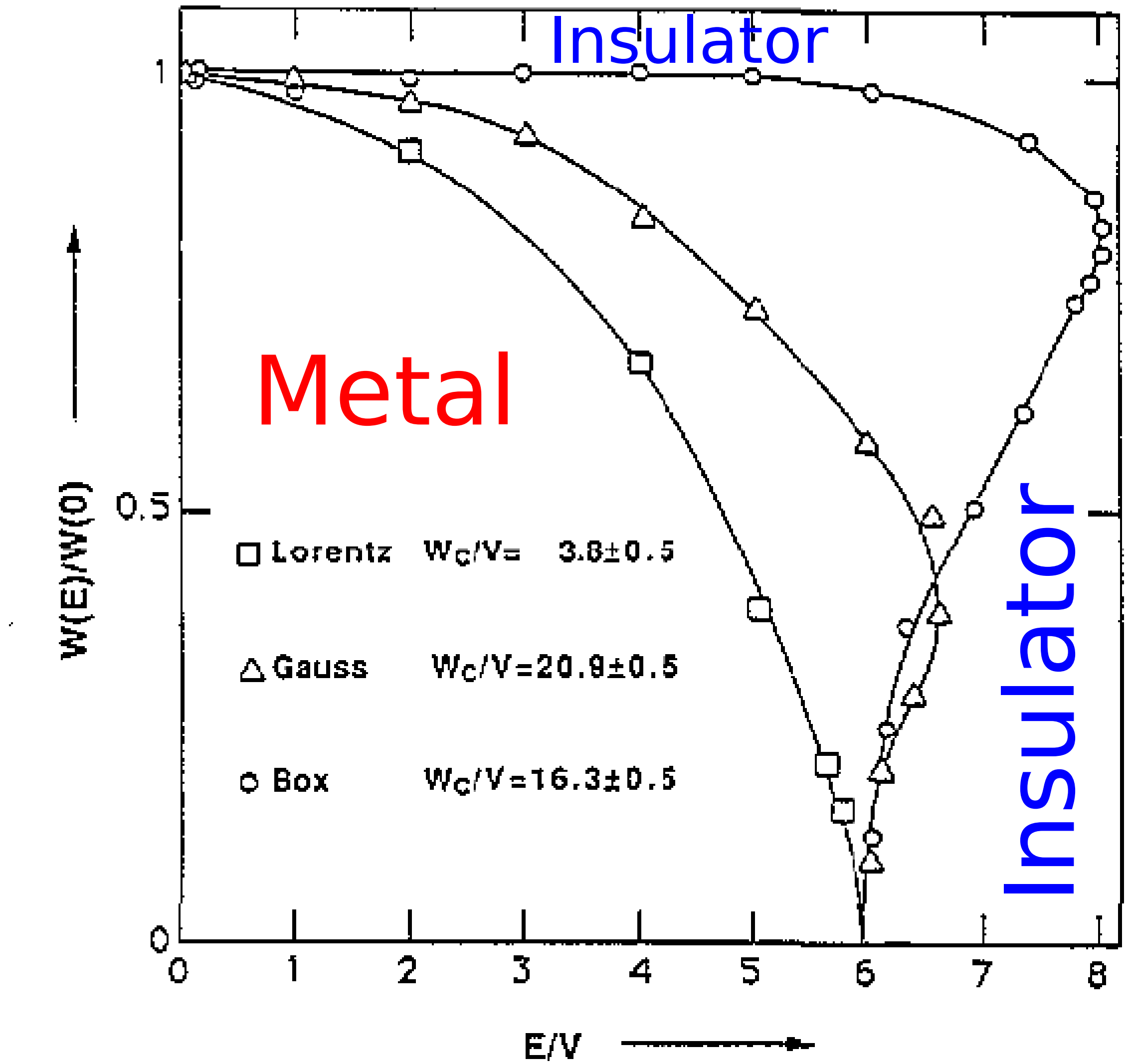
Localized regime
(large disorder)

$\ln(|\psi(x)|^2)$

Exponential shape

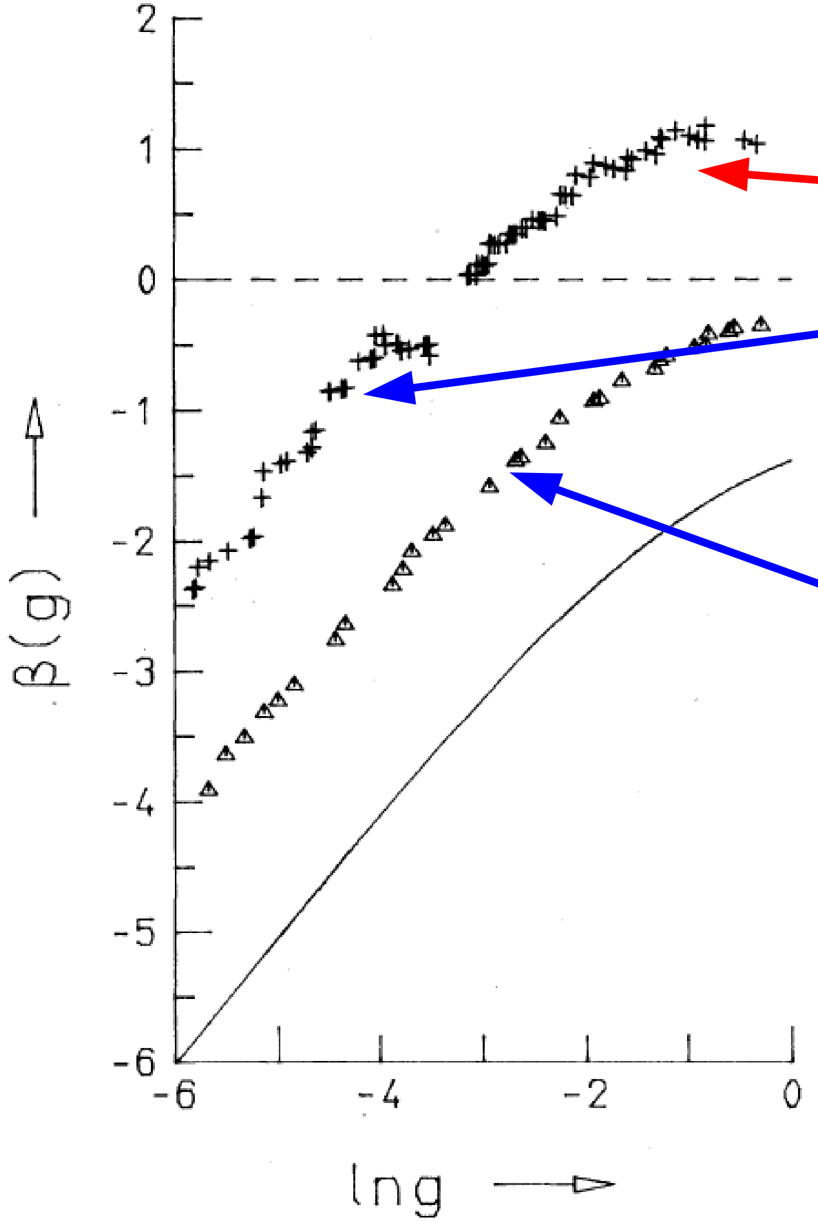
T. Schwartz et al, Nature, 446, 52 (2007)

Phase diagram of the 3d Anderson model



B. Kramer and A. McKinnon, Phys. Rep. 56, 1469 (1993)

Numerical experiments on the Anderson model



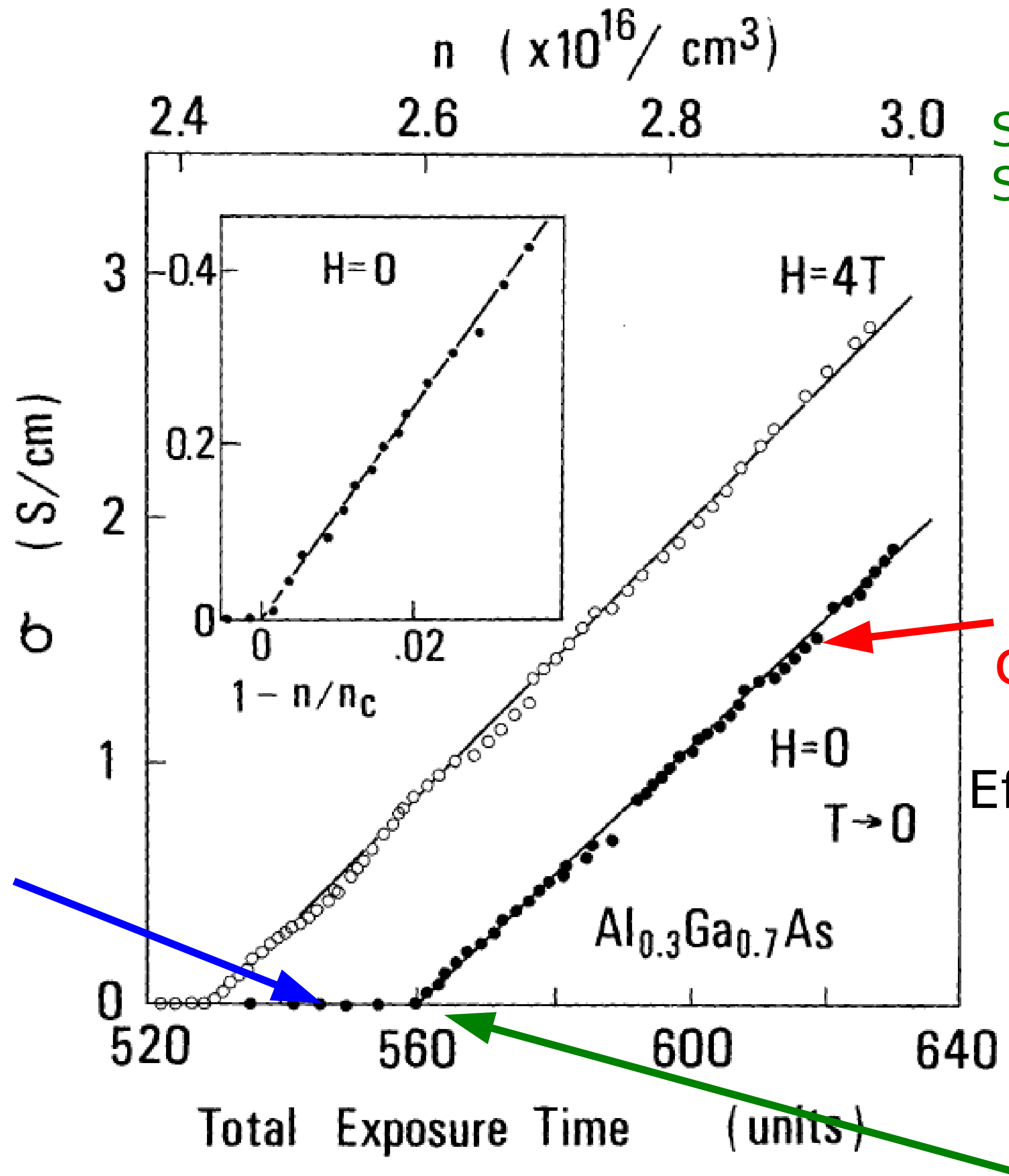
d=3: either localized or delocalized depending on the disorder strength

d=2: always localized

Direct numerical computation of the scaling function in d=2 and d=3 (transfer matrix method)

A. MacKinnon and B. Kramer, PRL, 47, 1546 (1981)

Metal-insulator transition d=3



S. Katsumoto et al, J. Phys. Soc. Japan, 56, 2259 (1987)

Conductivity

Linear behaviour
critical exponent=1

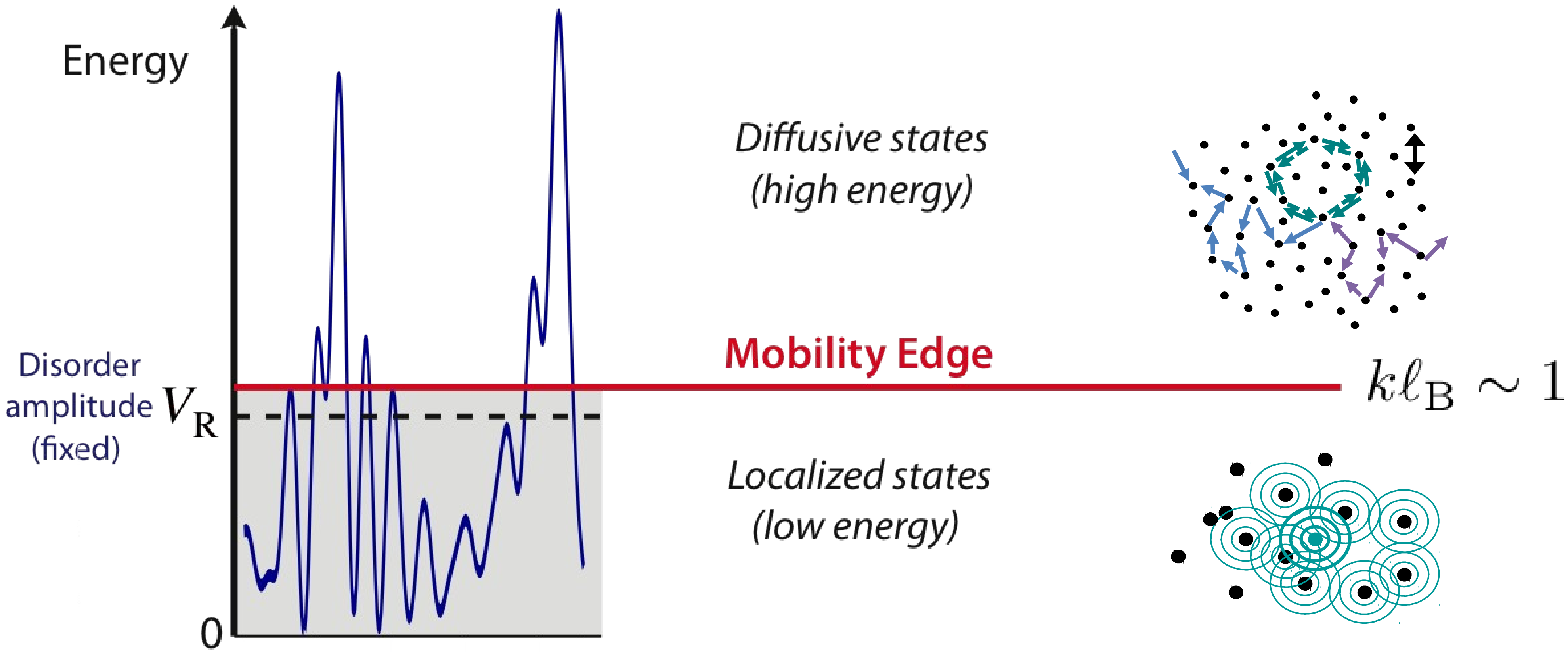
Effect of electron-electron interaction?

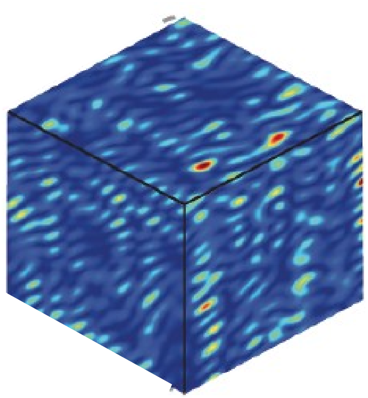
Anderson insulator

Critical point

Impurity concentration

3D Anderson transition

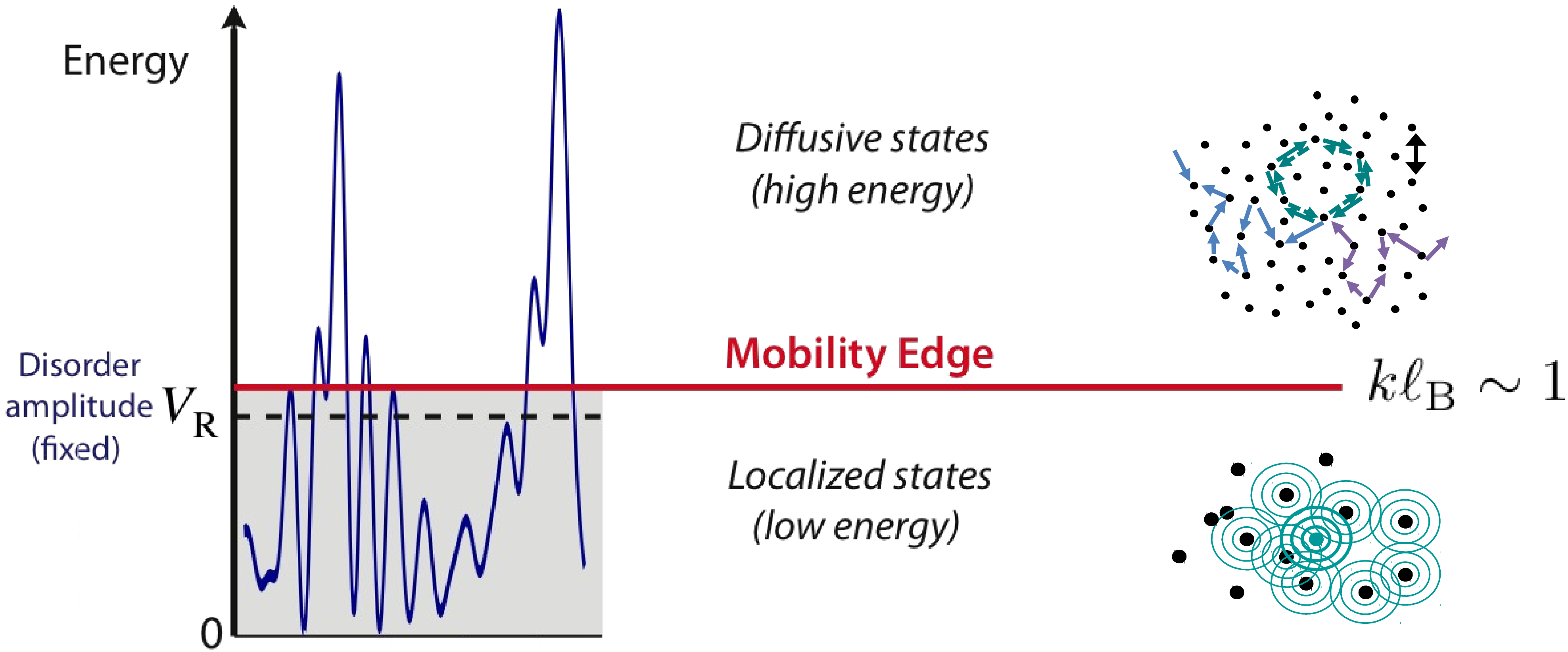


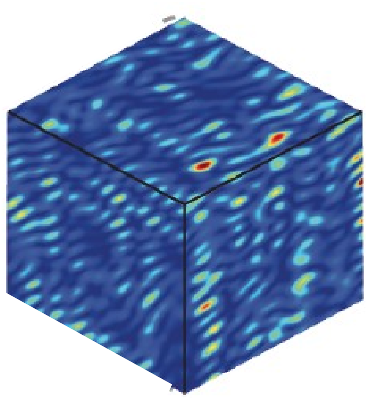


3D speckle disorder potential
(repulsive potential)

⇨ *A transition from diffusive to localized states in strong disorder*

3D Anderson transition



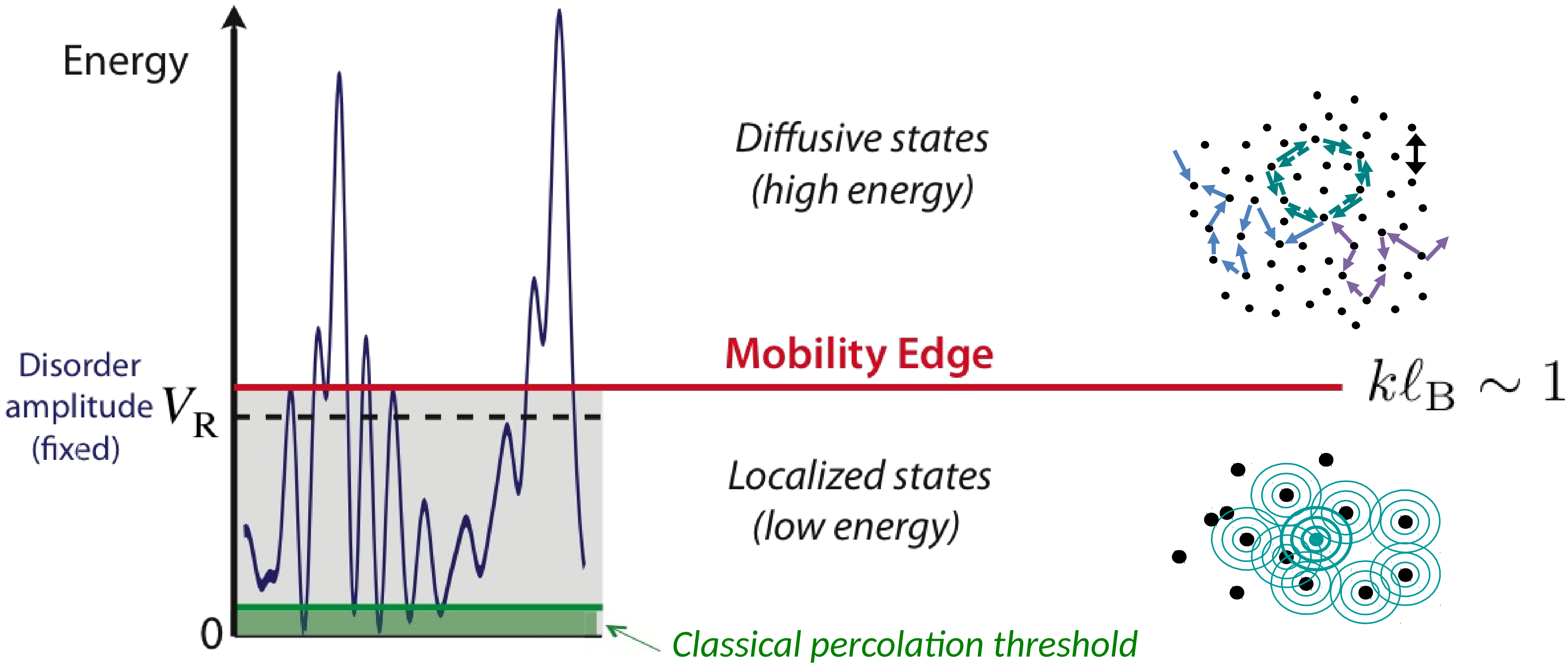


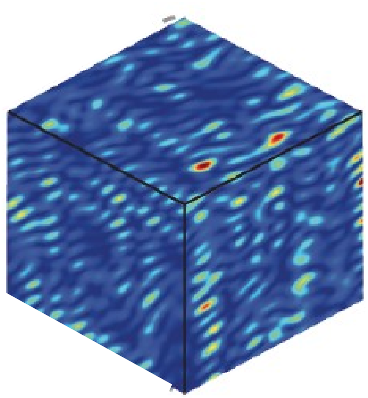
3D speckle disorder potential
(repulsive potential)

Objectives

- Observe 3D Anderson localization
- Estimate the position of the mobility edge and compare with quantitative predictions
- Study critical regime (longer term)
  (critical exponents, multifractality….)

3D Anderson transition





3D speckle disorder potential (repulsive potential)

3D laser speckle disorder

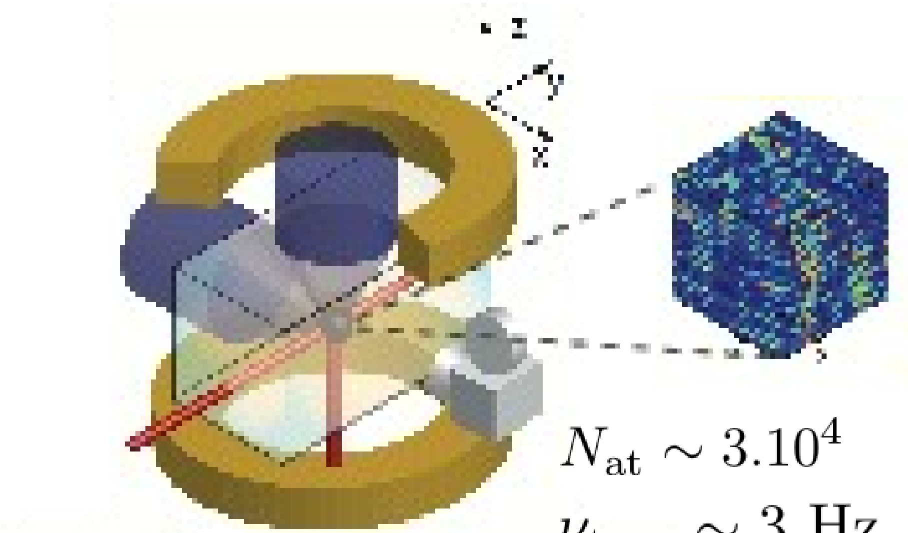
Extremely low classical trapping probability

Atoms “turn around” the speckle grains

$$E_{\mathrm{cl}} \sim 10^{-4} V_R$$

Pilati et al. PRL (2009), NJP (2010)

Palaiseau's experiment (2012)



$$N_{\rm at} \sim 3.10^4$$

$$\nu_{\rm trap} \sim 3 \text{ Hz}$$

Palaiseau, ^{87}Rb (bosons)

3D laser speckle disorder + <u>levitation</u>

Jendrzejewski et al. Nat. Phys. (2012)

"Quantum disorder" regime for the matterwave propagation

(extremely dilute and very cold BEC)

$$T \sim 3 \text{ nK}$$

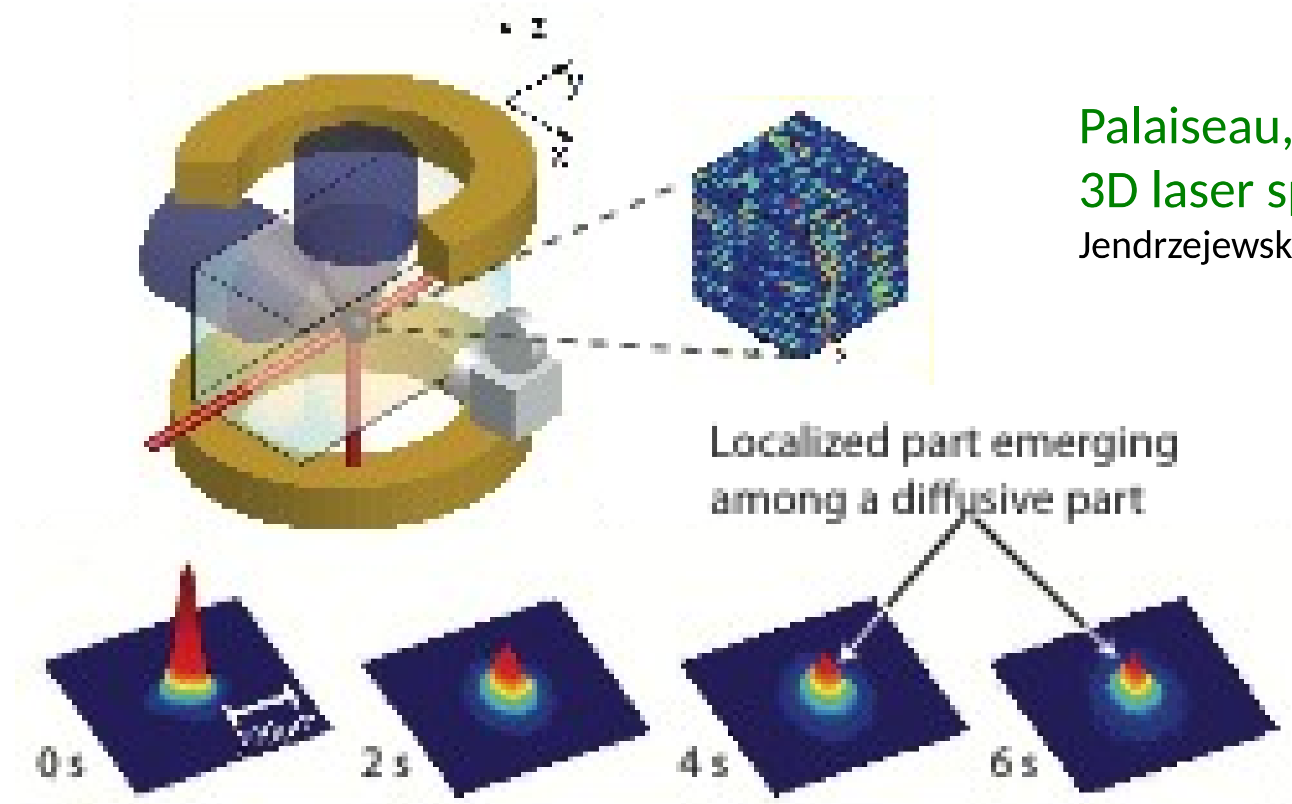
$$\mu_{\rm in} = 40 \text{ Hz}$$

- Large deBroglie wavelength $\lambda_{\rm dB} \sim 6\ \mu\text{m}$
- Thin speckle grain $\sigma_{\rm R} \sim 0.25\ \mu\text{m}$

⇨ Can we observe localization when pushing the system towards the strong disorder regime ?

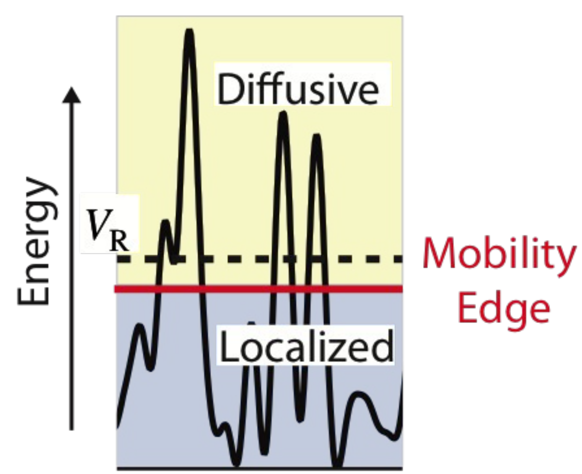
Palaiseau's experiment (2012)

Palaiseau, ^{87}Rb (bosons)
3D laser speckle disorder + levitation
Jendrzejewski et al. Nat. Phys. (2012)



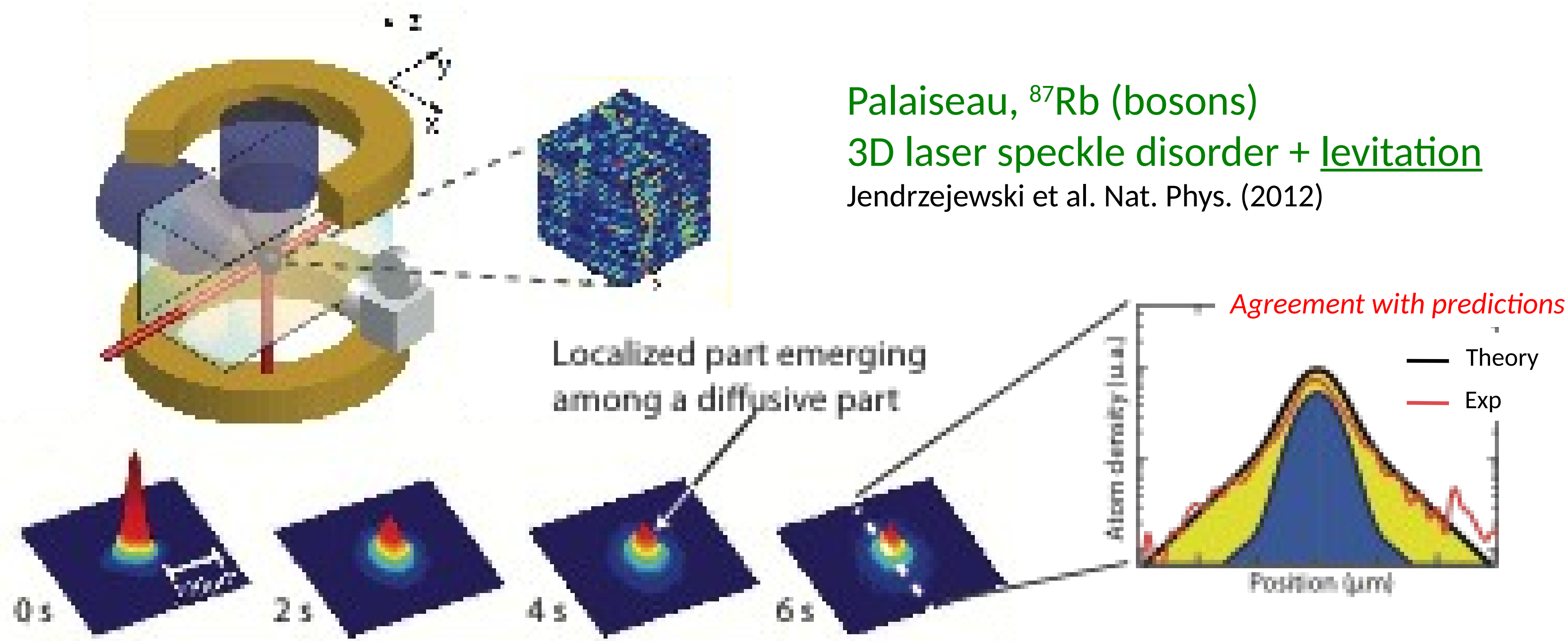
$$V_{\mathrm{R}}/h = 680\ \mathrm{Hz} \gg \mu_{\mathrm{in}}$$

⇨ A good news: observation of an 3D Anderson localized component



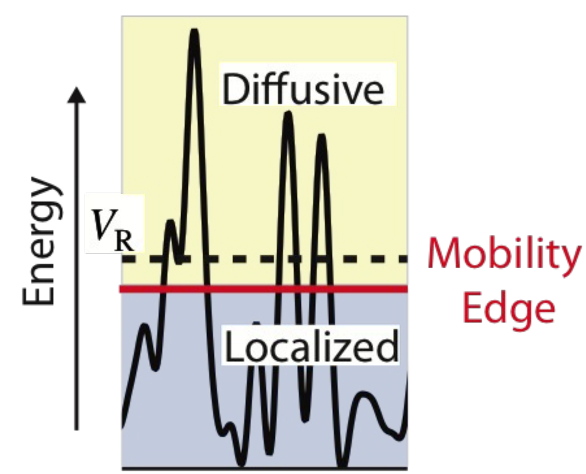
Palaiseau's experiment (2012)

Palaiseau, ^{87}Rb (bosons)
3D laser speckle disorder + levitation
Jendrzejewski et al. Nat. Phys. (2012)



⇨ A good news: observation of an 3D Anderson localized component

Important: Need very long propagation times to differenciate between very slow diffusion and localization

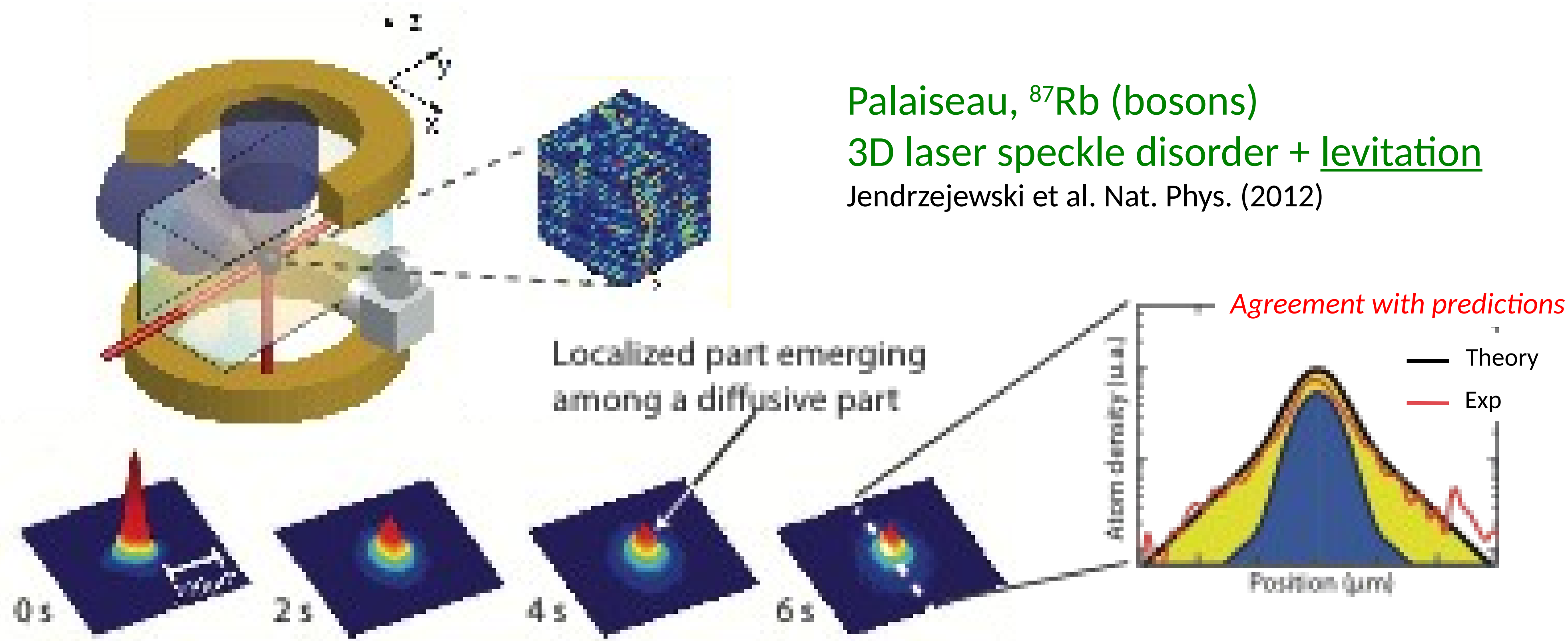


Palaiseau's experiment (2012)

Palaiseau, ^{87}Rb (bosons)

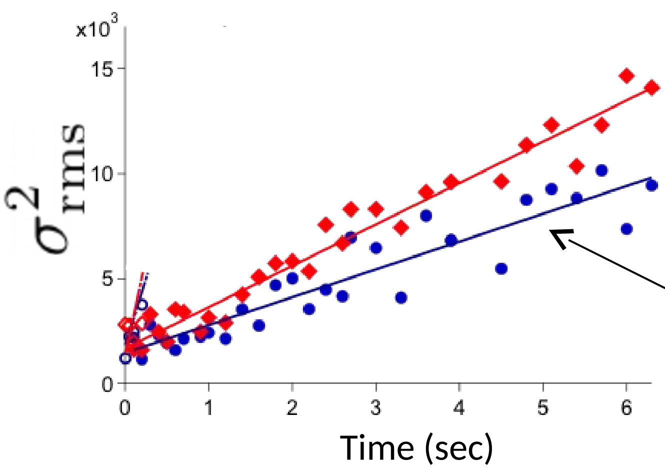
3D laser speckle disorder + levitation

Jendrzejewski et al. Nat. Phys. (2012)



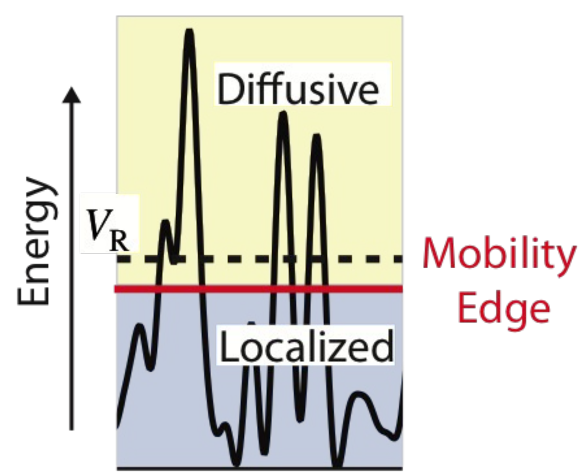
⇨ A good news: observation of an 3D Anderson localized component

⇨ A bad news: a diffusive component at the same time



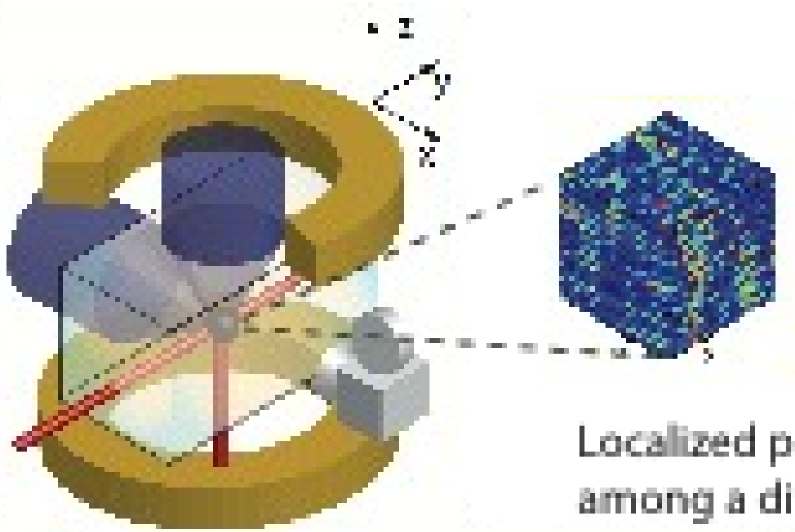
Signature of diffusion from expansion in the wings

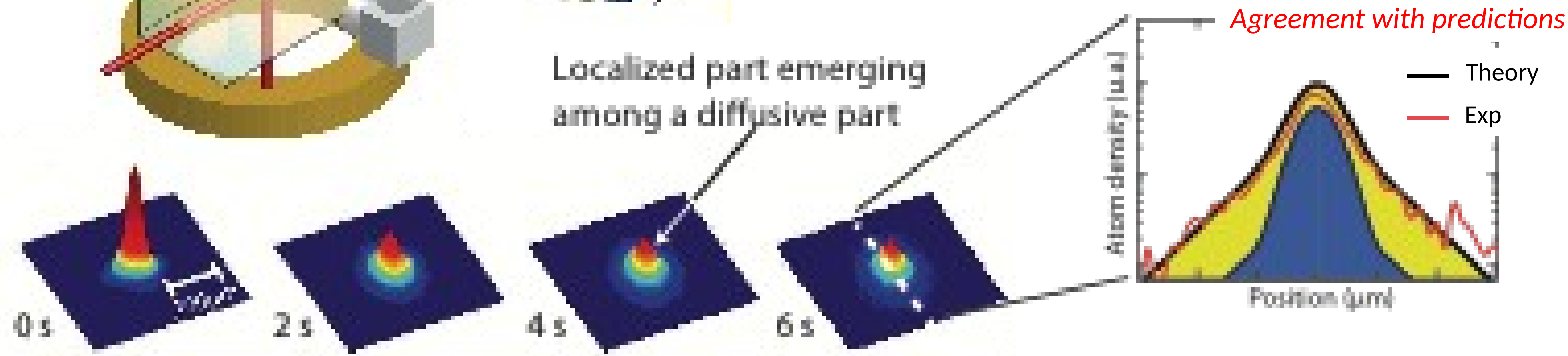
$$\sigma^2_{\rm rms}(t) = \sigma_0^2 + 2Dt$$



Palaiseau's experiment (2012)

Palaiseau, ^{87}Rb (bosons)
3D laser speckle disorder + levitation
Jendrzejewski et al. Nat. Phys. (2012)





⇨ A good news: observation of an 3D Anderson localized component

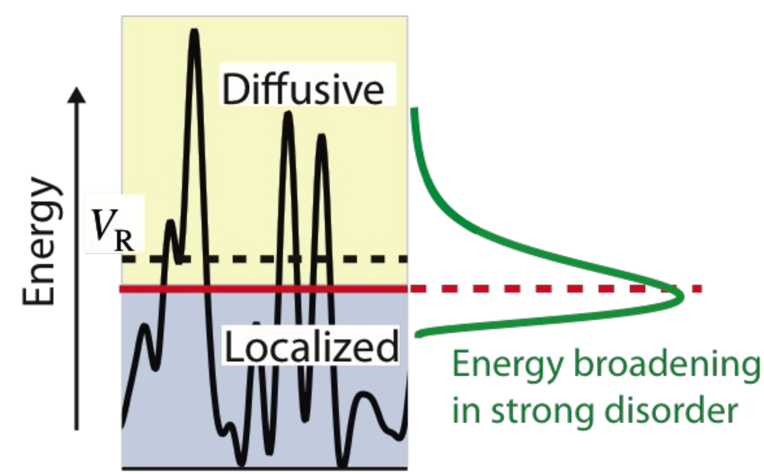
⇨ A bad news: a diffusive component at the same time

Strong disorder
to achieve localization
(Hard to localize)

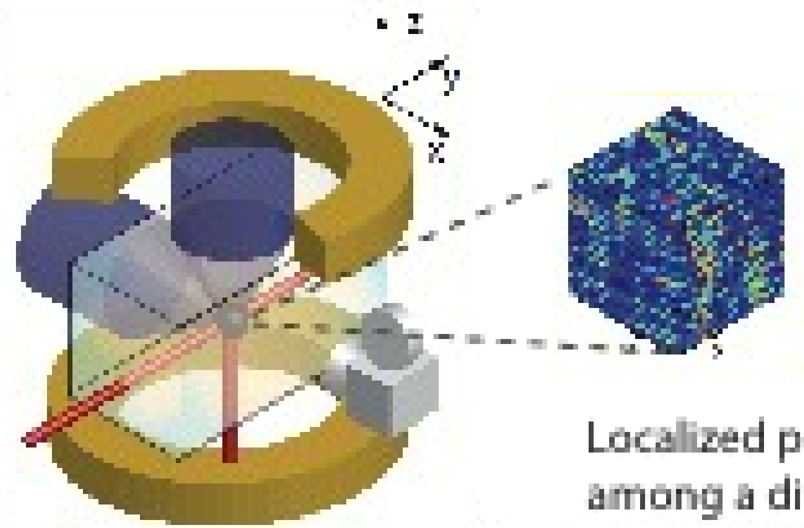
$$kl_{\rm B} \sim 1$$

⇨

Spreading
of the energy
Distribution
(fast switch on)



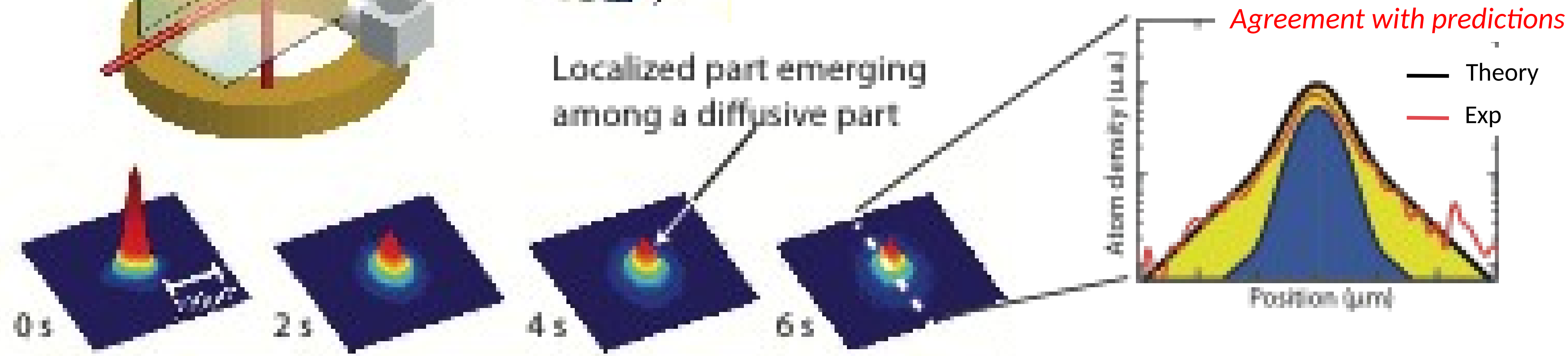
Palaiseau's experiment (2012)



Palaiseau, ^{87}Rb (bosons)

3D laser speckle disorder + levitation

Jendrzejewski et al. Nat. Phys. (2012)

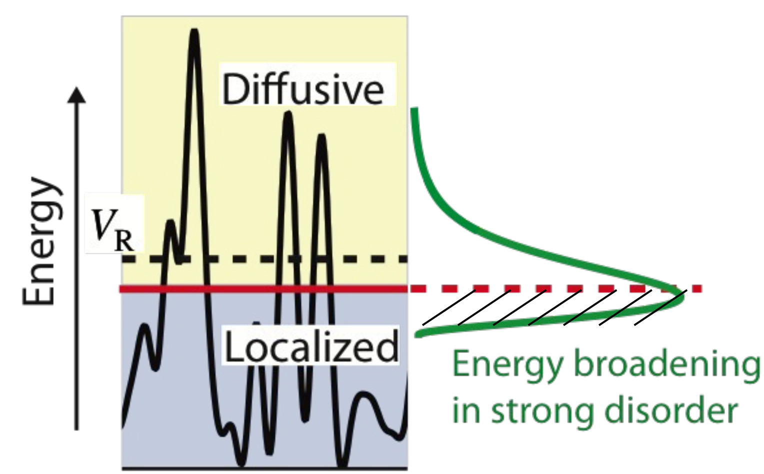


⇨ A good news: observation of an 3D Anderson localized component

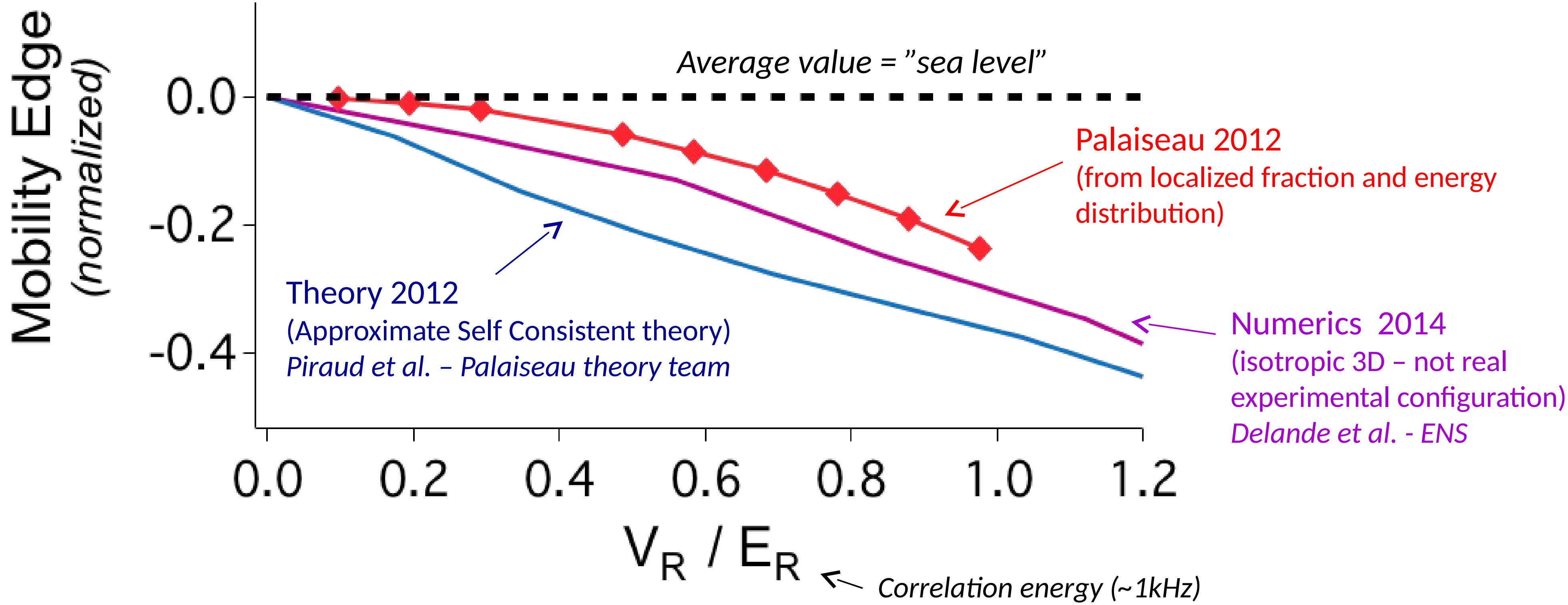
⇨ A bad news: a diffusive component at the same time

⇨ Estimation of the mobility edge from:

1. Localized fraction (experimental profiles)
2. Energy distribution (from numerics)



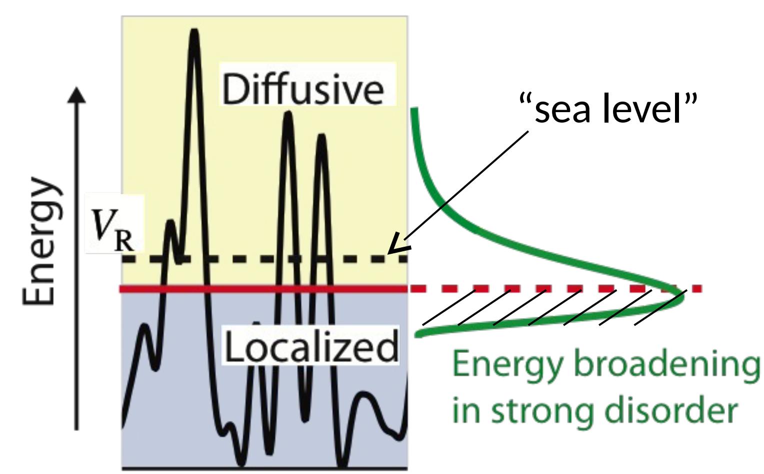
Palaiseau's experiment (2012)



⇨ "Not so bad" but :

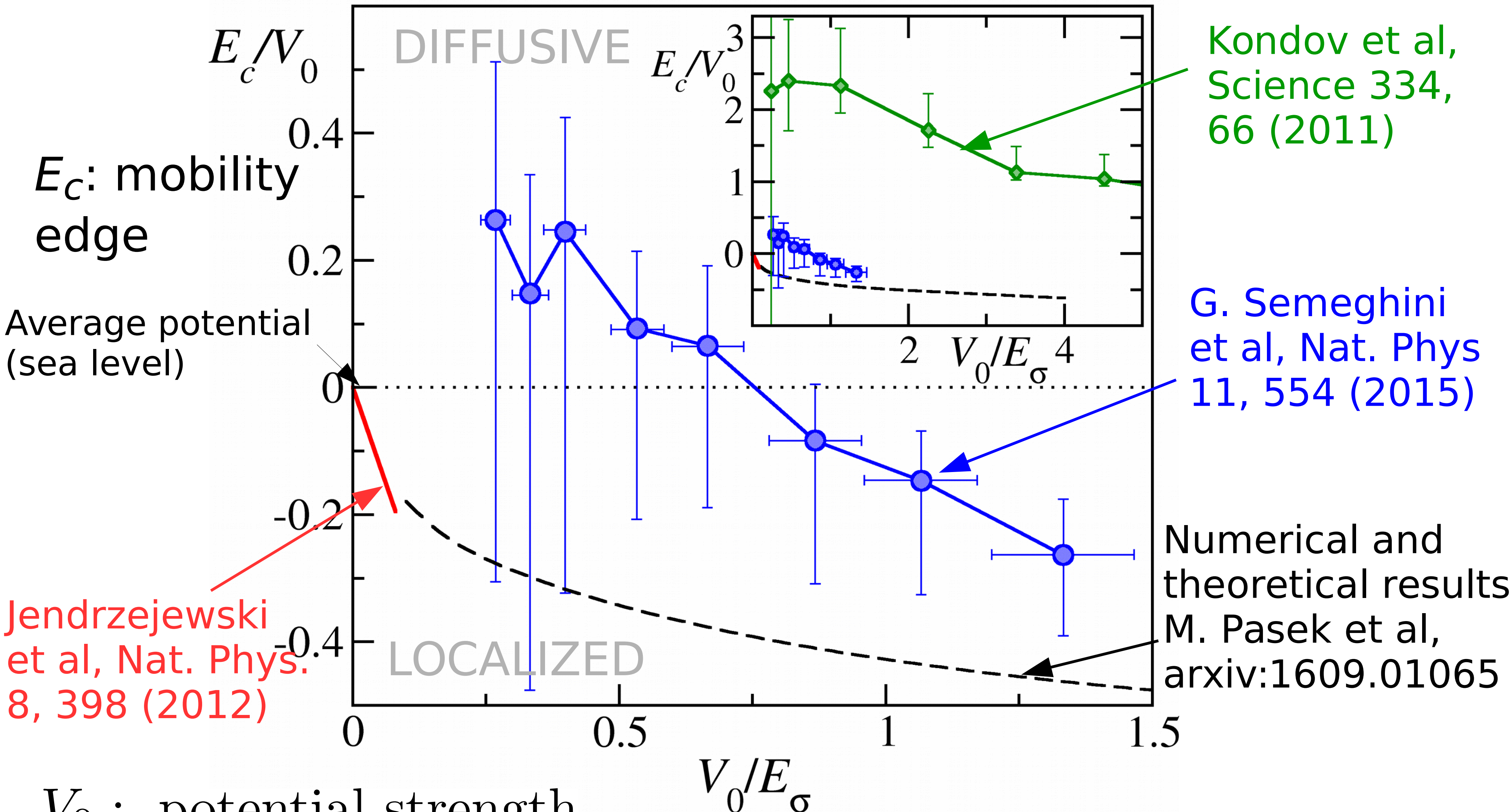
- too early to conclude
- Indirect measurement

(need to know the energy distribution from numerics)



Mobility edge for cold atoms in a disordered optical potential

Experimental results

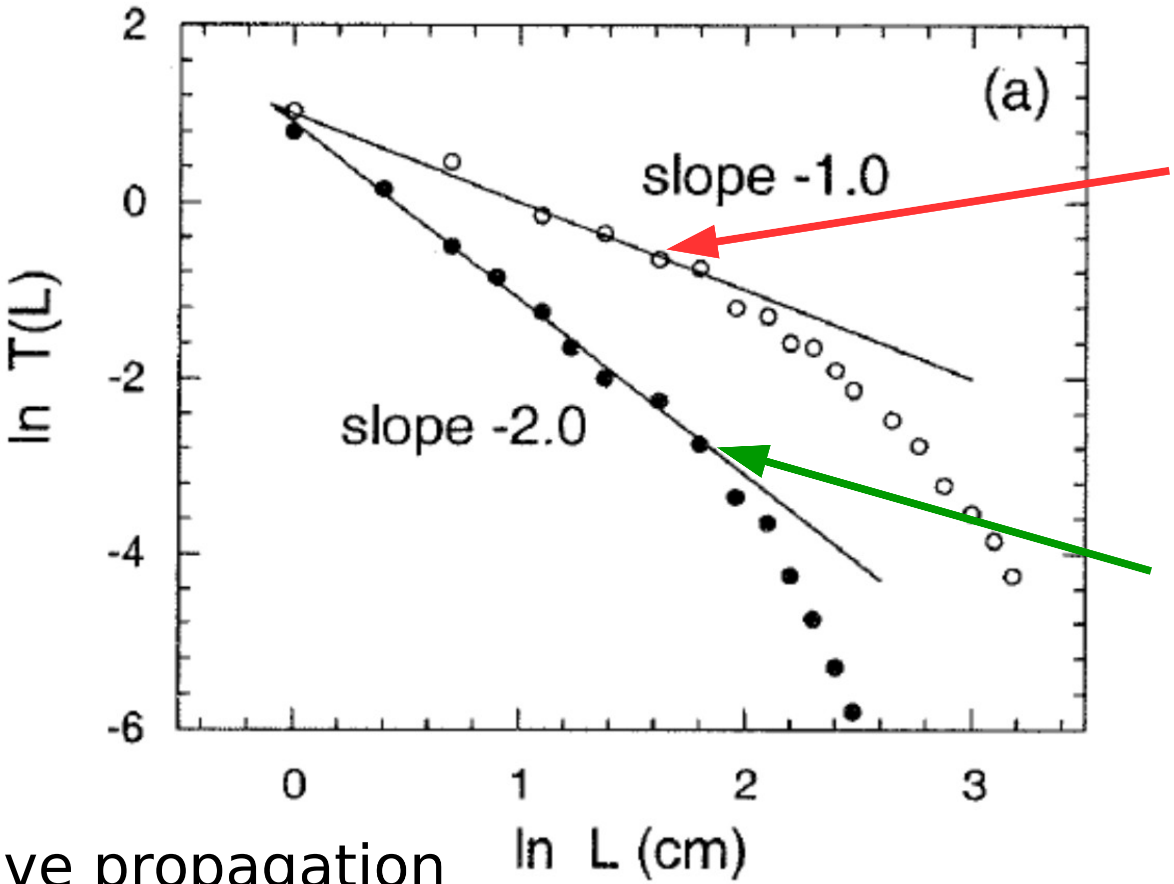


V_0 : potential strength

$$E_\sigma = \frac{\hbar^2}{m\sigma^2} \text{ : correlation energy } (\sigma : \text{potential correlation length})$$

Anderson localization of electromagnetic waves

- All claims of experimental observation of Anderson localization of light have been withdrawn, see S.E. Skipetrov and J. Page, NJP 18, 021101 (2016)
- Observation with microwaves (transmission and fluctuations):



Diffusive regime (Ohm's law)

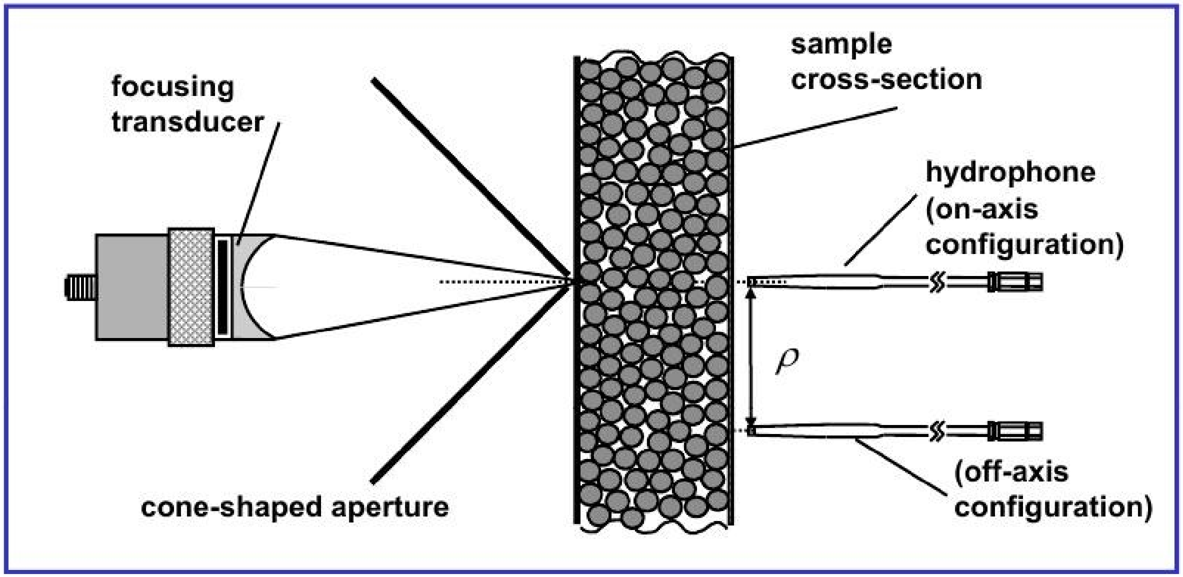
Critical regime $1/L^2$

Microwave propagation in a mixture of Teflon and Aluminium spheres

N. Garcia and A.Z. Genack, PRL, 66, 1850 (1991)

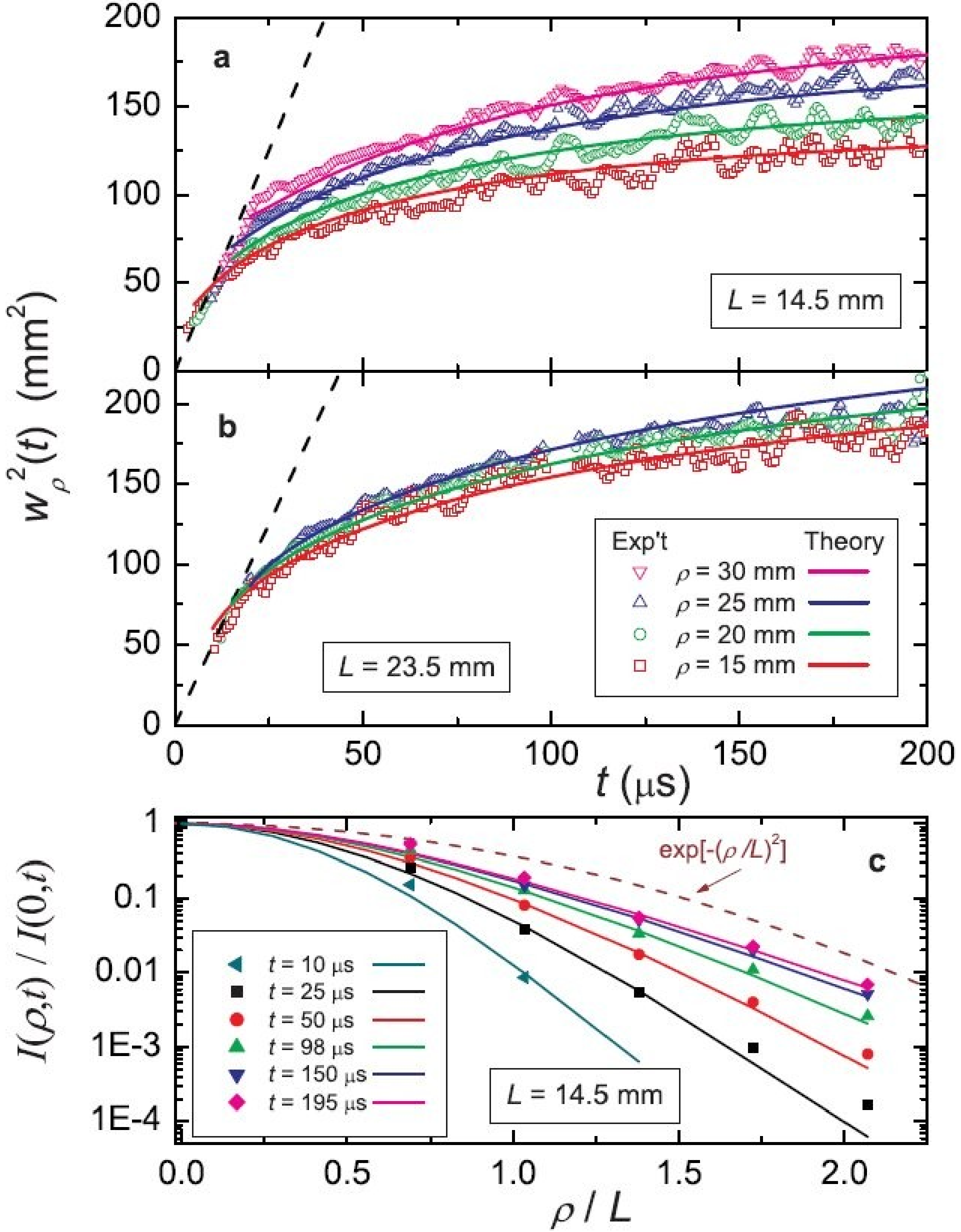
Anderson localization of acoustic waves

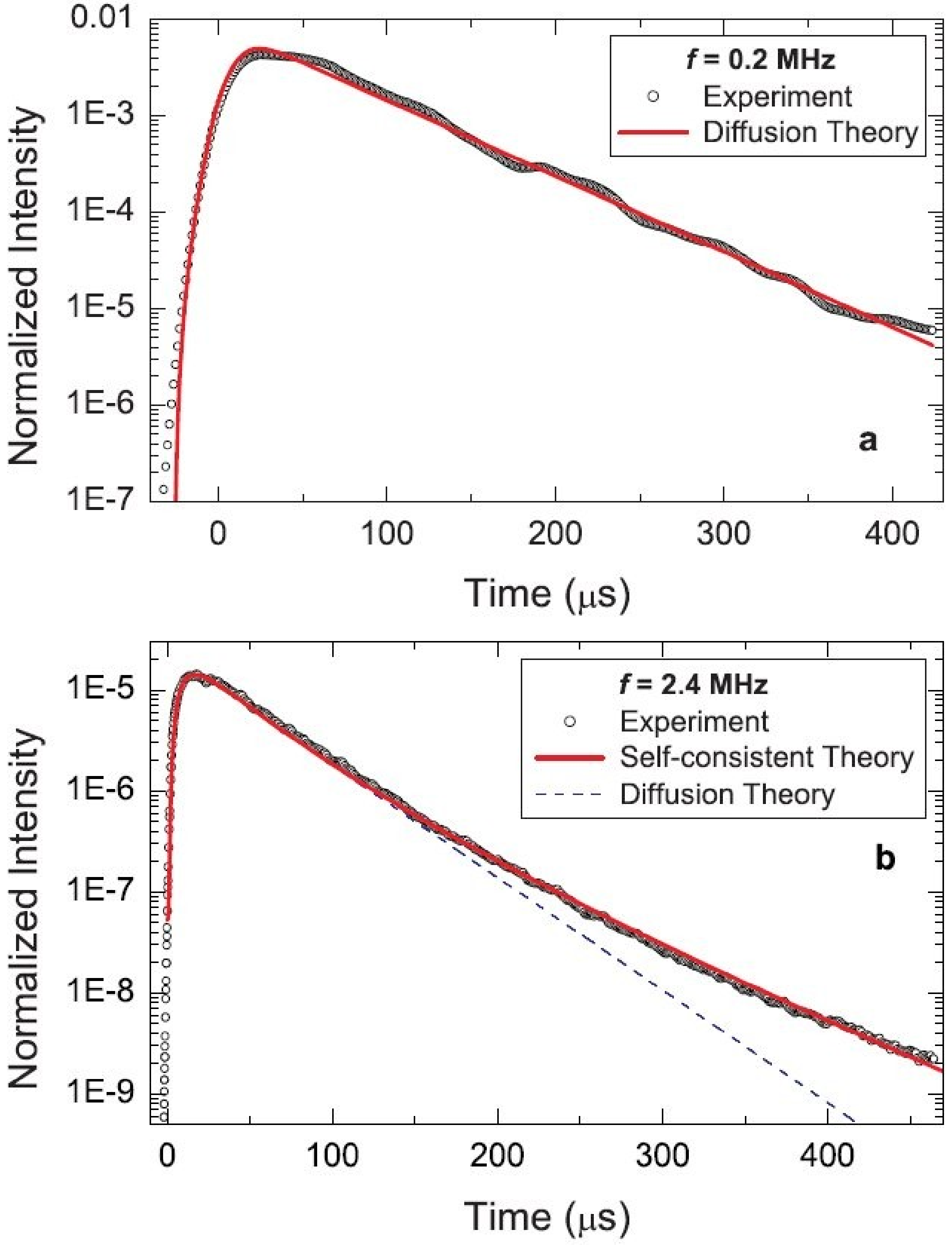
- Packed (disordered) aluminium beads
- Inject acoustic wave at a given point



- Look at the spatial profile of the transmitted intensity
  - In the diffusive regime, expect a Gaussian profile (even in the presence of absorption!)
  - Theory uses a position-dependent diffusion coefficient (B. v. Tiggelen et al, LPMMC Grenoble)
  - Experiment in the group of J. H. Page (Winnipeg)

Anderson localization of acoustic waves





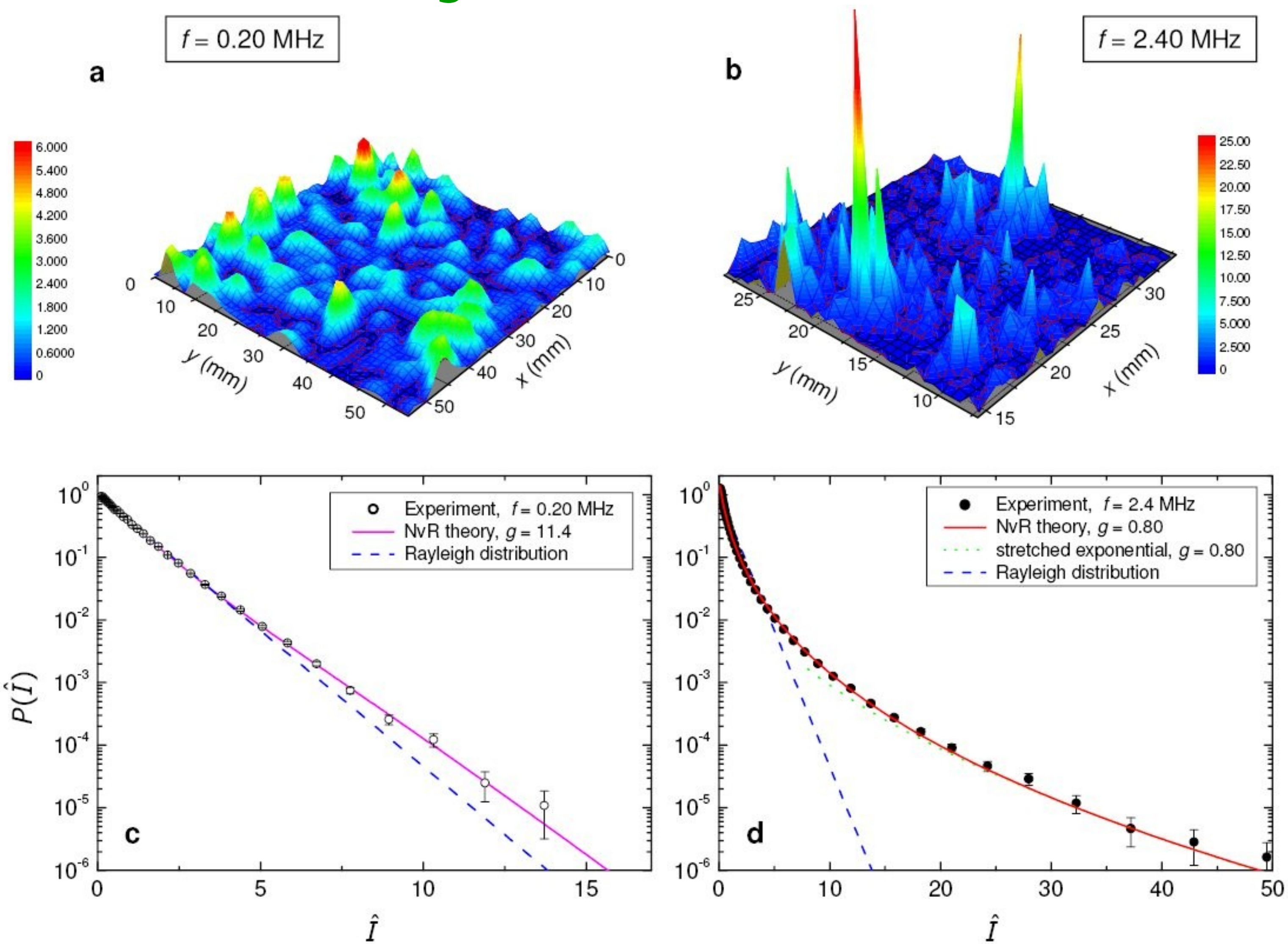
Spatial profile on the outgoing face

Total transmitted intensity

H. Hu et al, Nat. Phys. 4, 495 (2008), group of J. H. Page (Winnipeg)

Anderson localization of acoustic waves: fluctuations

Diffusive regime   Localized regime



H. Hu et al, Nat. Phys. 4, 495 (2008), group of J. H. Page (Winnipeg)

See also multifractality of the intensity distribution:
S. Faez et al, PRL 103, 155703 (2009)

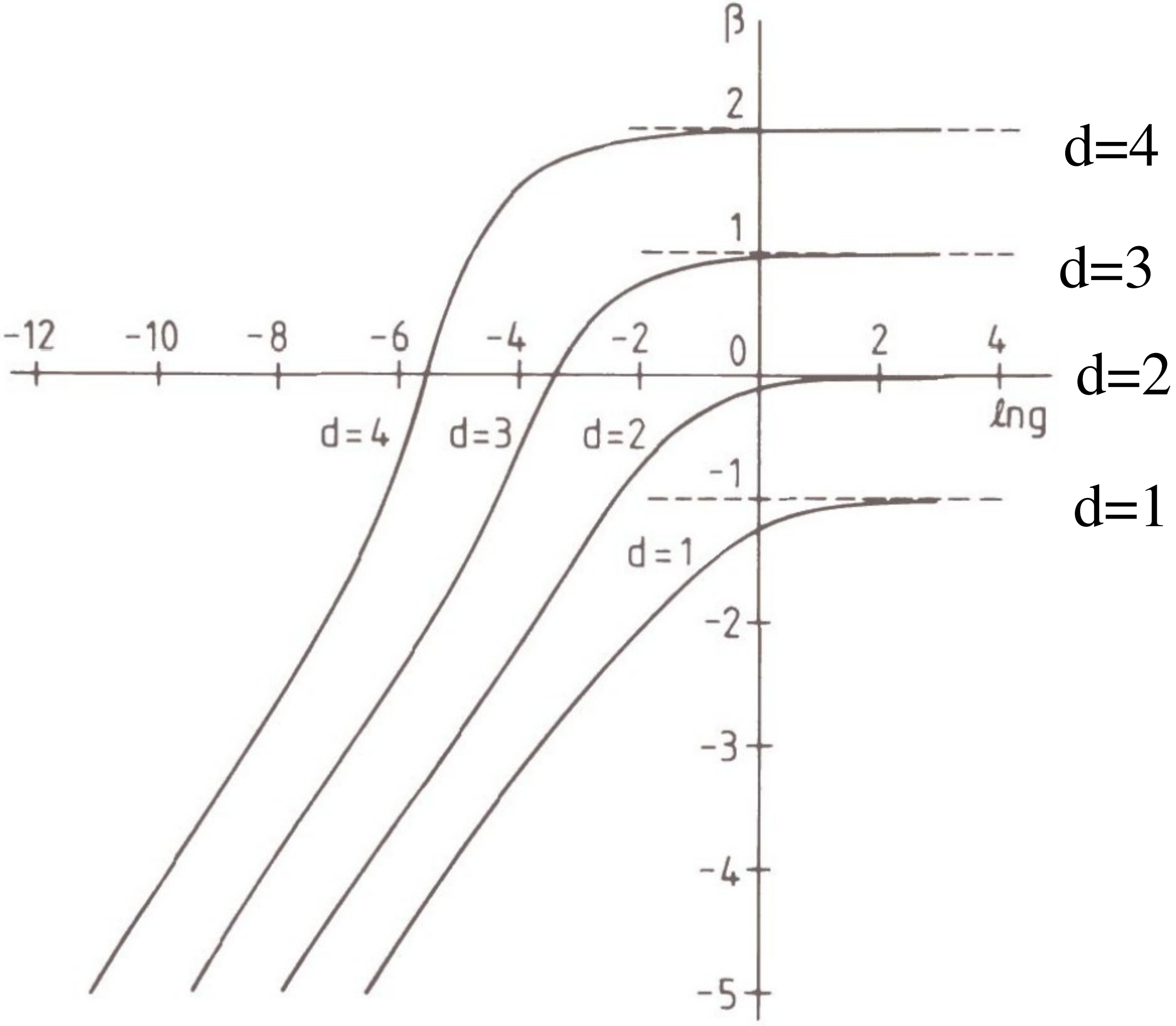
The atomic kicked rotor: a simple system to study transport and localization

- 1D Hamiltonian
  - Standard kicked rotor: $$H = \frac{p^2}{2} + K \cos x \sum \delta(t-n)$$
  - Quasi-periodically kicked rotor:

$$H = \frac{p^2}{2} + K \cos x \left[1 + \varepsilon \cos(\omega_2 t) \cos(\omega_3 t)\right] \sum \delta(t-n)$$

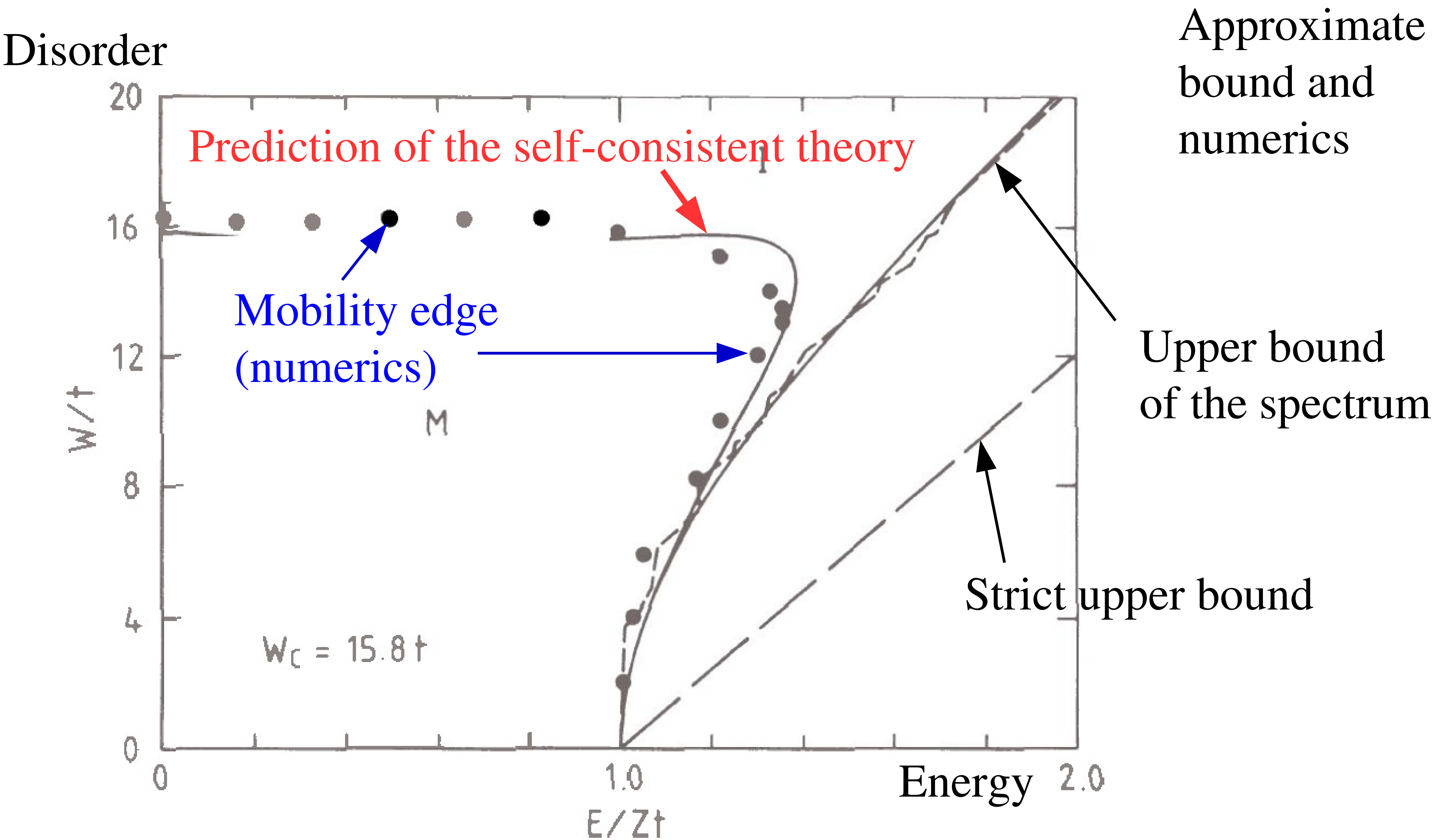
- Classically chaotic dynamics => (deterministic) pseudo-disordered system.
  - Standard kicked rotor: 1D Anderson-like model
  - With (*d*-1) quasi-periods : Anderson model in dimension *d*.
- Experiment with cold Caesium atoms in the group of J.C. Garreau and P. Szrifgiser (Phlam, Lille).
  - Observation of 1D localization: F. Moore et al, PRL 73, 2974 (1994)
  - 3D localization and metal-insulator transition with the first measure of the critical exponent: J. Chabé et al, PRL 101,255702 (2008)
  - Universality of the critical exponent: M. Lopez et al, PRL, 108, 095701 (2012)
  - Dynamics at the critical point: G. Lemarié et al, PRL 105, 090601 (2010)
  - Exponential dependence of the localization length in 2D: I. Manai et al, PRL 115, 240603 (2015)
  - Controled breaking of time-reversal symmetry and observation of enhanced return to origin: C. Hainaut et al, arxiv: 1606.07237

Scaling function deduced from self-consistent theory of localization



P. Wölfle and D. Vollhardt, *Self-Consistent Theory of Anderson Localization: General Formalism and Applications,* arXiv: 1004.3238

Self-consistent theory of localization for the Anderson model (box disorder)



D. Vollhardt and P. Wölfle, *Self-consistent theory of Anderson Localization*, in: W. Hanke and Y. V. Kopaev, editors, *Electronic phase transitions* (Elsevier, Amsterdam, 1992)