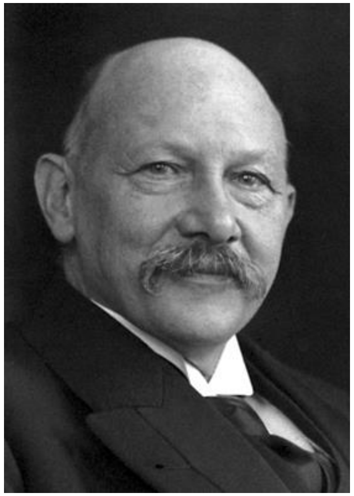
Disordered Superconductors

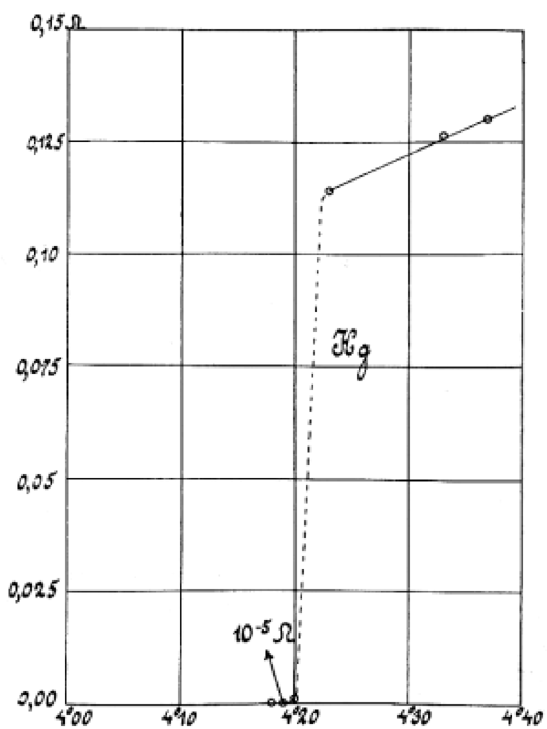
Claude Chapelier,
INAC-PHELIQS, CEA-Grenoble

INSTITUT NANOSCIENCES
ET CRYOGÉNIE

UNIVERSITÉ
Grenoble
Alpes

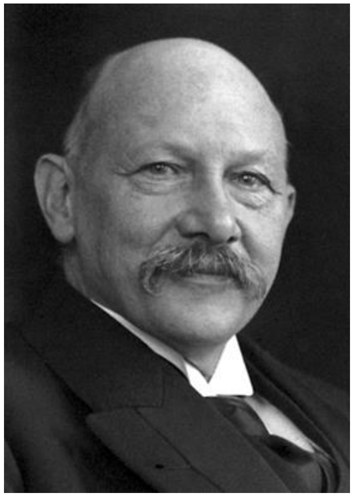
Superconductivity in pure metals

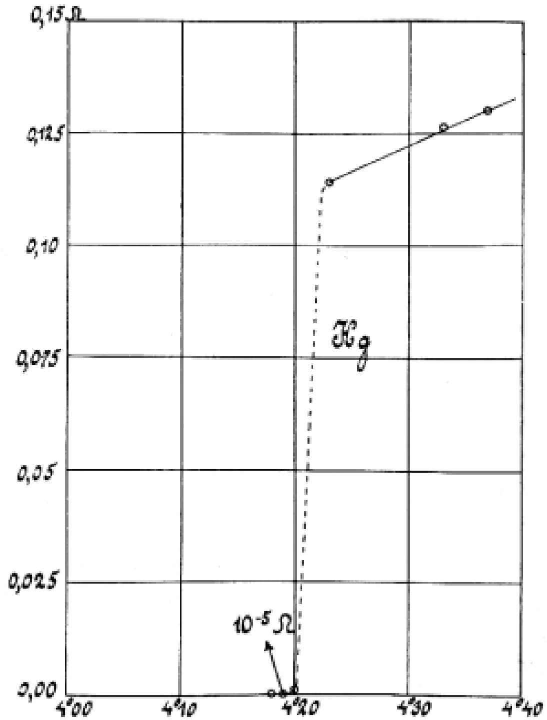




Kamerlingh Onnes, H., "Further experiments with liquid helium. C. On the change of electric resistance of pure metals at very low temperatures, etc. IV. The resistance of pure mercury at helium temperatures." *Comm. Phys. Lab. Univ. Leiden*; No. 120b, 1911.

Superconductivity in pure metals *and alloys …*





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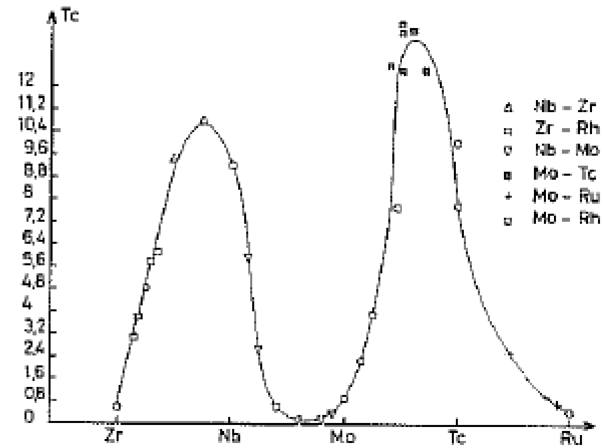
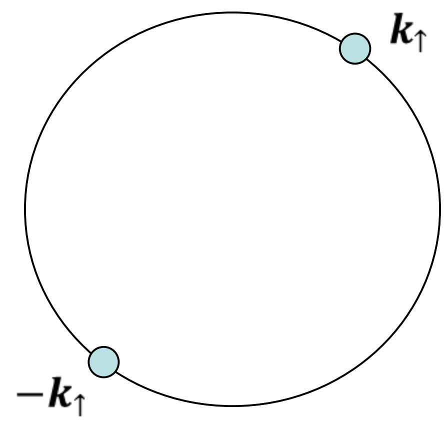


FIG. 1.11. — *Variation de T_c dans les alliages à base de* Zr, Nb, Mo, Tc *et* Ru (voir ROBERTS).

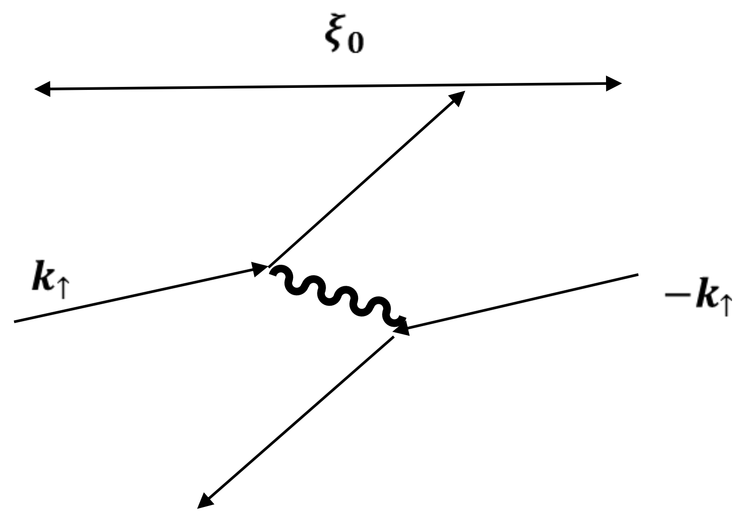
J.P. Burger
la supraconductivité des métaux, des alliages et des films minces (Ed.Masson)



Bloch plane waves

$$\phi_{k,\sigma} = \frac{1}{\sqrt{V}}\, e^{ikr}$$

$$\phi_{-k,-\sigma} = \frac{1}{\sqrt{V}}\, e^{-ikr}$$



J. Bardeen, L.N. Cooper and J.R. Schrieffer, *Phys. Rev. B.* ***108***, 1175, (1957)
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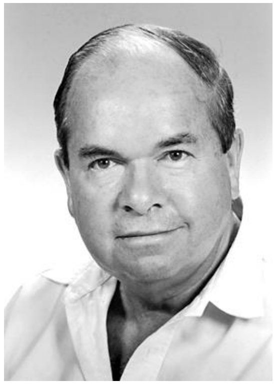
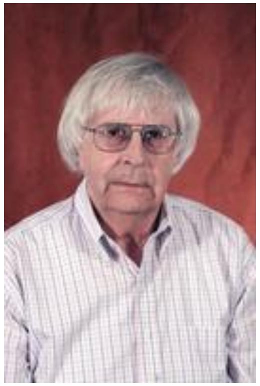
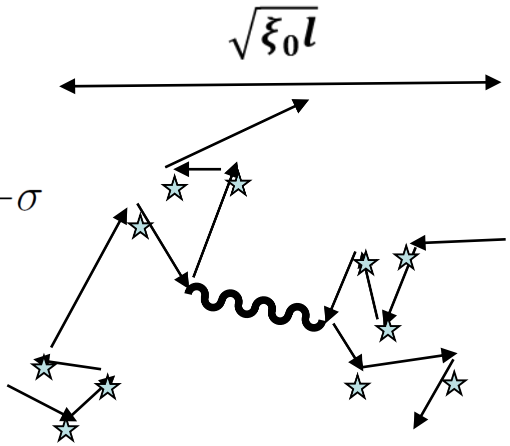
Theory of dirty superconductors





$$\psi_{n,\sigma} = \sum_{n} < n \mid k > \phi_{k,\sigma}$$

$$\psi^{*}_{n,\sigma} = \sum_{n} < n \mid k >^{*} \phi_{-k,-\sigma}$$



The scatterers are non-magnetic (time-reversed symmetry)

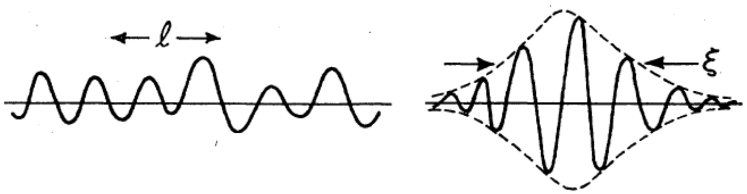
P.W. Anderson, *J. Phys. Chem. Solids.* ***11***, 26, (1959)

A.A. Abrikosov & I.P. Gorkov, *Sov. Phys. JETP* ***8***, 1090, (1959)

In superconducting grains, superconductivity disappears when the mean level spacing between different electronic states becomes greater than the superconducting gap

Anderson criterion for superconductivity : $\nu \, \Delta \, L^3 > 1$

Localization in disordered metals



$$\phi_{k,\sigma} = \frac{1}{\sqrt{V}}\, e^{ikr}\, e^{-\frac{|r-r_0|}{\xi}}$$

P.W. Anderson,
Absence of diffusion in certain random lattices
Phys. Rev. ***109***, 1492(1958)

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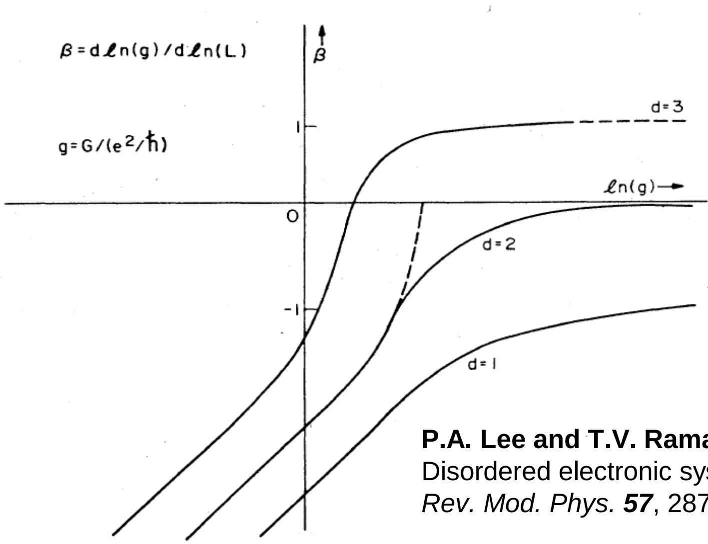

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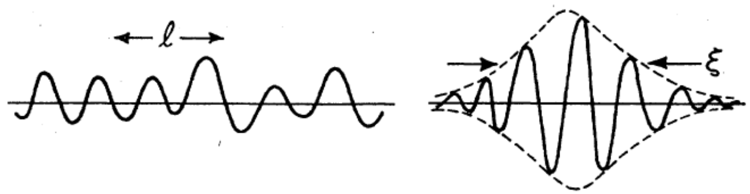

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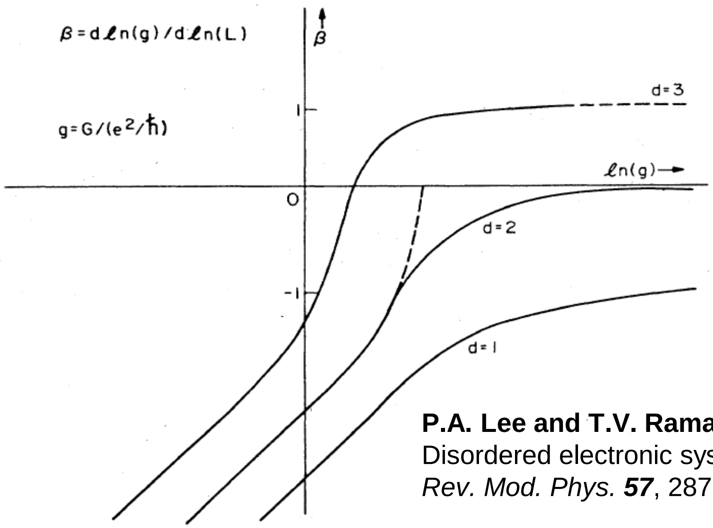

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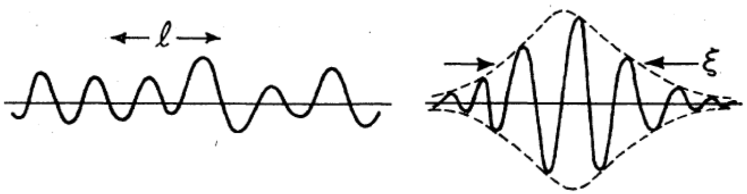

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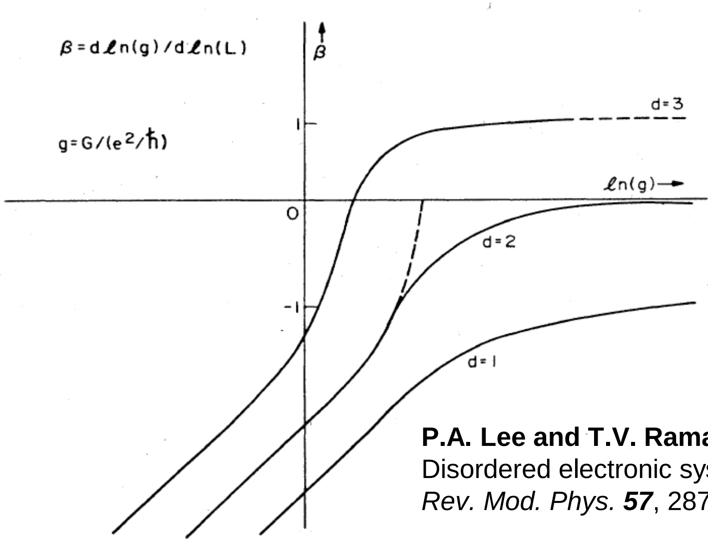

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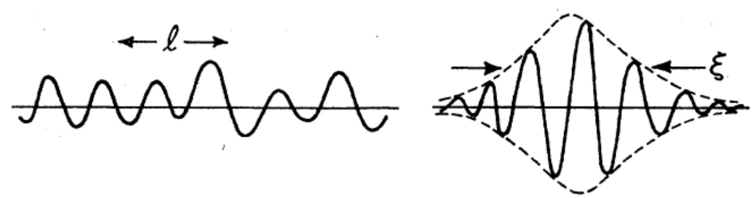

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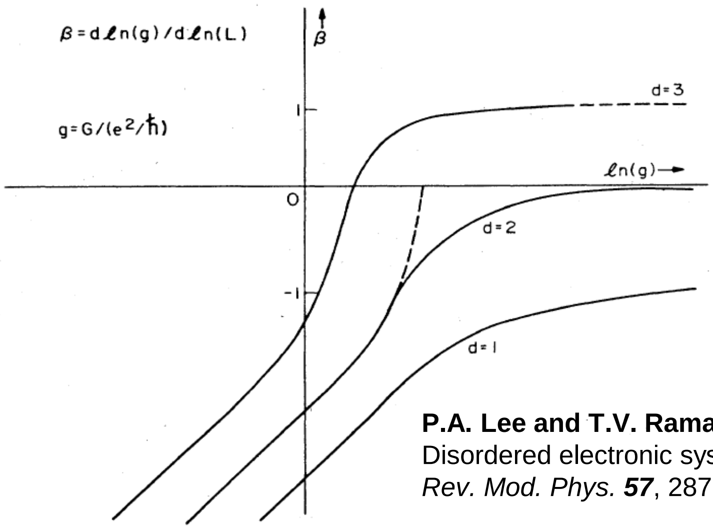

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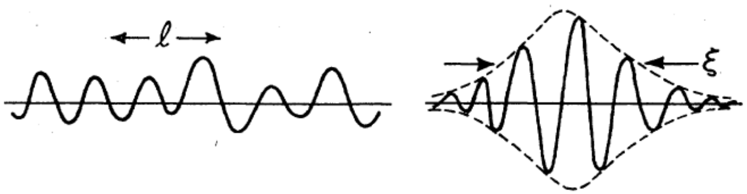

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Matthew P. A. Fisher and G. Grinstein
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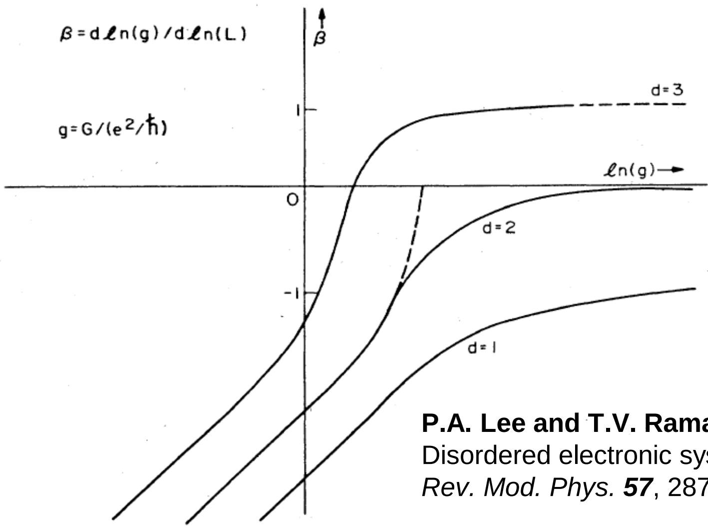
(Received 17 November 1989)



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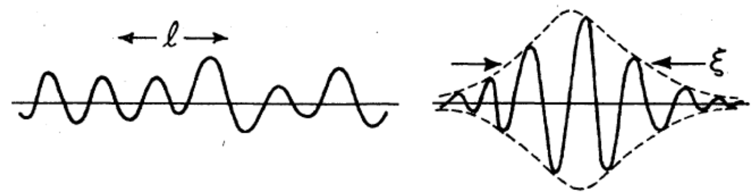

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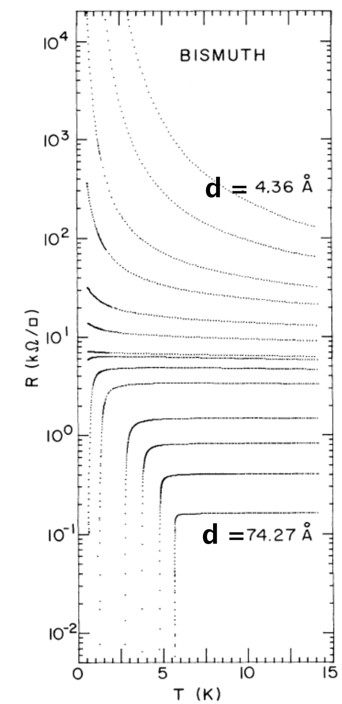

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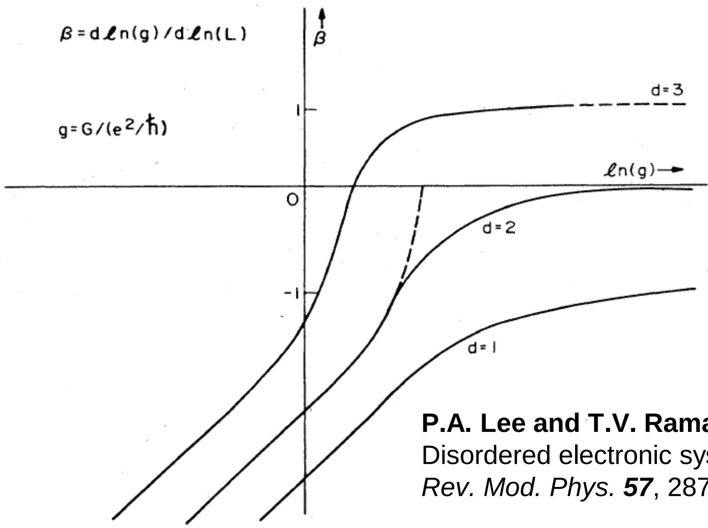

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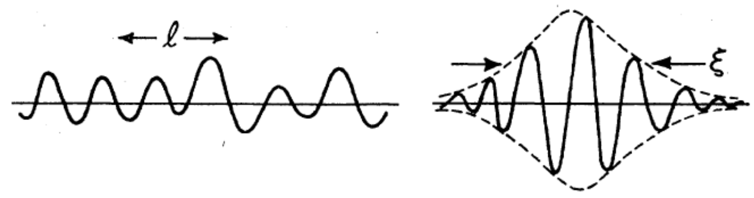

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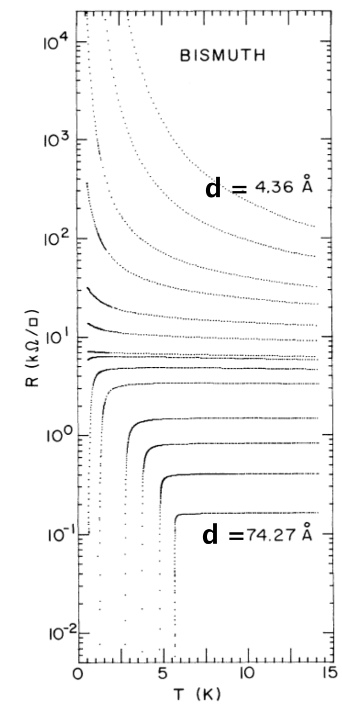

- Strong localization, fluctuations and Coulomb interactions

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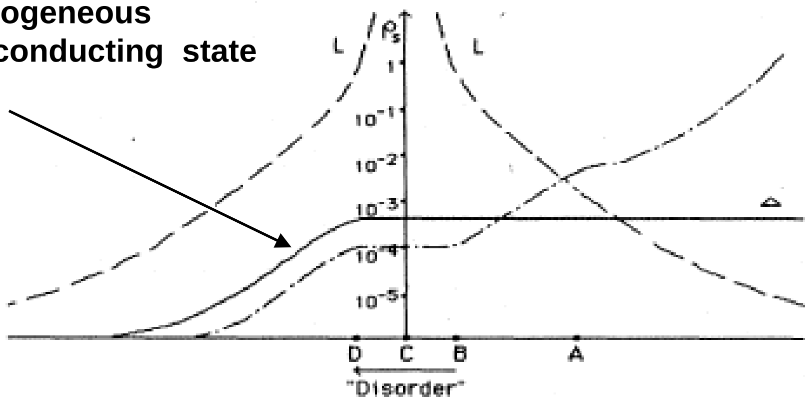
3D localization

Insulator **Metal**

Inhomogeneous superconducting state



$k_F\, l \ll 1$ $\nu\, \Delta\, \xi_{\rm loc}^3 \sim 1$ $k_F\, l \sim 1$ $k_F\, l \gg 1$

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M. Ma, P.A. Lee, *Phys. Rev. B* ***32***, 5658, (1985)
G. Kotliar, A. Kapitulnik, *Phys. Rev. B* ***33***, 3146 (1986)
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Localization in disordered superconductors

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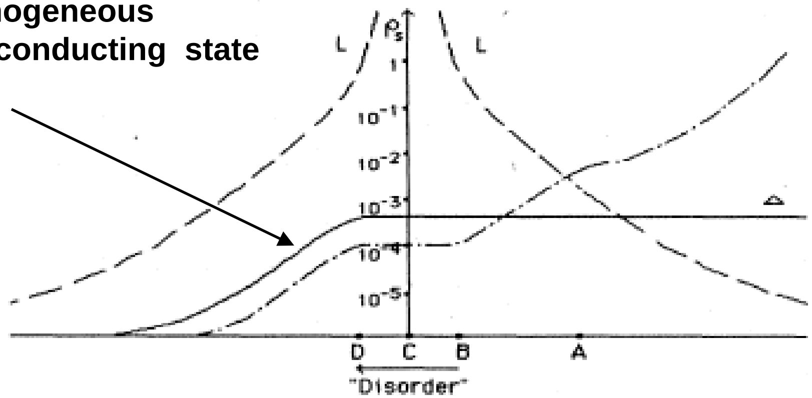

$k_F\, l \ll 1$ $\quad \nu\, \Delta\, \xi_{\mathrm{loc}}^3 \sim 1$ $\quad k_F\, l \sim 1$ $\quad k_F\, l \gg 1$

2D numerical simulation

$$H_{\mathrm{int}} = -\lambda \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$H_0 = -t \sum_{<i,j>,\sigma} \left(c_{i\sigma}^{+} c_{j\sigma} + h.c. \right) + \sum_{i,\sigma} \left(V_i - \mu \right) n_{i,\sigma}$$

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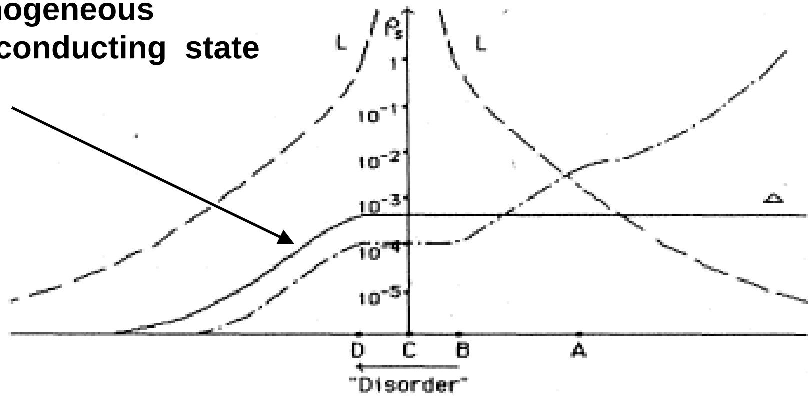

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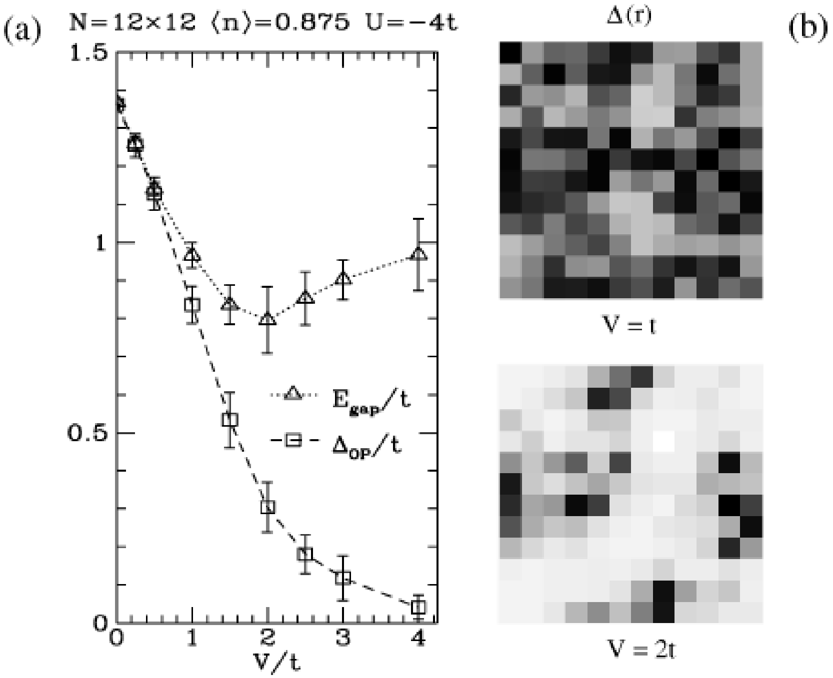

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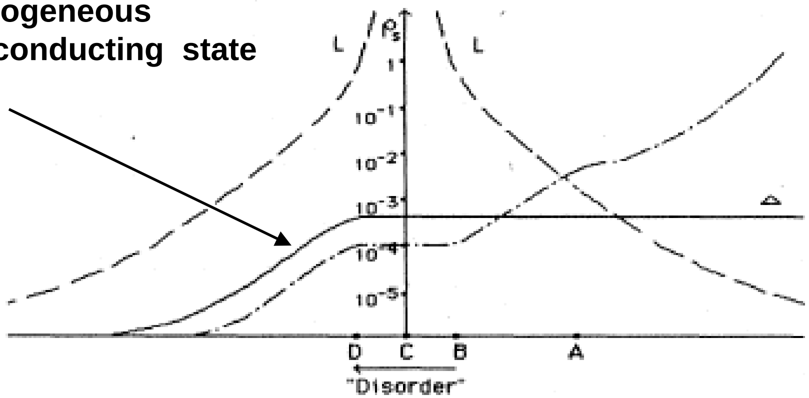
Localization in disordered superconductors

3D localization

Insulator **Metal**

Inhomogeneous superconducting state



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2D Monte Carlo calculation

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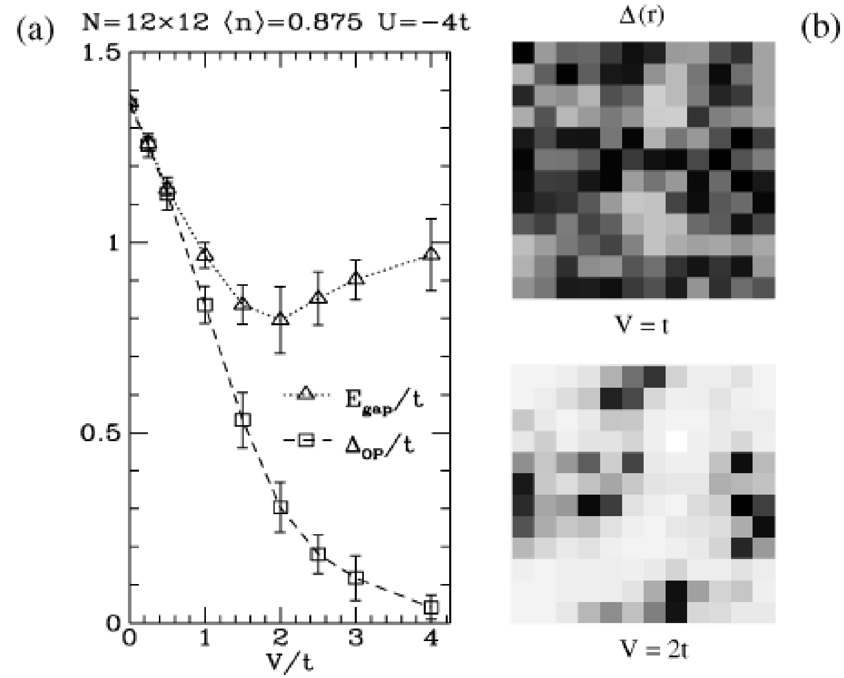

- Cooper pairing beyond the mobility edge leads to an inhomogeneous superconductor
- The transition to an insulator requires quantum fluctuations

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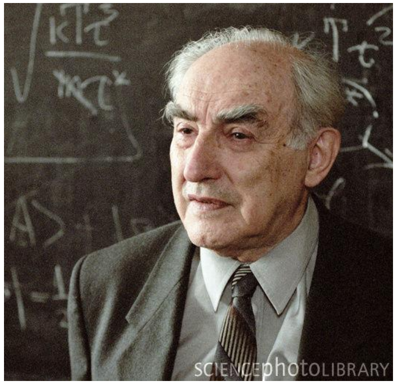
Superconducting fluctuations

Thermal fluctuations $\psi_{op} = \Delta(T)\, e^{i\varphi(T)}$



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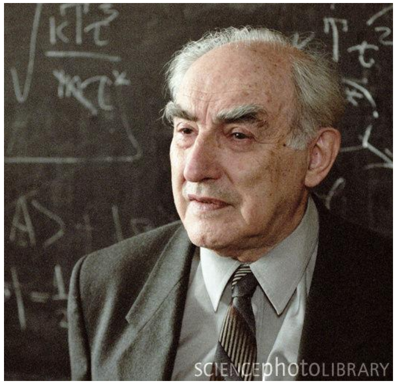
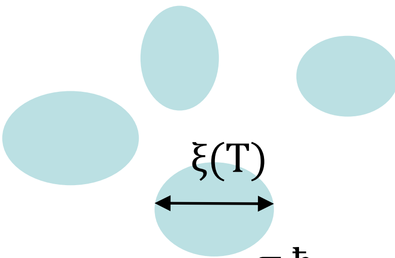


$T > T_c$ Amplitude fluctuations



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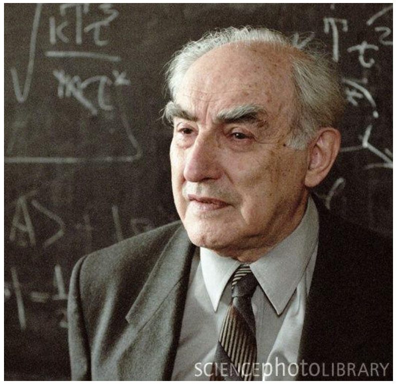
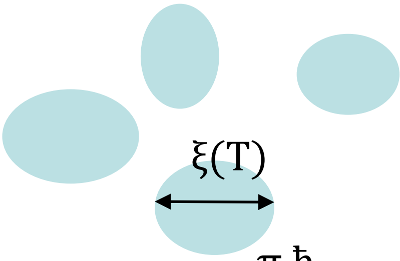


$T > T_c$ Amplitude fluctuations

$T \gtrsim T_c$ Amplitude and phase fluctuations



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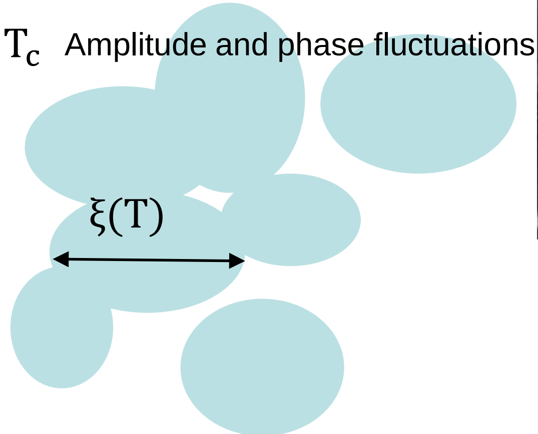

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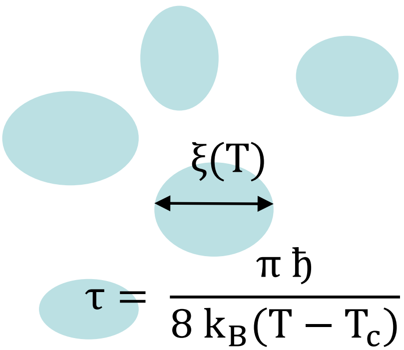

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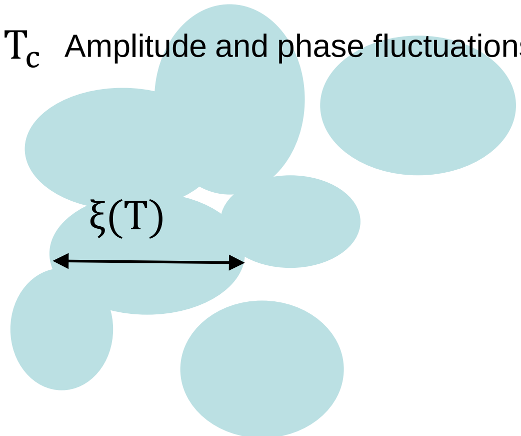
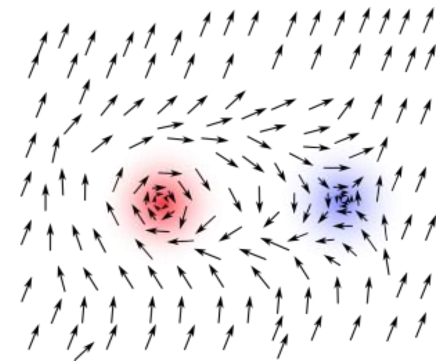


2D : Berezinskii – Kosterlitz - Thouless



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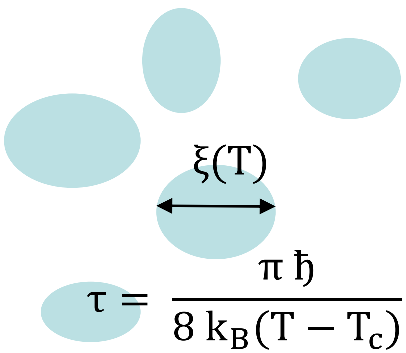

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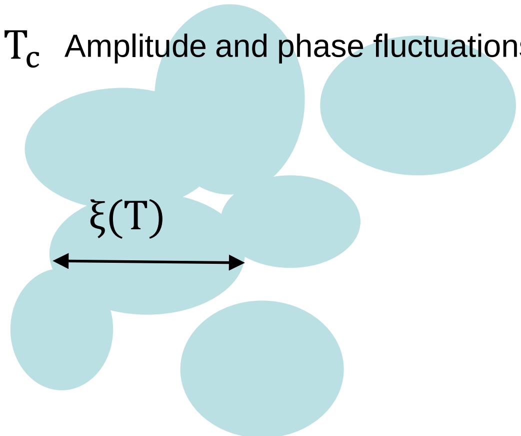
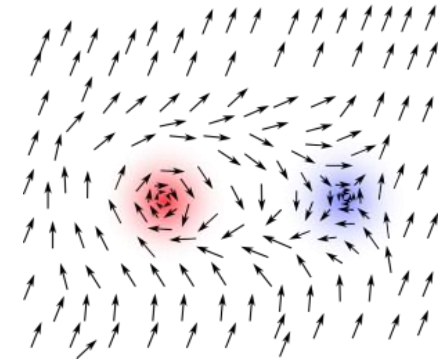
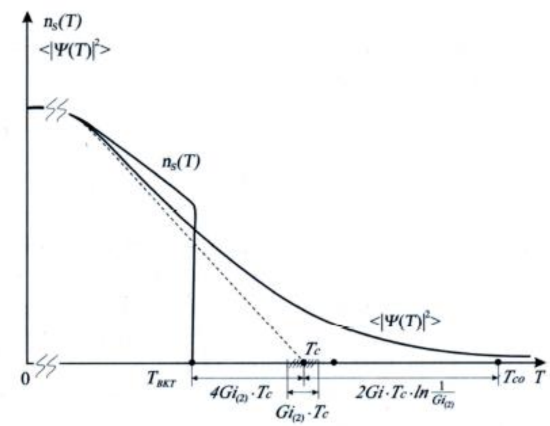
$T \gtrsim T_c$ Amplitude and phase fluctuations



2D : Berezinskii – Kosterlitz - Thouless





$$\frac{\delta T_c}{T_c} \sim G_{i(d)} \sim \left(\frac{1}{\nu\, k_B T_c\, \xi_0^d}\right)^{\frac{2}{4-d}}$$

Clean 3D superconductor : $G_{i(3)} \sim 80 \left(\frac{k_B T_c}{E_F}\right)^4 \sim 10^{-12} - 10^{-14}$

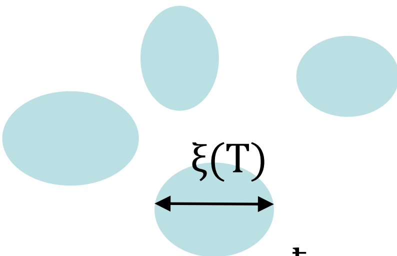
Dirty 2D superconductor : $G_{i(2)} \sim \frac{e^2}{23\,\hbar} R_\square$

3D localized superconductor : $\nu\, \Delta\, \xi_{loc}^3 \sim 1$ $\xi_0 \sim \xi_{loc}$ $G_i \sim 1$

A. Larkin and A. Varlamov,
Theory of fluctuations in superconductors,
Oxford University Press (2006)

Thermal fluctuations $\psi_{op} = \Delta(T)\, e^{i\varphi(T)}$

$T > T_c$ Amplitude fluctuations



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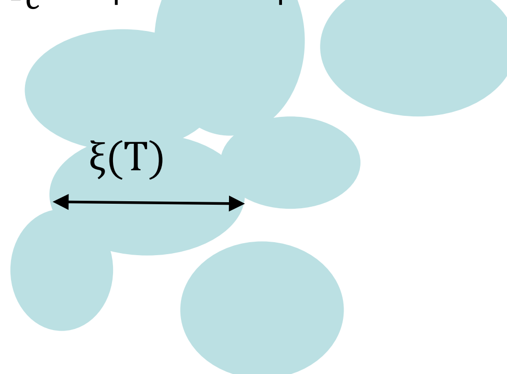

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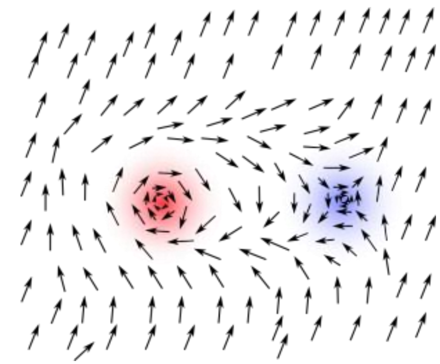

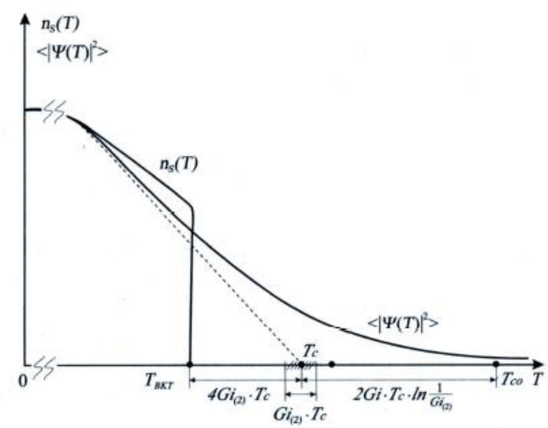

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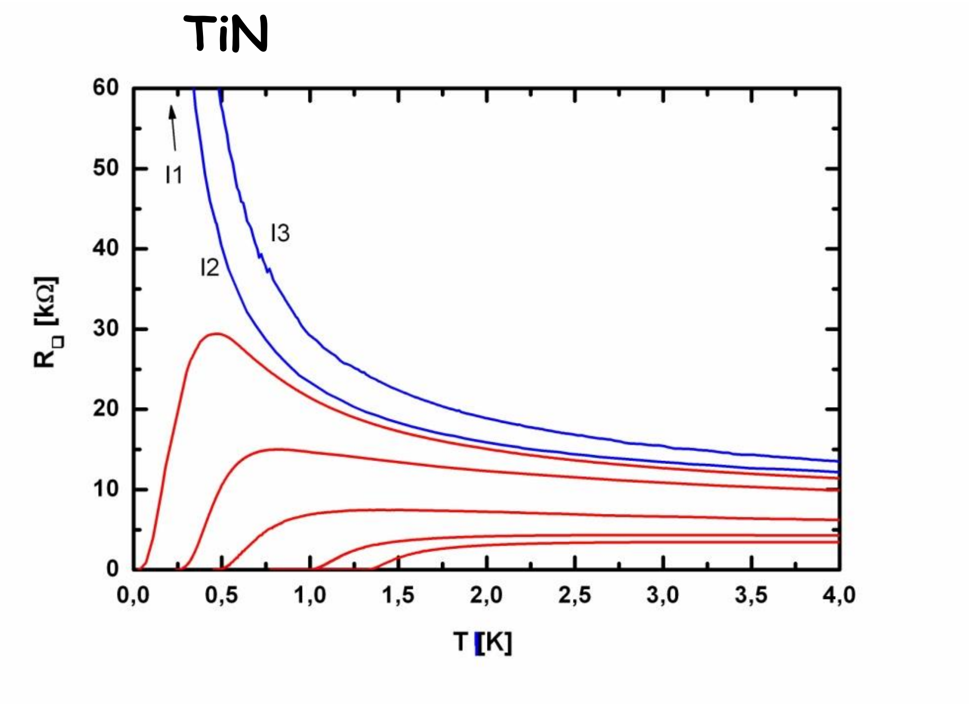
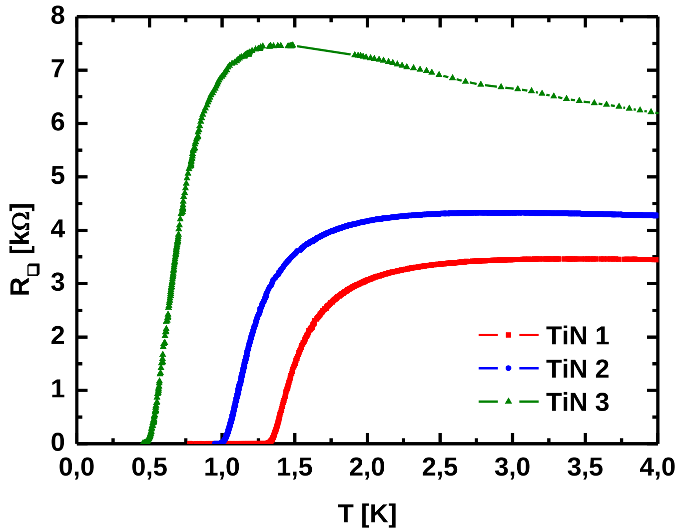
➢ Disorder drastically enhances thermal fluctuations

A. Larkin and A. Varlamov,
Theory of fluctuations in superconductors,
Oxford University Press (2006)

TIN Superconductor-Insulator transition



T. I. Baturina, et al.PRL *99, 257003 (2007)*



Sacépé et al., *PRL **101**, 157006 (2008)*

TIN Superconductor-Insulator transition

$$\Delta\sigma = \Delta\sigma^{WL} + \Delta\sigma^{AA} + \Delta\sigma^{DoS} + \Delta\sigma^{AL} + \Delta\sigma^{MT}$$

$$\sigma_{00} = \frac{e^2}{2\pi\hbar}$$

$$\frac{\Delta\sigma^{WL} + \Delta\sigma^{AA}}{\sigma_{00}} = A \ln(T\tau)$$

$$\frac{\Delta\sigma^{DoS}}{\sigma_{00}} = \ln\left(\frac{\ln\left(\frac{T_c}{T}\right)}{\ln(T_c\tau)}\right)$$

$$\frac{\Delta\sigma^{AL}}{\sigma_{00}} = \frac{\pi^2}{8}\left(\ln\left(\frac{T}{T_c}\right)\right)^{-1}$$

$$\frac{\Delta\sigma^{MT}}{\sigma_{00}} = \beta\left(\frac{T}{T_c}\right) \ln\left(\frac{\pi}{8\delta}\right)$$

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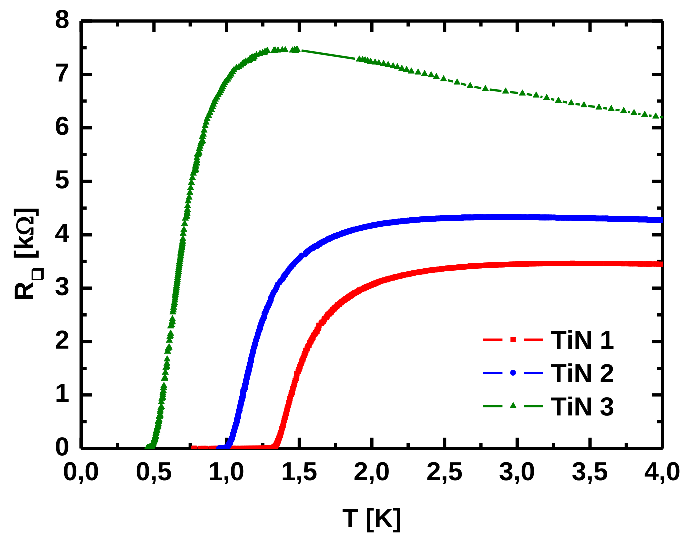

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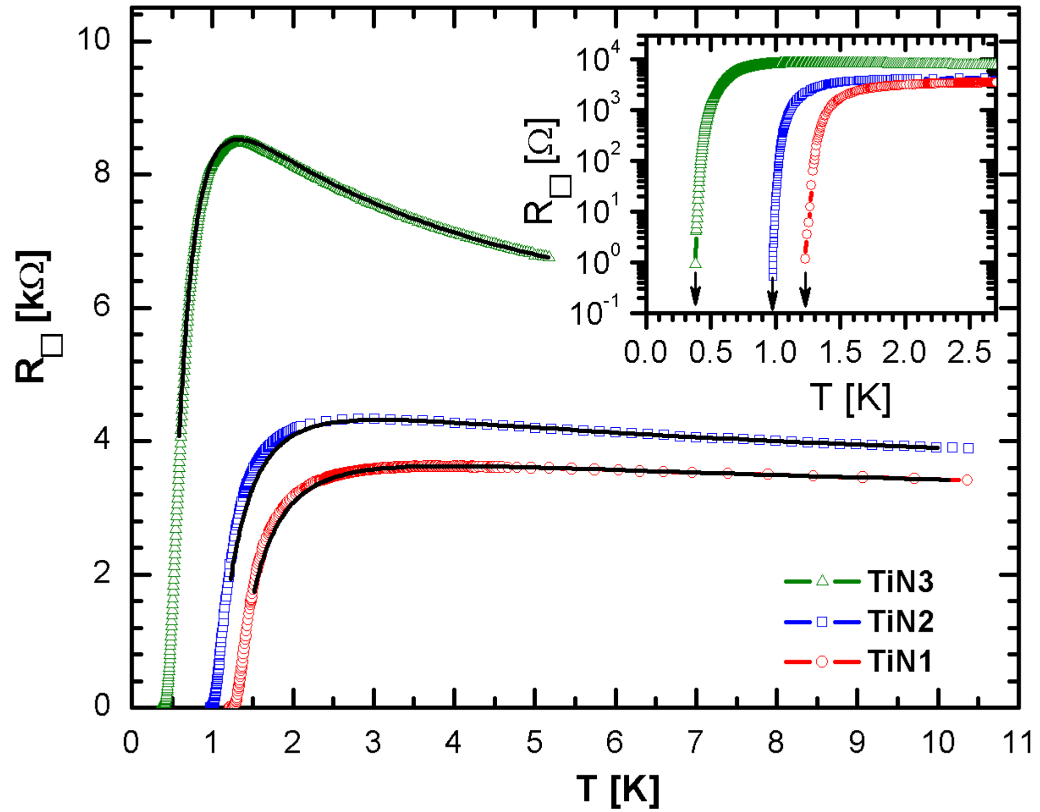

Sacépé et al., *PRL* ***101****, 157006 (2008)*

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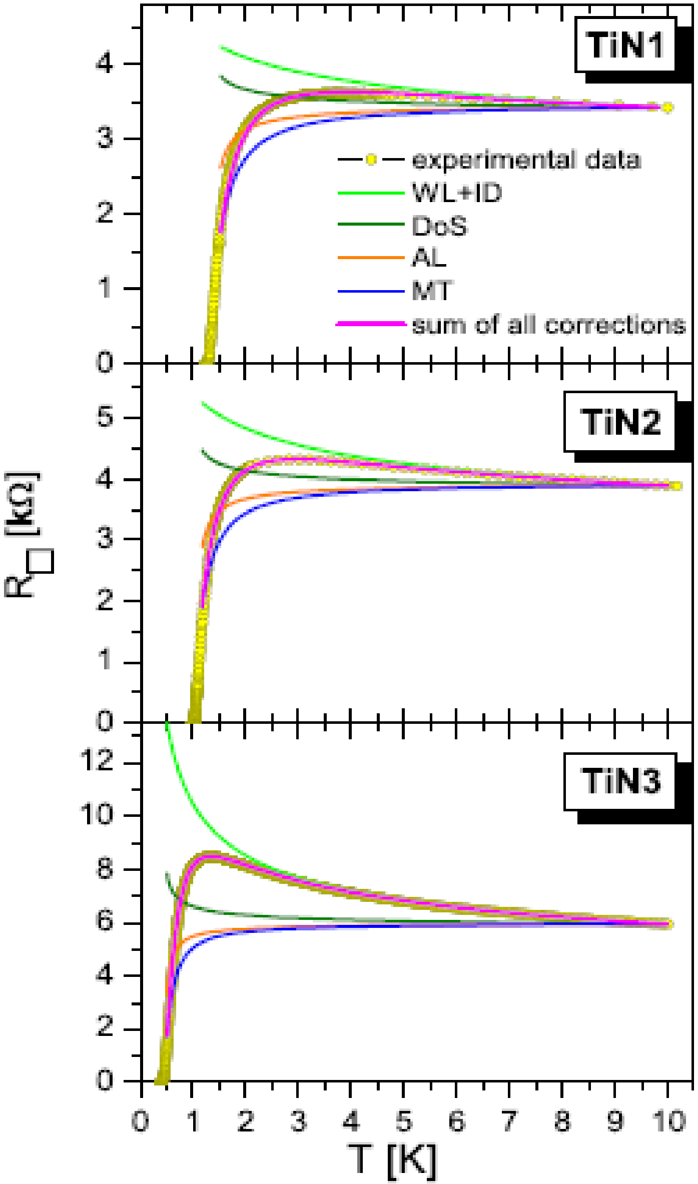

Sacépé et al., *PRL* ***101***, *157006 (2008)*

Superconducting fluctuations

Quantum fluctuations $\psi_{op} = \Delta\, e^{i\varphi}$

K.B. Efetov,
Phase transition in granulated superconductors,
JETP ***51****, 1016 (1980)*

K.A. Matveev and A.I. Larkin
Parity effect in ground state energies of ultrasmall superconducting grains,
Phys. Rev. Lett. ***78****, 3749(1997)*

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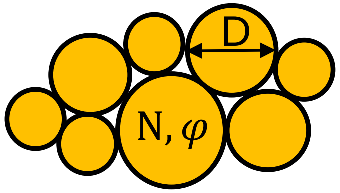
$$\Delta N\, \Delta\varphi > \frac{1}{2}$$



$$E_J = \frac{h\,\pi\,\Delta}{8\,e^2\,R_t}\,\mathrm{th}\left(\frac{\Delta}{2T}\right)$$

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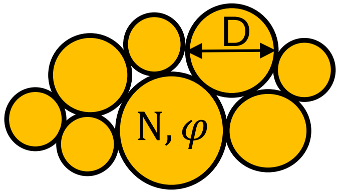

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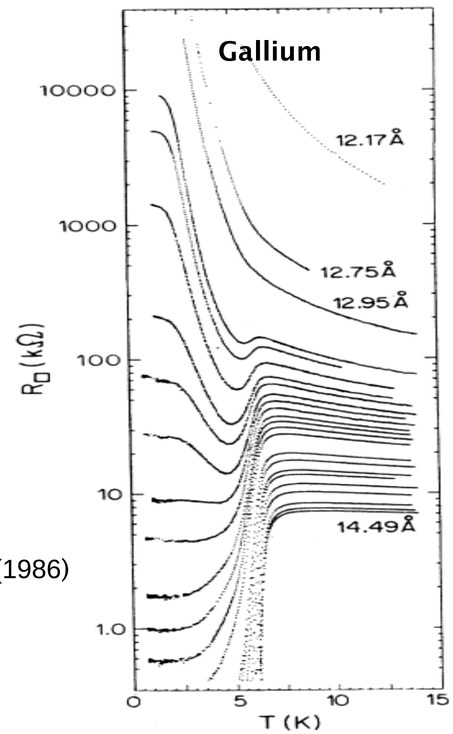

H. M. Jaeger, et al.
Phys.Rev.B 34, 4920 (1986)

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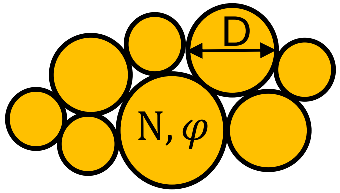

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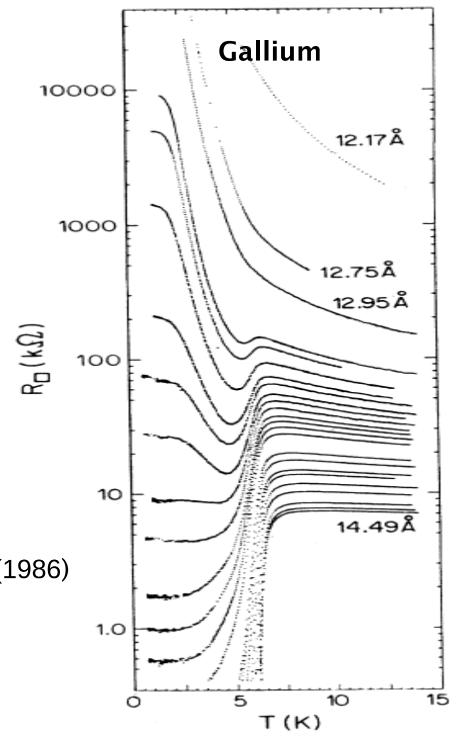

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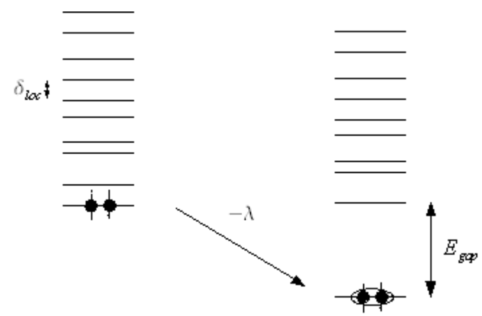
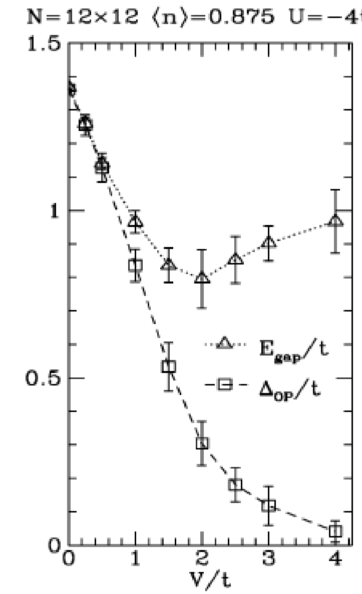
2D Monte Carlo calculation

$$H_{int} = -\lambda \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$H_0 = -t \sum_{<i,j>,\sigma} \left(c_{i\sigma}^+ c_{j\sigma} + h.c.\right) + \sum_{i,\sigma} (V_i - \mu)\, n_{i,\sigma}$$





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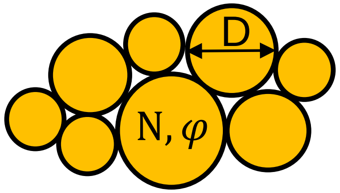

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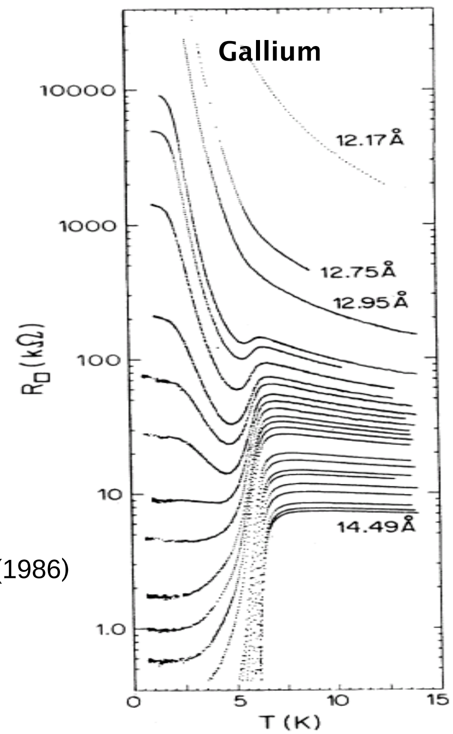

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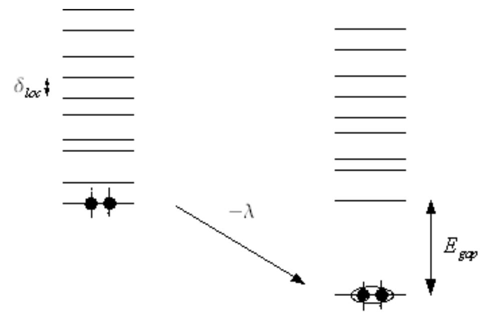

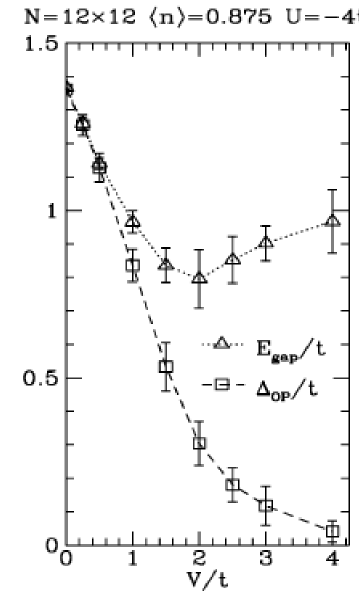

Parity gap

$$\Delta_p = \frac{\delta}{2\ln\frac{\delta}{\Delta}}$$

$$\Delta = 1.76\, k_B\, T_c$$

$$E_g = \Delta + \Delta_p$$

K.A. Matveev and A.I. Larkin
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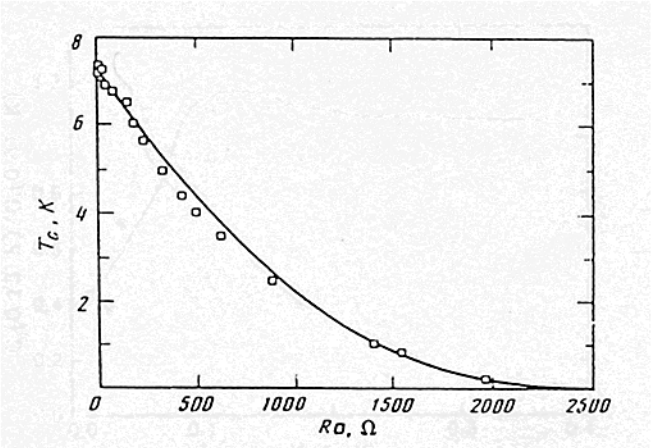
Coulomb depairing in disordered superconductors



A.M Finkelstein
Pis'sma Zh. Esk. Theor. Fiz., **45**, 46 (1987**)**

$$\frac{T_c}{T_c^{bulk}} = e^{\gamma}\left(\frac{1/\gamma - \sqrt{r/2} + r/4}{1/\gamma + \sqrt{r/2} + r/4}\right)^{1/\sqrt{2r}}$$

$$r = \frac{R_\square e^2}{\pi h}$$

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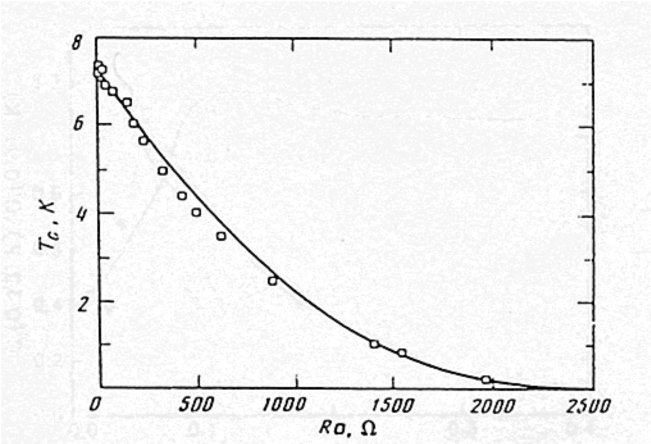

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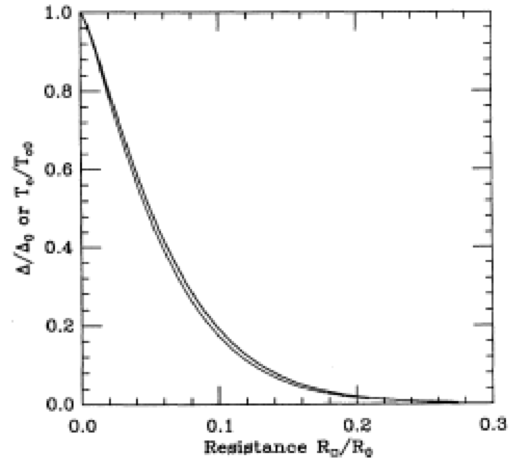

R. A. Smith, M.Y. Reizer, and J. W. WIlkins
Phys. Rev. B **51**, 6470(1995**)**

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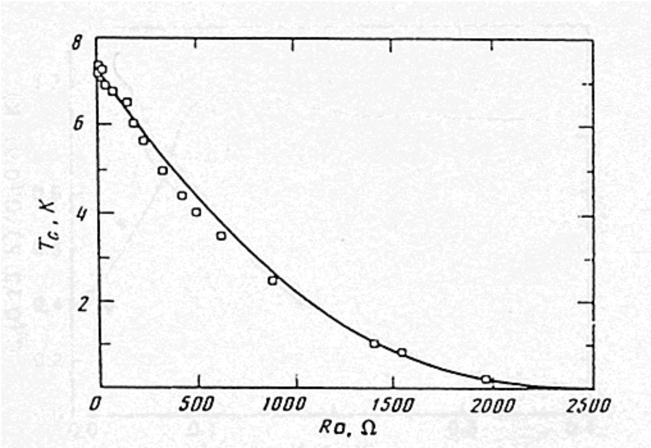

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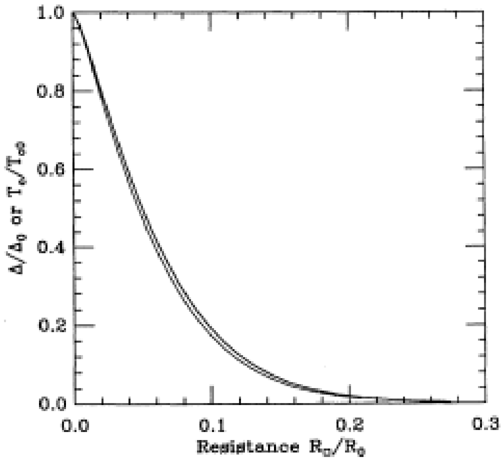

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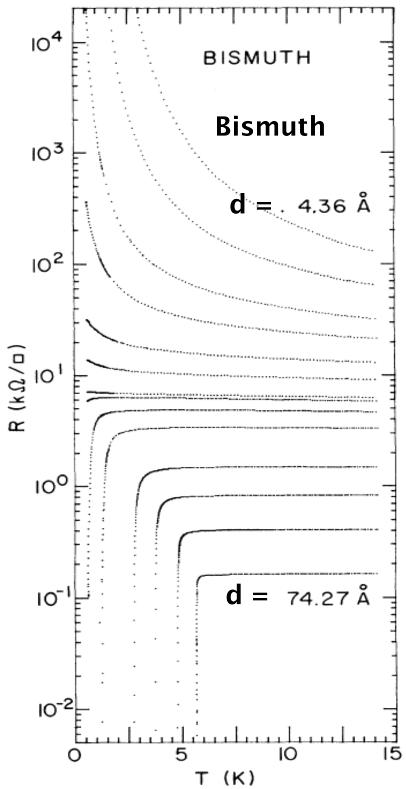
- Short range Coulomb interaction continuously decreases Tc and Δ in the same proportion

Superconductor-insulator transition : two scenarios

Amorphous films



D.B. Haviland, Y. Lui, A.M. Goldman, *PRL* ***62***, 2180 (1989)

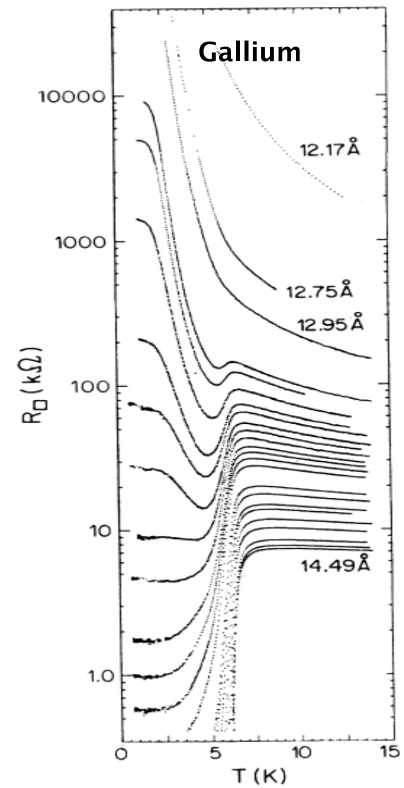
- Continuous decrease of T_c
- Cooper pairing suppressed at the SIT

$$\psi_{op} = \Delta(T)\, e^{i\varphi(T)}$$

Bosonic

Fermionic

Granular films

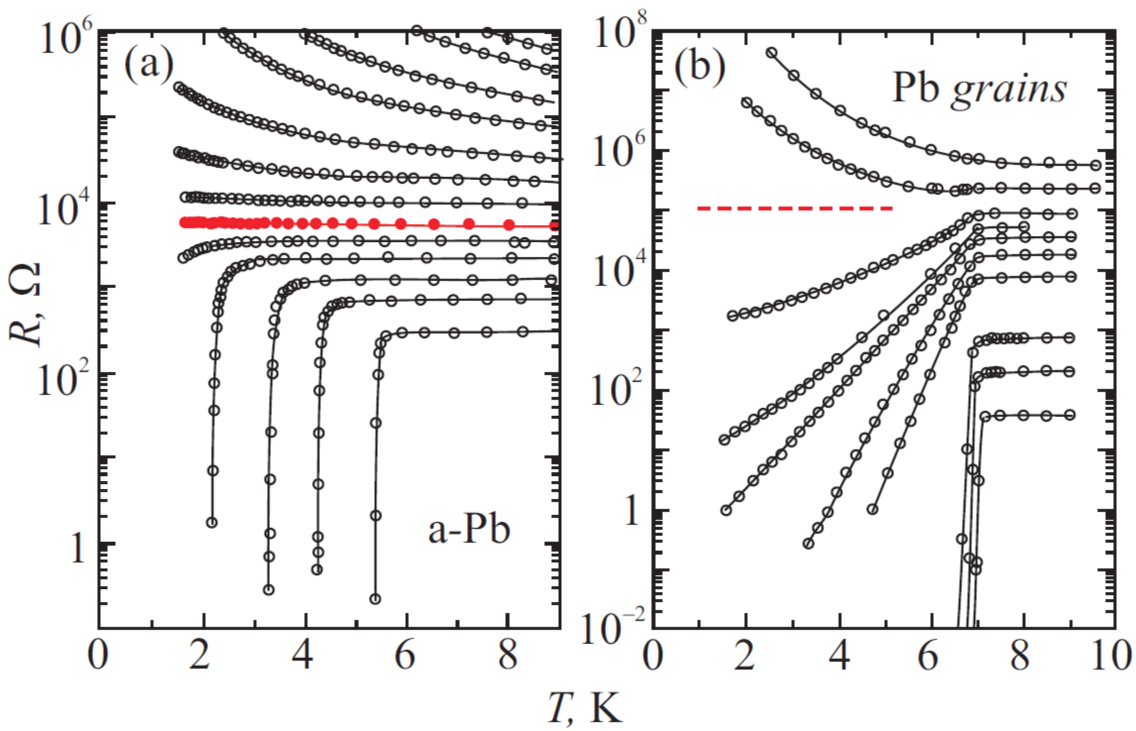


H. M. Jaeger, et al. Phys.Rev.B 34, 4920 (1986)

- Competition between E_C and E_J
- Cooper pairs localized in grains

Superconductor-insulator transition : two scenarios

Amorphous films | Granular films



Frydman, A., *Physica C : Superconductivity* ***391***, 189 (2003)

Hsu, S.-Y., and Valles, J. M. *Phys. Rev. B* ***48***, 4164 (1993)

- Continuous decrease of T_c
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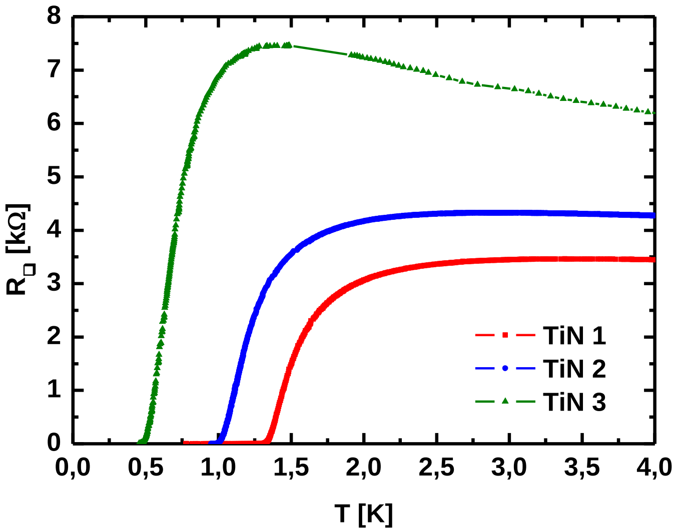
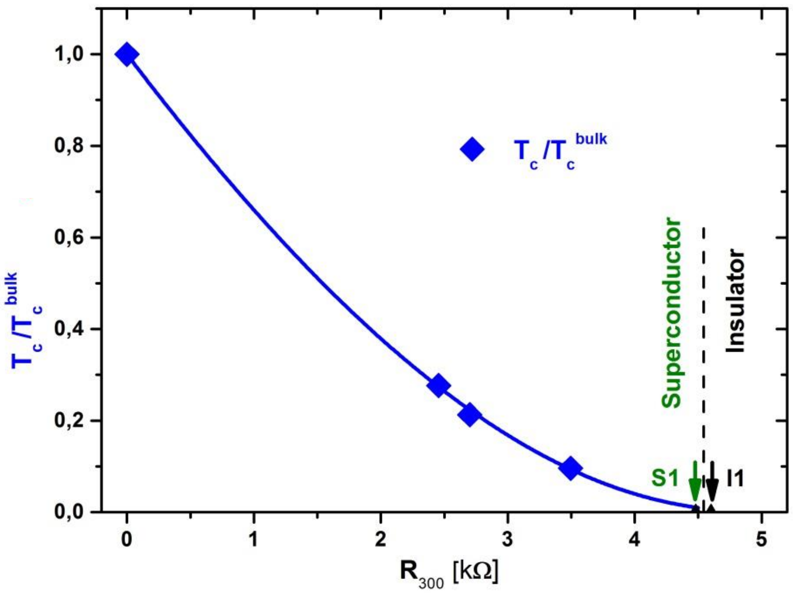
- SIT due to phase fluctuations
- Cooper pairs localized in grains

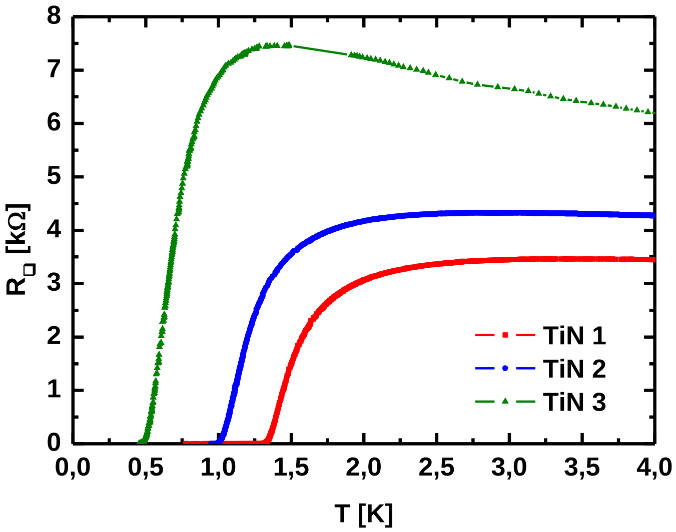
TiN



Sacépé et al., *PRL **101**, 157006 (2008)*

TiN

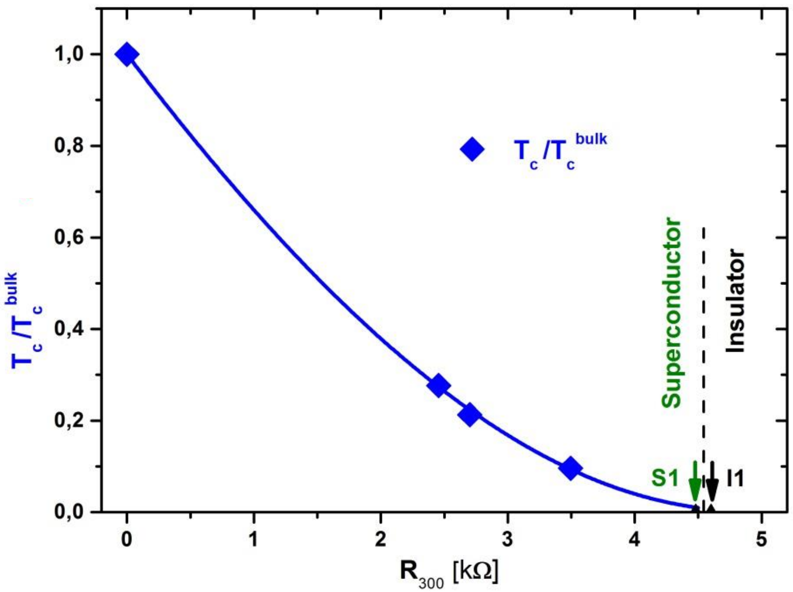


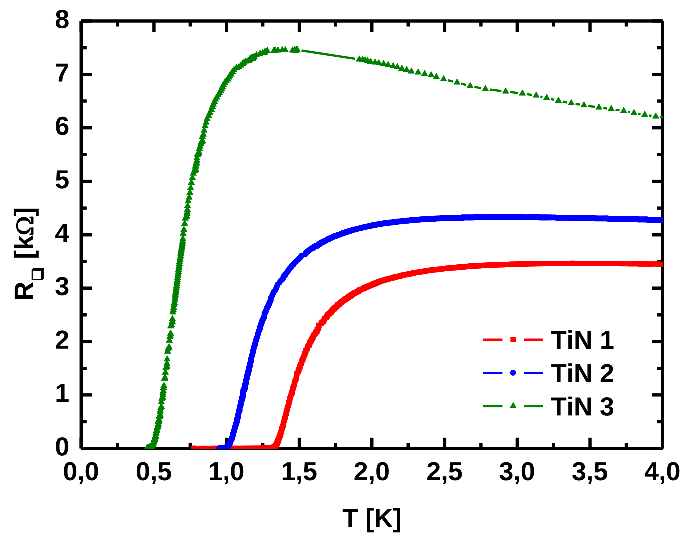


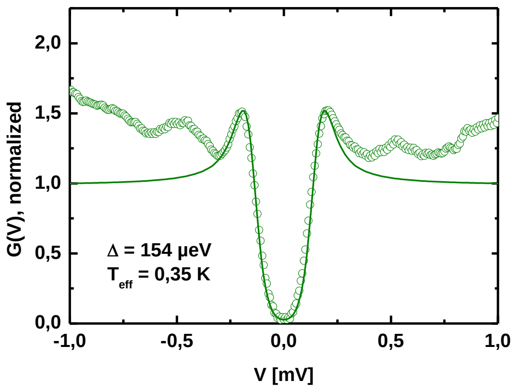
Sacépé et al., *PRL* ***101****, 157006 (2008)*

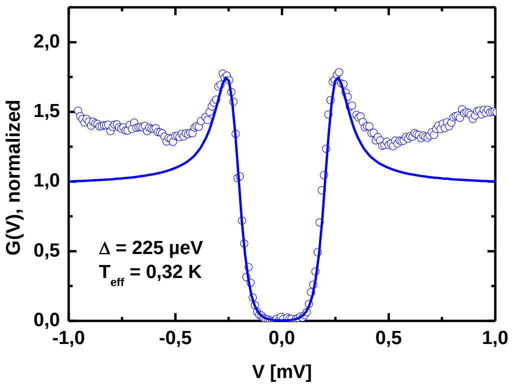
TIN Superconductor-Insulator transition

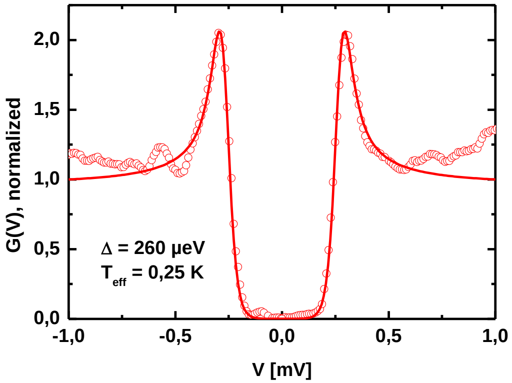
TiN







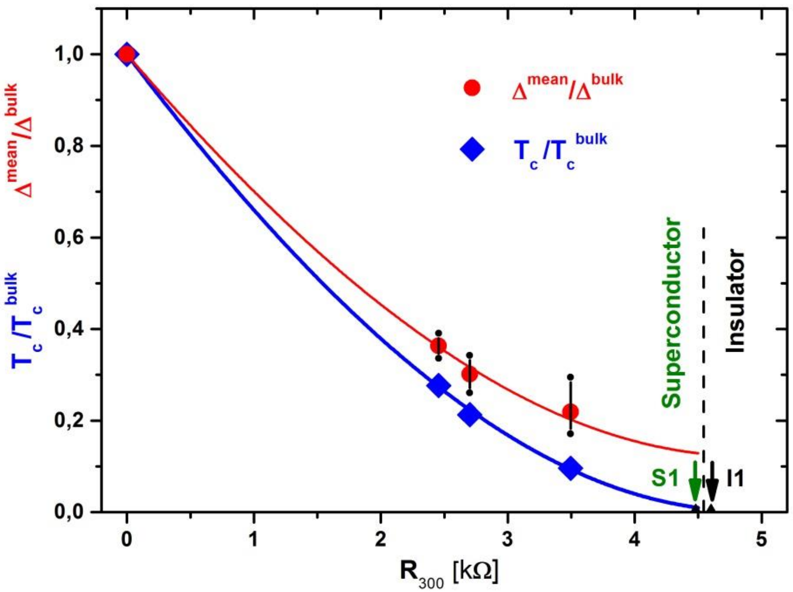


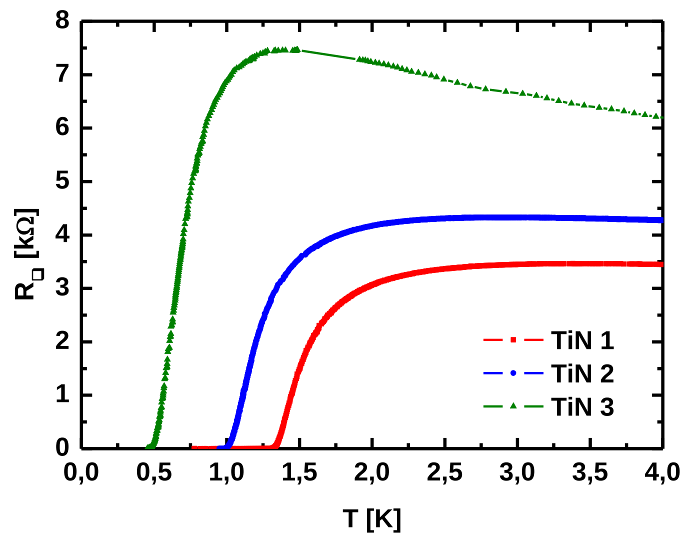


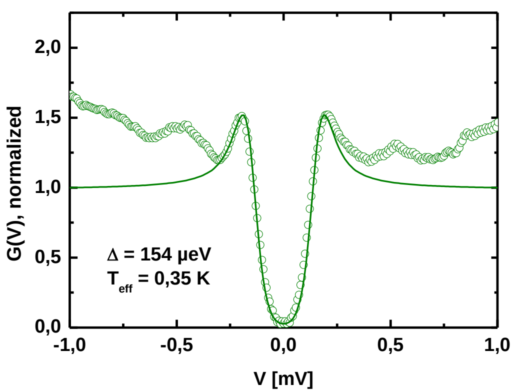
Increasing disorder

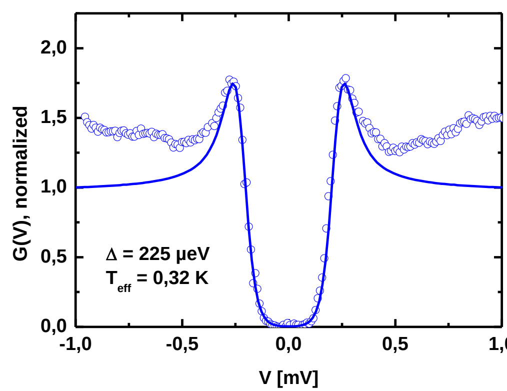
Sacépé et al., *PRL* ***101****, 157006 (2008)*

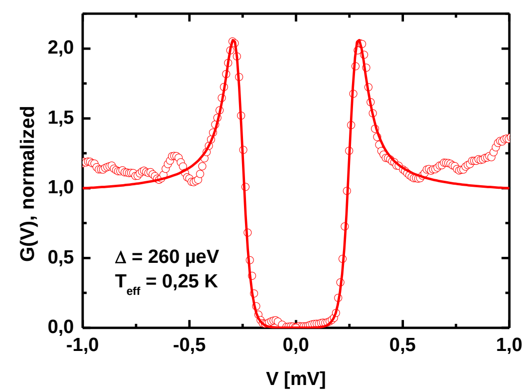
TiN









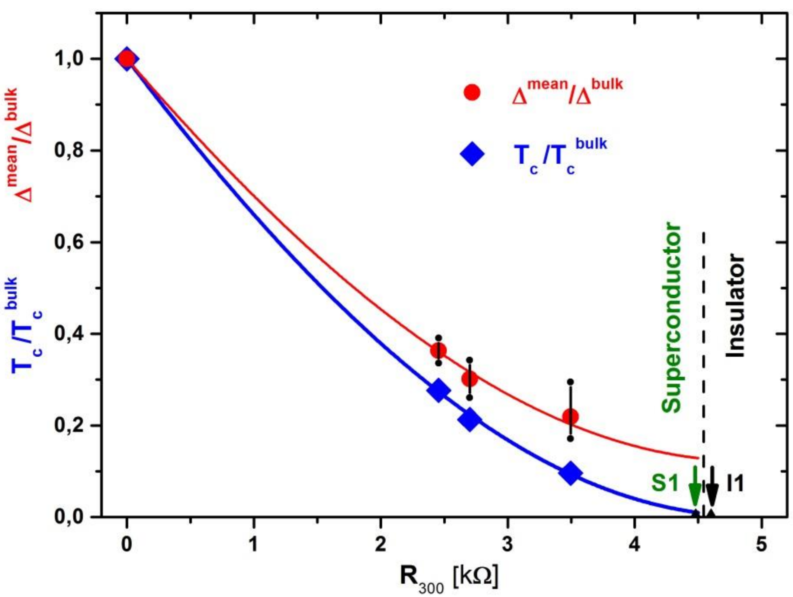


Increasing disorder

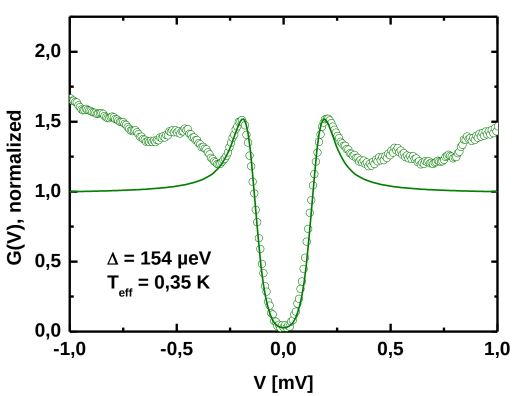
Sacépé et al., *PRL* ***101***, *157006 (2008)*

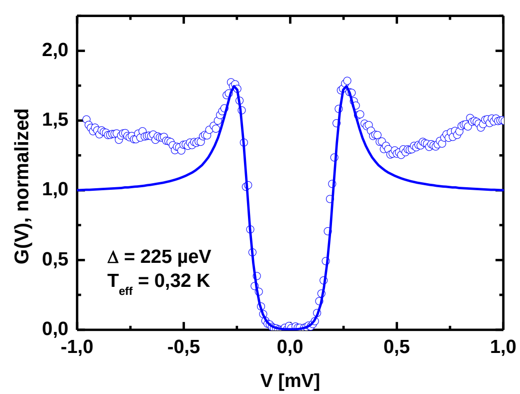
TIN Superconductor-Insulator transition

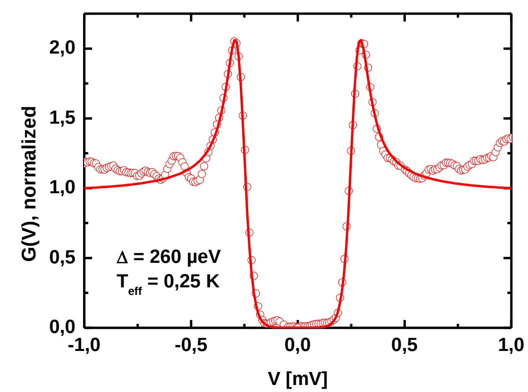
TiN



T_c [K]	Δ/T_c
4.7	1.8
1.3	2.3
1	2.6
0.45	4



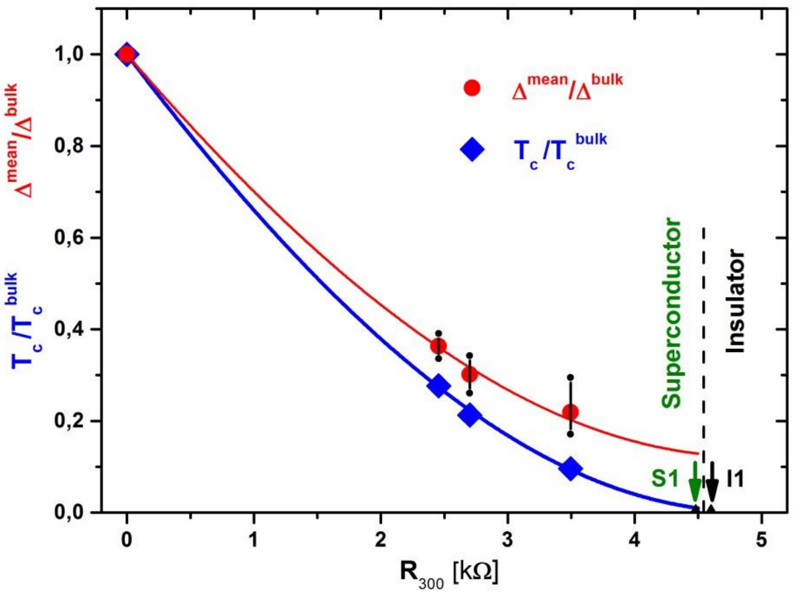




Increasing disorder

Sacépé et al., *PRL* ***101****, 157006 (2008)*

TIN Superconductor-Insulator transition



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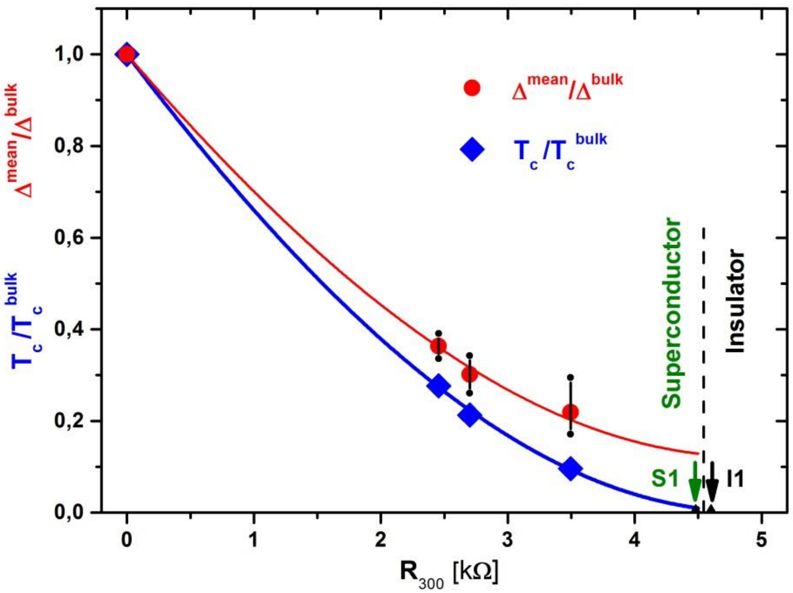
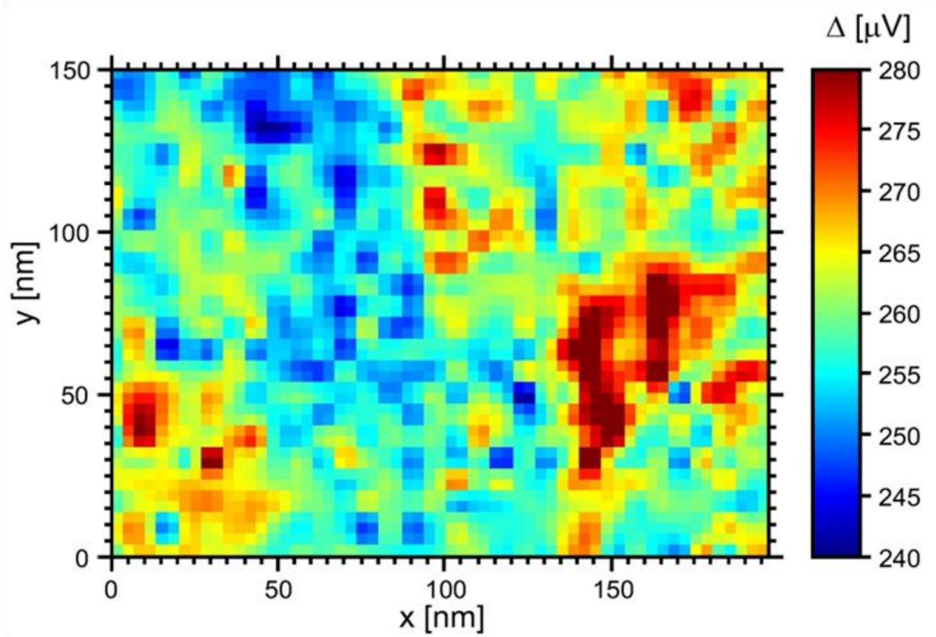
Sacépé et al., *PRL* ***101****, 157006 (2008)*





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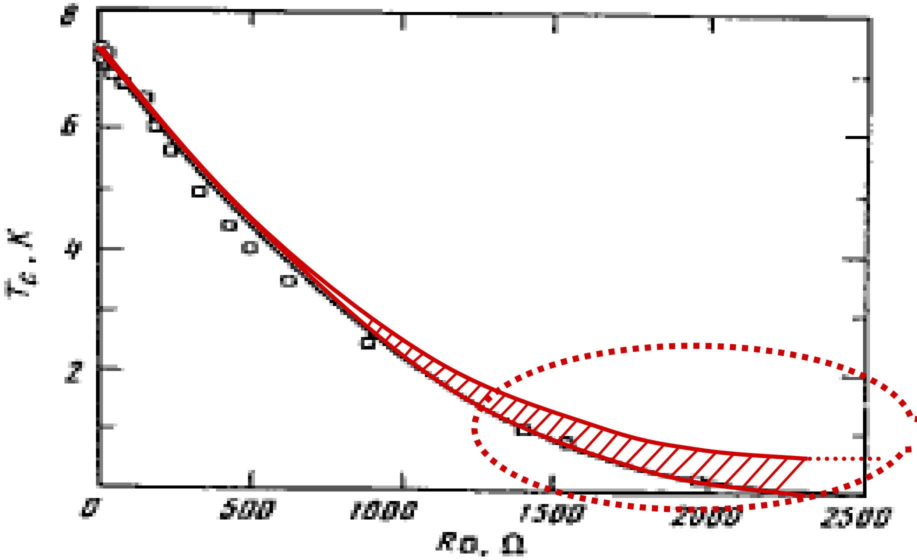
Sacépé et al., *PRL* ***101***, *157006 (2008)*

Superconductivity and Coulomb interaction

M. A. Skvortsov and M. V. Feigel'man, *Phys. Rev. Lett.* ***95****, 057002, (2005)*

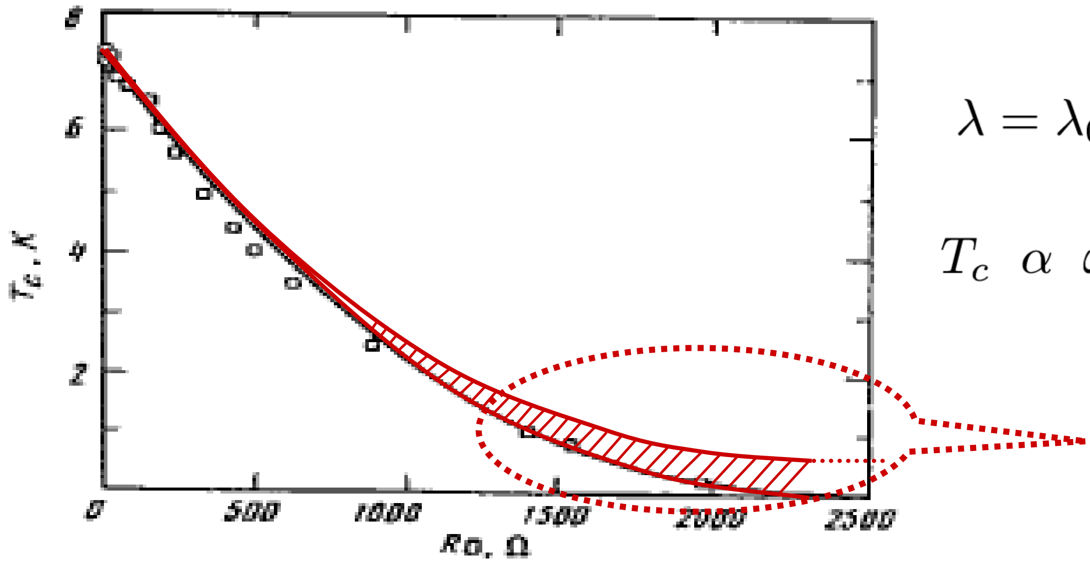


$$\lambda = \lambda_0 - \frac{1}{24\pi g}\log(\frac{1}{\epsilon\tau})$$

$$T_c \;\; \alpha \;\; \omega_D e^{-\frac{1}{\lambda N(E_F)}} \quad \Rightarrow \quad \frac{\delta T_c}{T_c} = \frac{\delta\lambda}{\lambda^2}$$

Spatial fluctuations of Tc

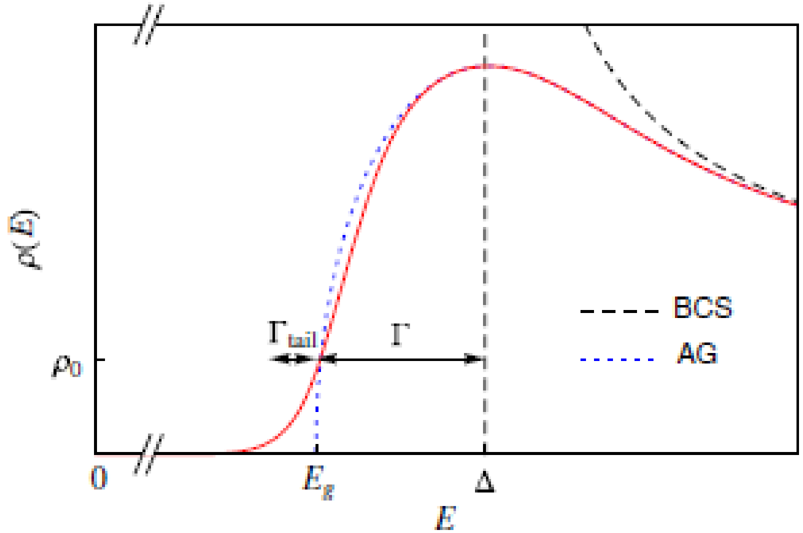
M. A. Skvortsov and M. V. Feigel'man, *Phys. Rev. Lett.* ***95****, 057002, (2005)*



$$\lambda = \lambda_0 - \frac{1}{24\pi g}\log\left(\frac{1}{\epsilon\tau}\right)$$

$$T_c \;\alpha\; \omega_D e^{-\frac{1}{\lambda N(E_F)}} \Rightarrow \frac{\delta T_c}{T_c} = \frac{\delta\lambda}{\lambda^2}$$

Spatial fluctuations of Tc

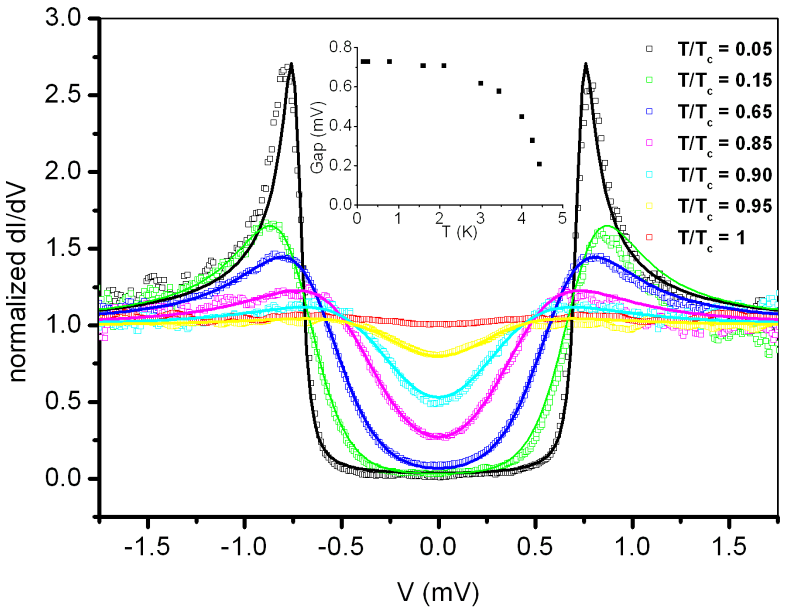


$$\frac{D}{2}\nabla^2\theta + iE\sin\theta + \Delta(\mathbf{r})\cos\theta - \Delta_0\,\eta\cos\theta\sin\theta = 0,$$

M.V. Feigelman and M.A. Skvortsov, *Phys. Rev. Lett.* ***109***, 147002 (2012)

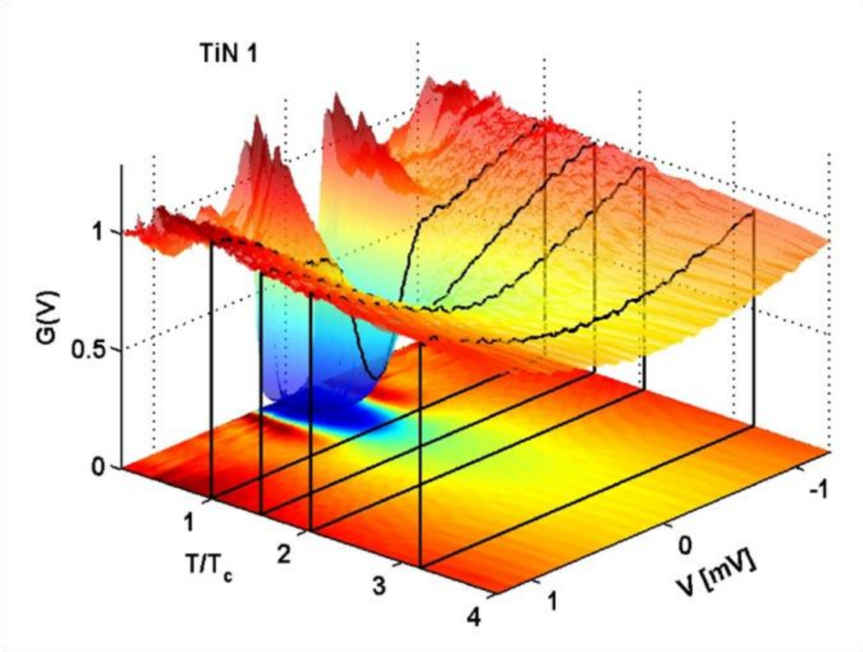
A.I. Larkin and Yu. N. Ovchinnikov, Sov. JETP **34**, 1144 (1972)

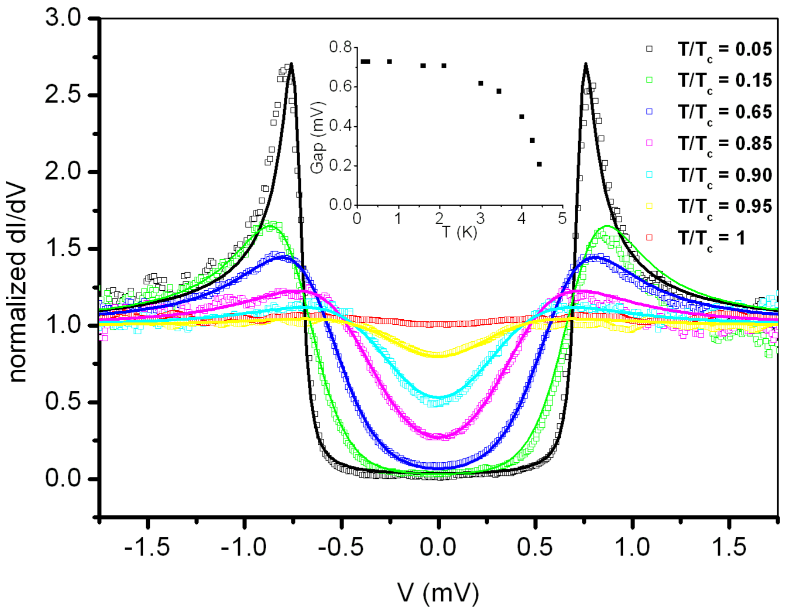
Thermal dependence of the Density of States



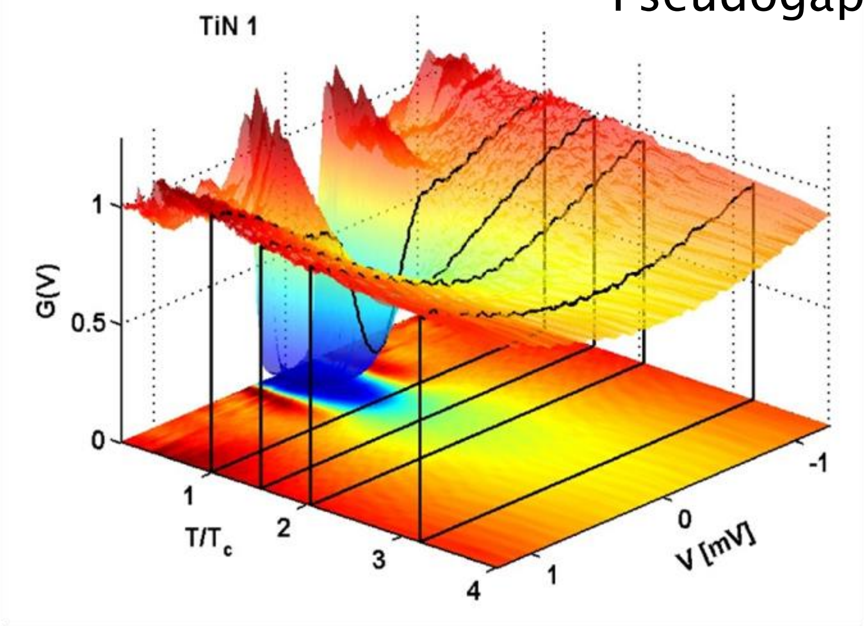
W. Escoffier, et al., *PRL **93**, 217005, (2004)*

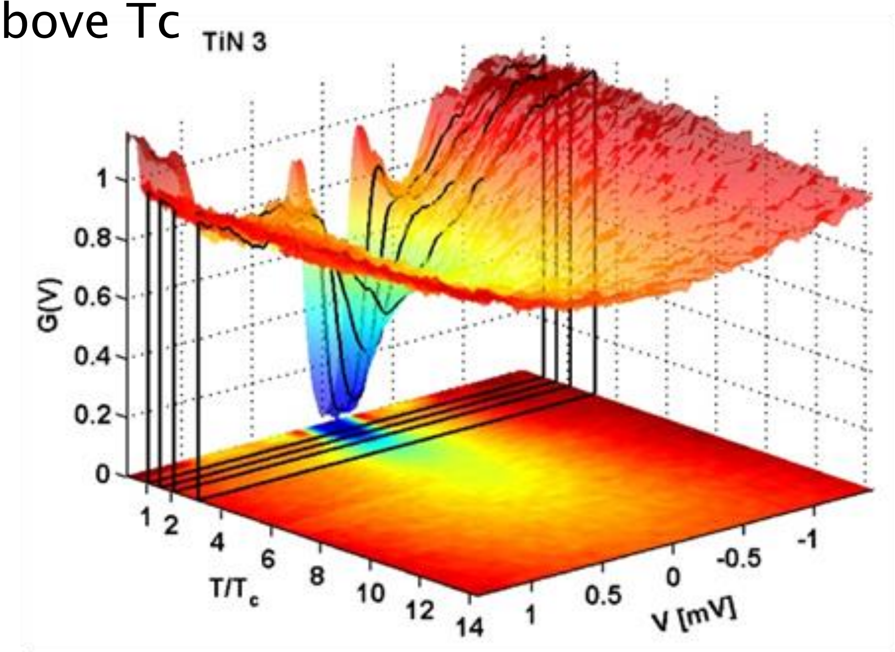
Thermal dependence of the Density of States





W. Escoffier, et al., *PRL **93**, 217005, (2004)* Pseudogap above Tc

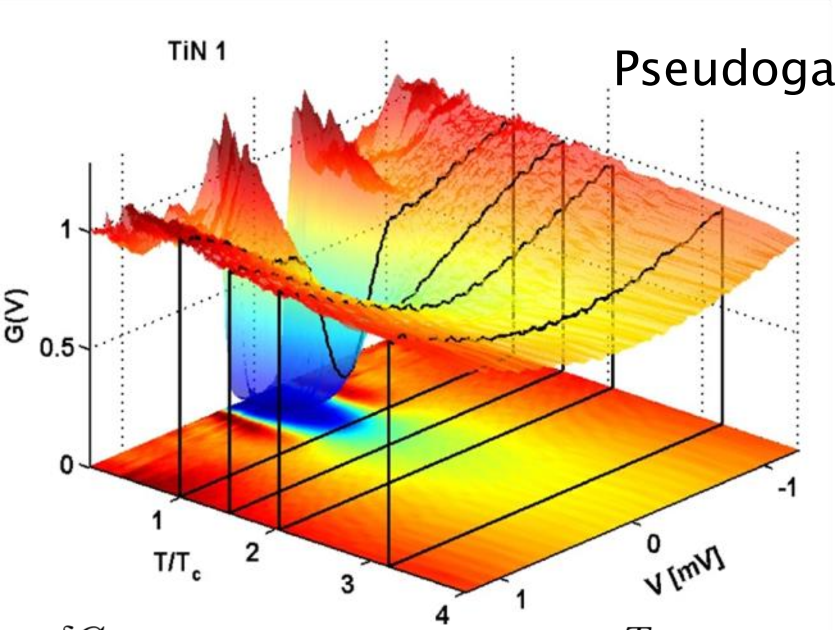


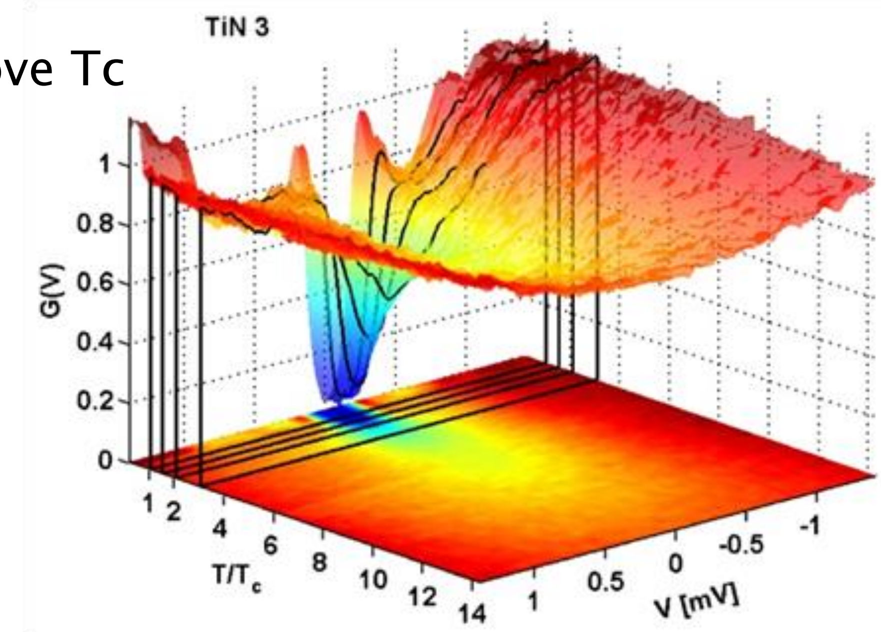


B. Sacépé et al., *Nat. Comm.*, (2010)

Thermal dependence of the Density of States

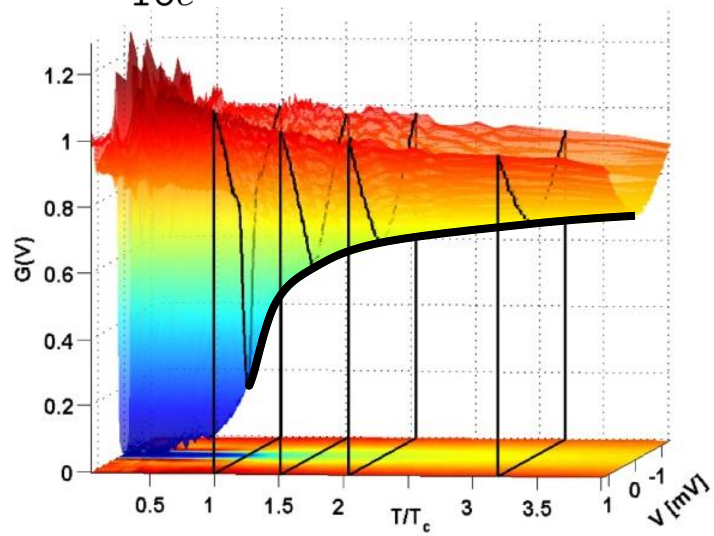
Pseudogap above Tc

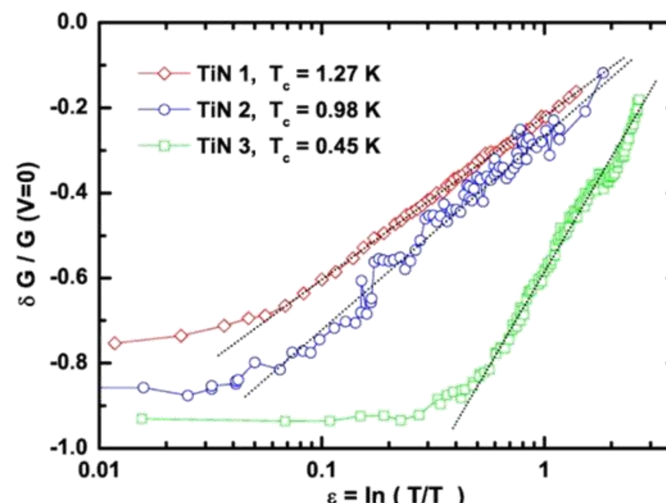




$$\frac{\delta G}{G}(eV=0) = -2Gi\,\ln(\ln\frac{T}{T_c})$$

$$Gi = \frac{\hbar}{16e^2}R_\square$$

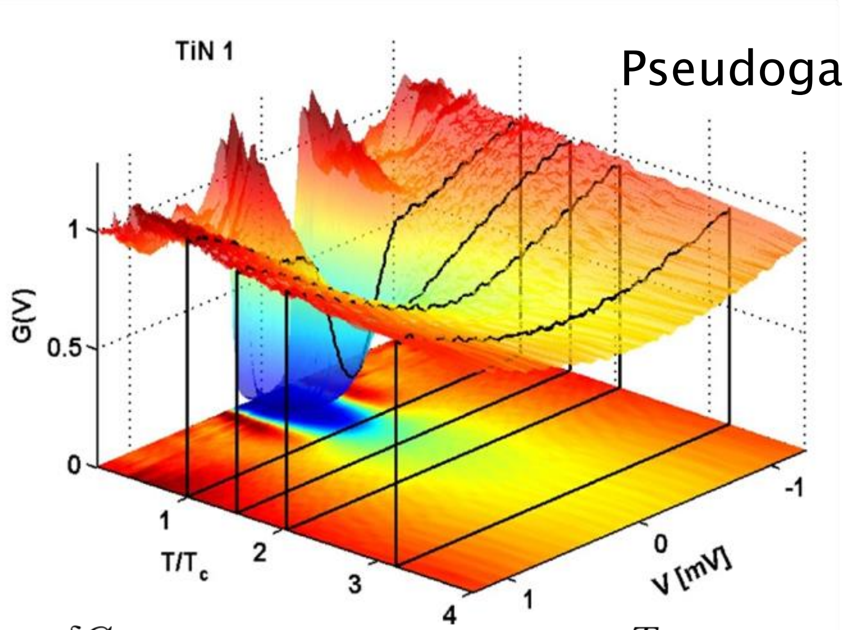


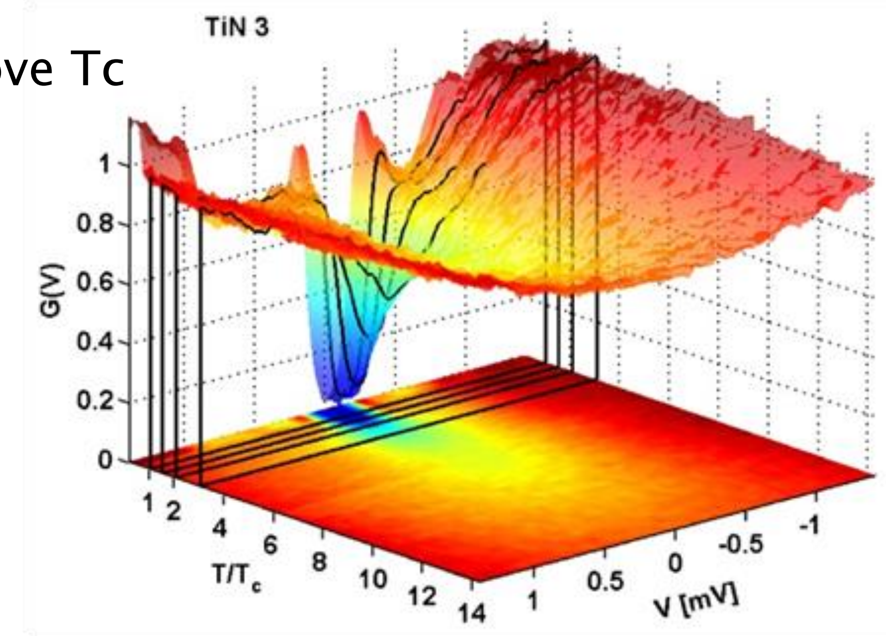


B. Sacépé et al., *Nat. Comm.*, (2010)

Thermal dependence of the Density of States

Pseudogap above Tc

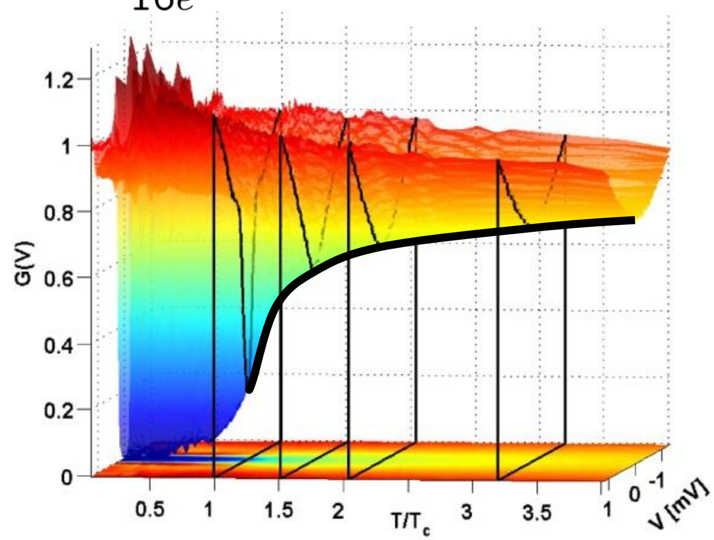


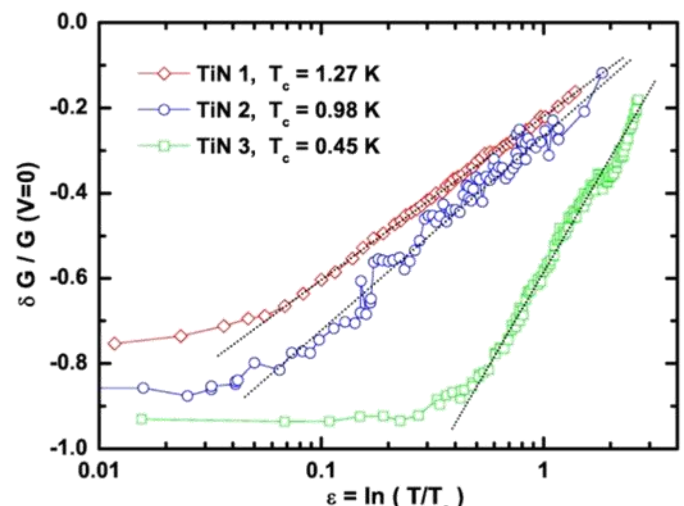


$$\frac{\delta G}{G}(eV = 0) = -2Gi\ln(\ln\frac{T}{T_c})$$

- Pseudogap is due to pre-formed Cooper pairs

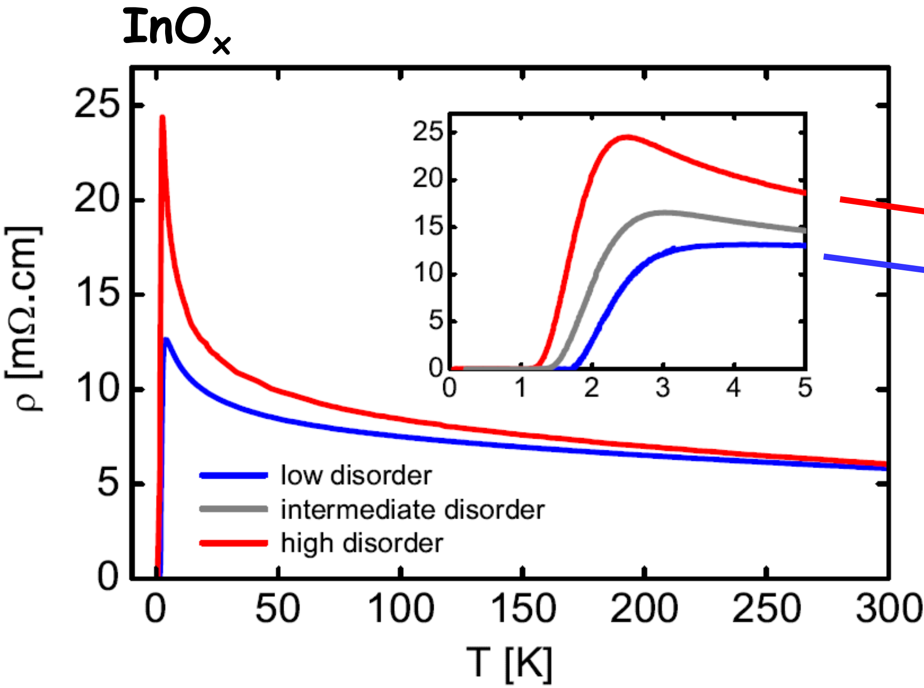
$$Gi = \frac{\hbar}{16e^2}R_\square$$

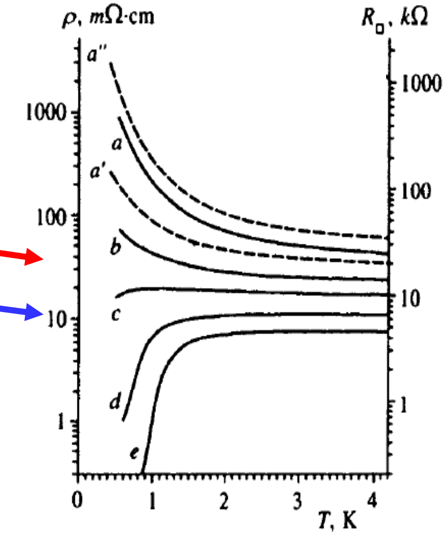




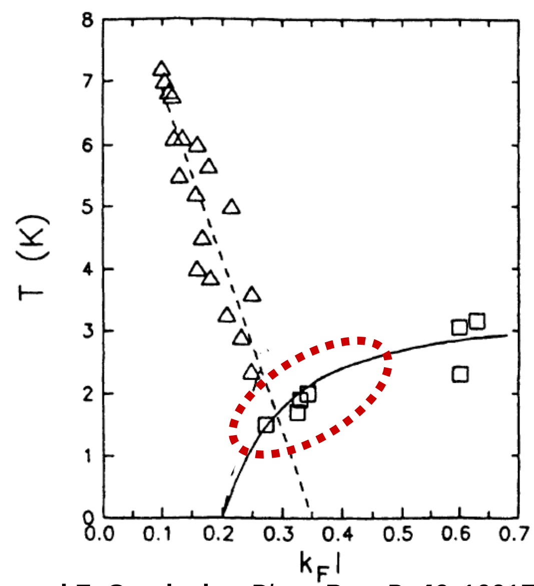
B. Sacépé et al., *Nat. Comm.*, (2010)

InOx Superconductor-Insulator transition



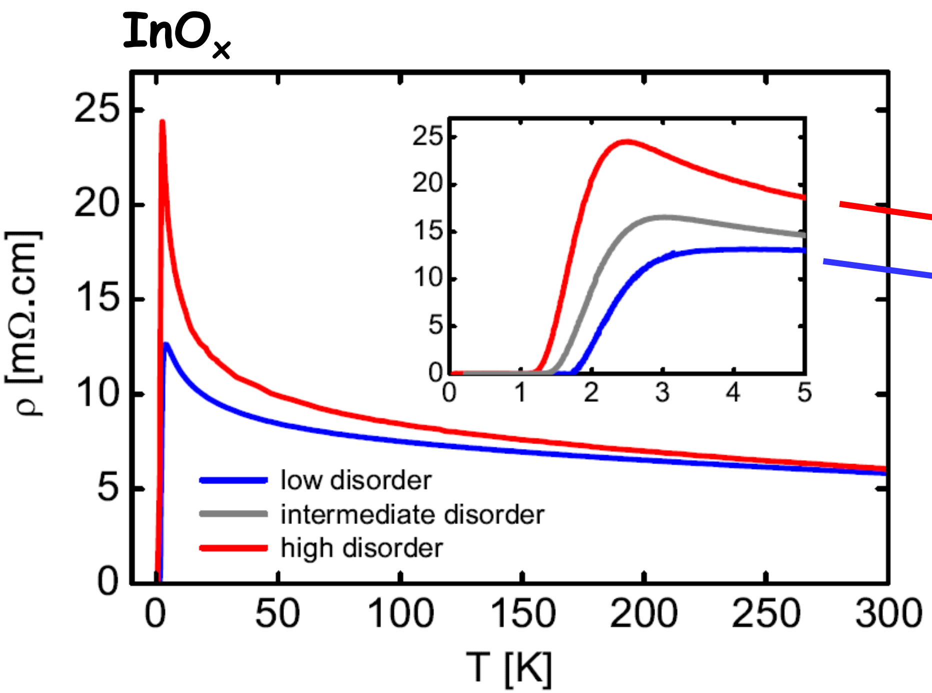


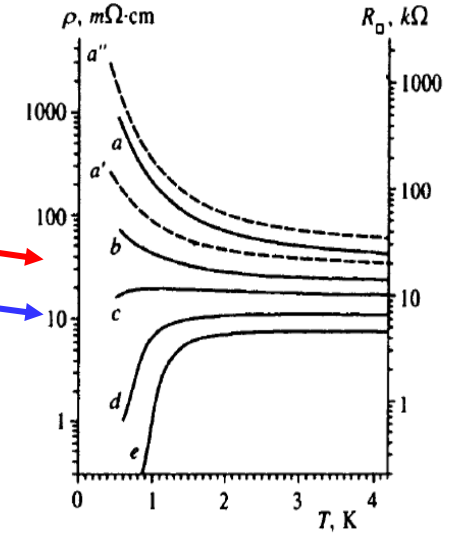
V. F. Gantmakher *et al.*, *JETP* ***82***, 951 (1996)



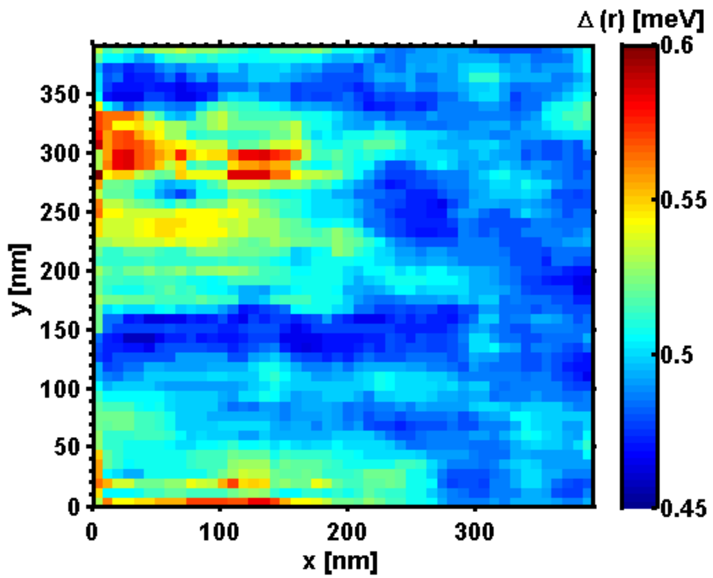
D. Shahar and Z. Ovadyahu, *Phys. Rev. B* ***46***, 10917 (1992)

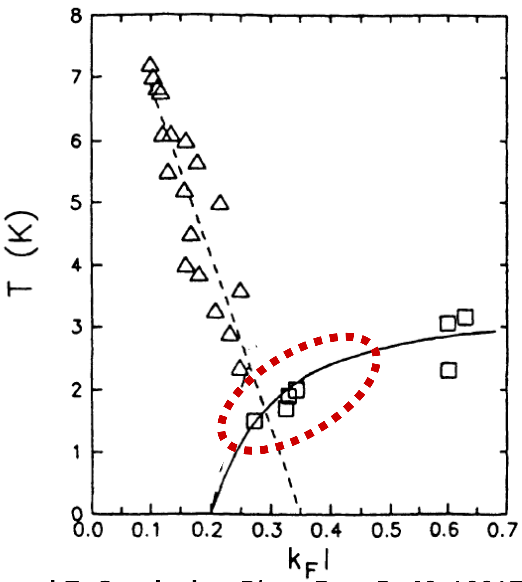
InOx Superconductor-Insulator transition





V. F. Gantmakher *et al.*, *JETP* ***82***, 951 (1996)



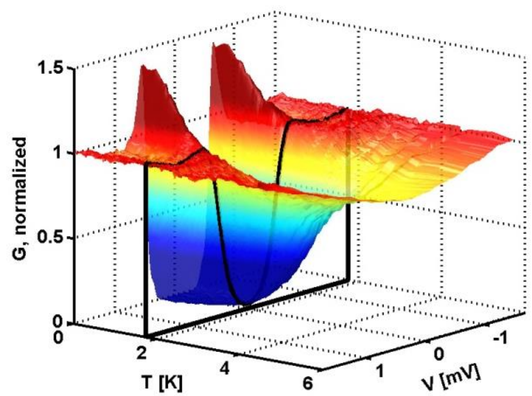


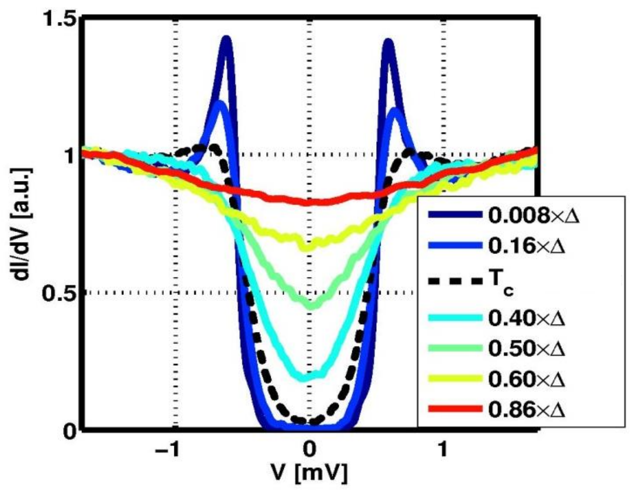
D. Shahar and Z. Ovadyahu, *Phys. Rev. B* ***46***, 10917 (1992)

Thermal dependence of the Density of States

InO_x

Pseudogap above Tc

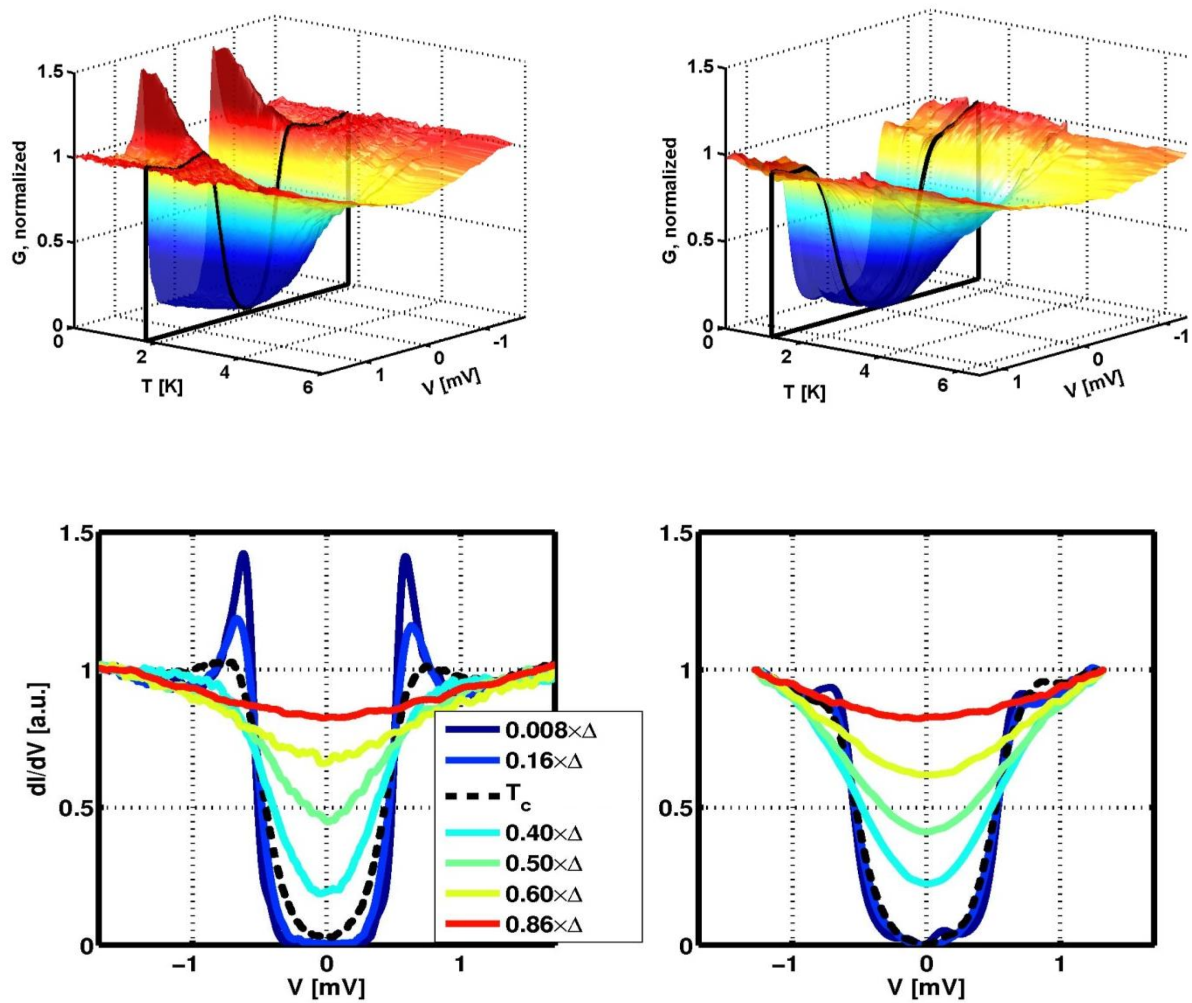




B. Sacépé et al., *Nat. Phys.* (2011)

Thermal dependence of the Density of States

InO_x Pseudogap above Tc

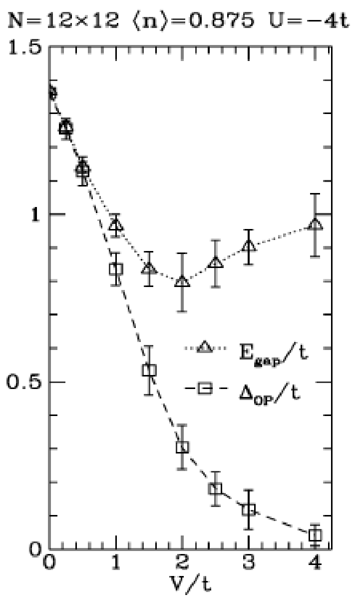


- ➢ Spectra without coherence peaks are the signature of localized pre-formed Cooper pairs

B. Sacépé et al., *Nat. Phys.* (2011)

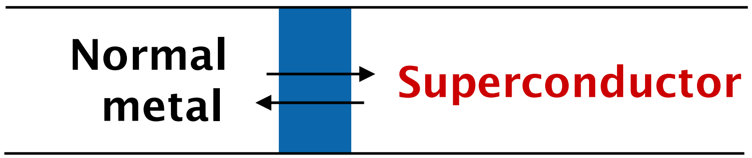
Parity gap

$$E_g = \Delta + \Delta_p$$



How to measure the order parameter ?

$$T = \frac{1}{1 + Z^2}$$



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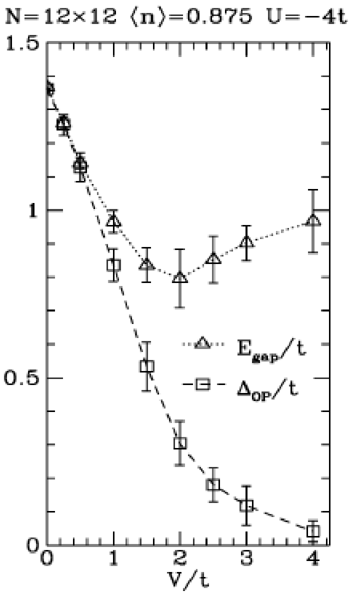

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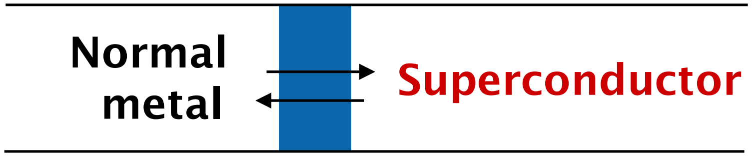
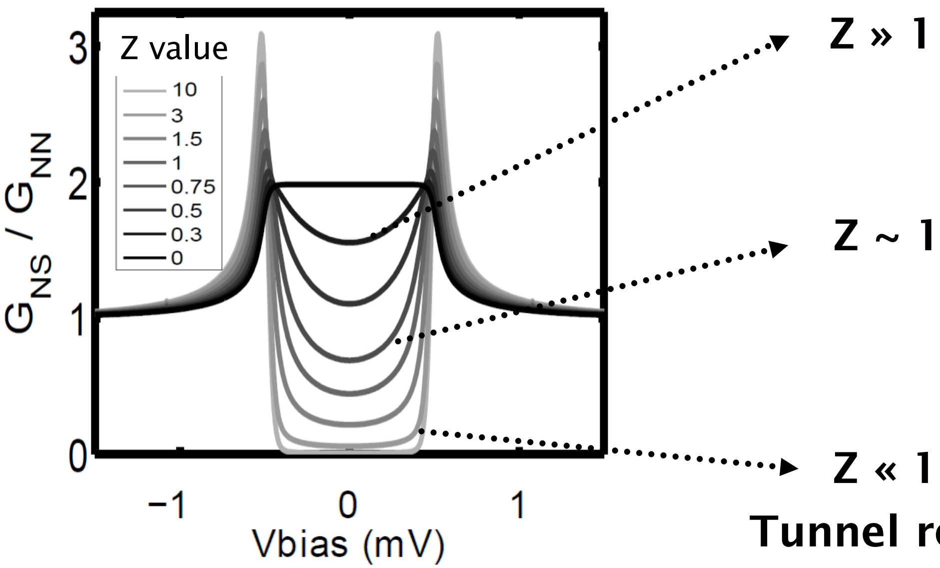


Blonder, G. E., Tinkham, M., and Klapwijk T.M.
Phys. Rev. B ***25***, 7 4515 (1982)

Contact regime



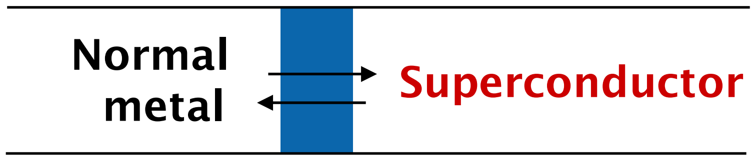
Tunnel regime

How to measure the order parameter ?

Parity gap

$$E_g = \Delta + \Delta_p$$

$$T = \frac{1}{1 + Z^2}$$



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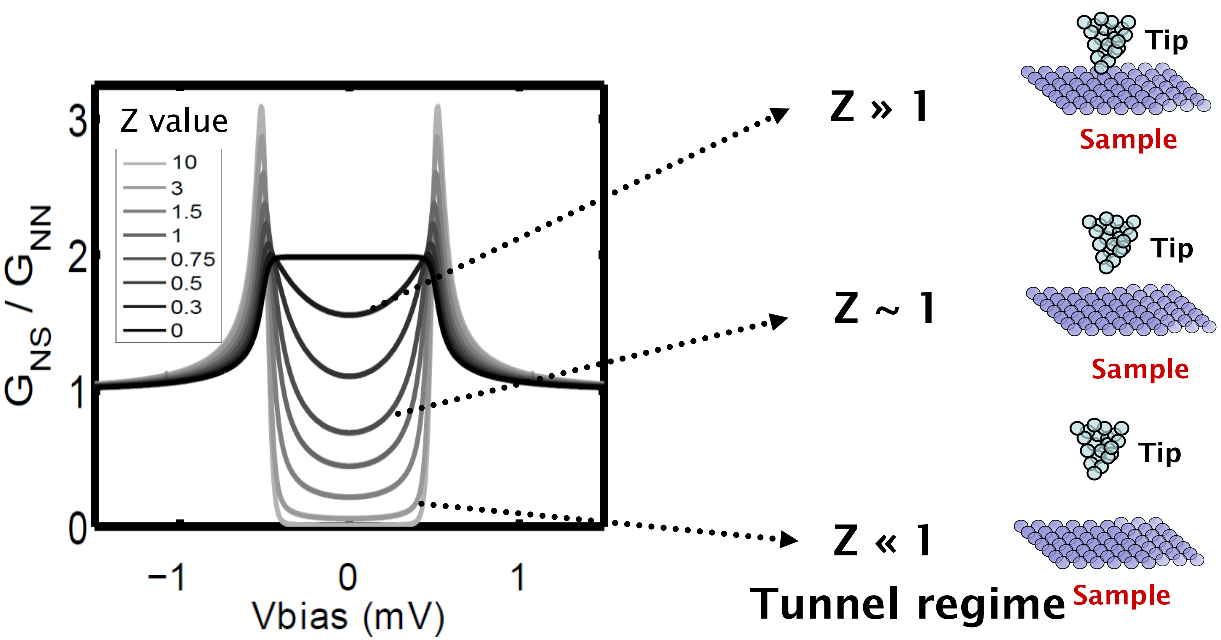
Phys. Rev. B ***25***, 7 4515 (1982)

Contact regime



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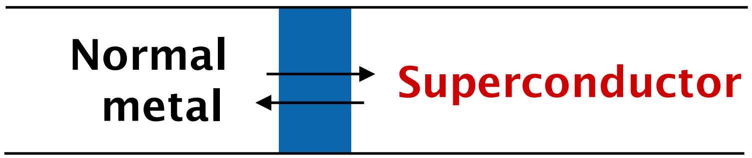

Blonder, G. E., Tinkham, M., and Klapwijk T.M.
Phys. Rev. B ***25***, 7 4515 (1982)

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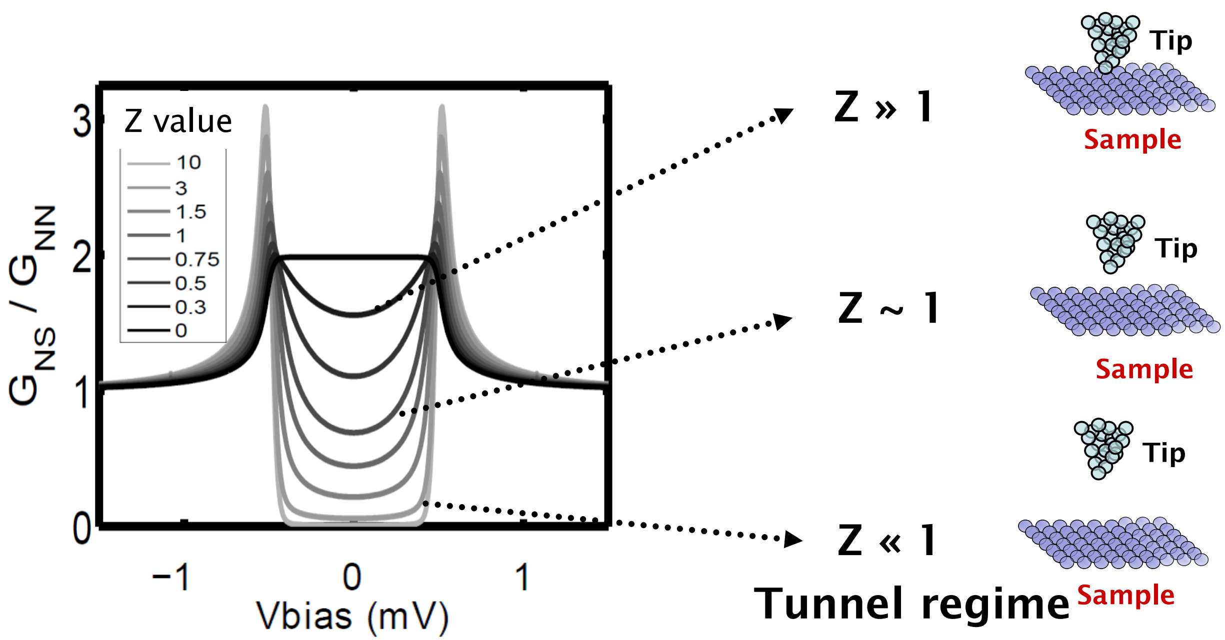


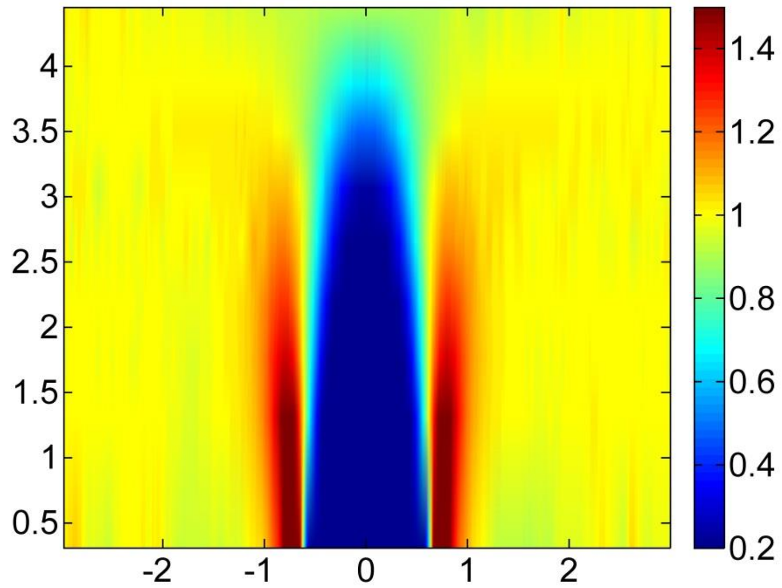
$$E_g = \Delta + \cancel{\Delta_p}$$

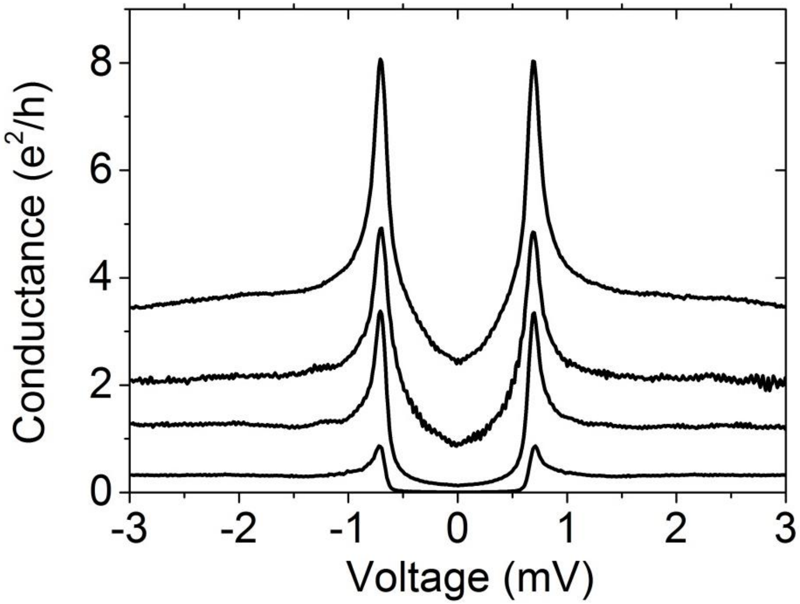
$$E_g = \Delta + \Delta_p$$

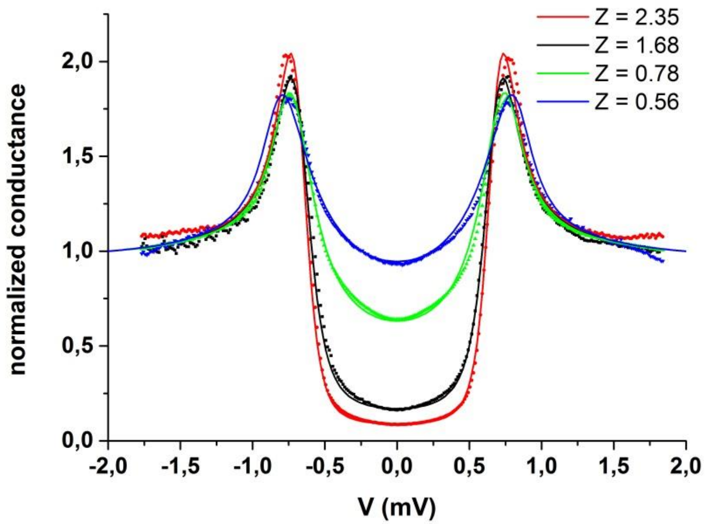
InO_x

InOx film far from the Superconductor-Insulator Transition : Tc = 3.5K

- Homogeneous
- No pseudogap

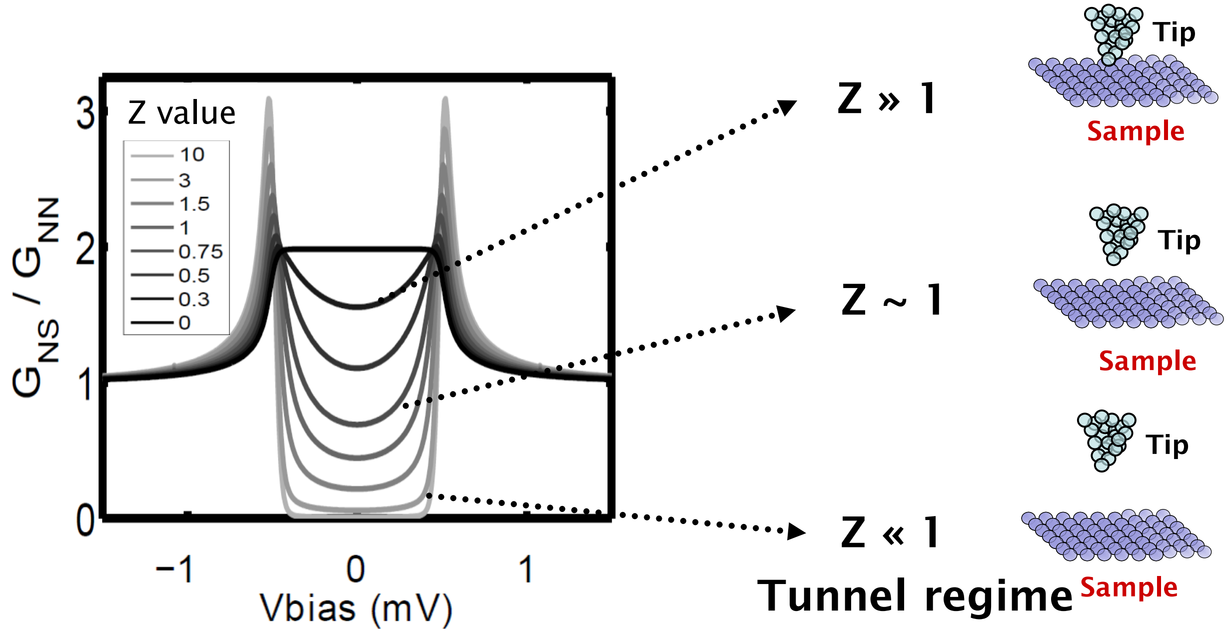






Close to the superconductor-insulator transition $E_g = \Delta + \Delta_p$

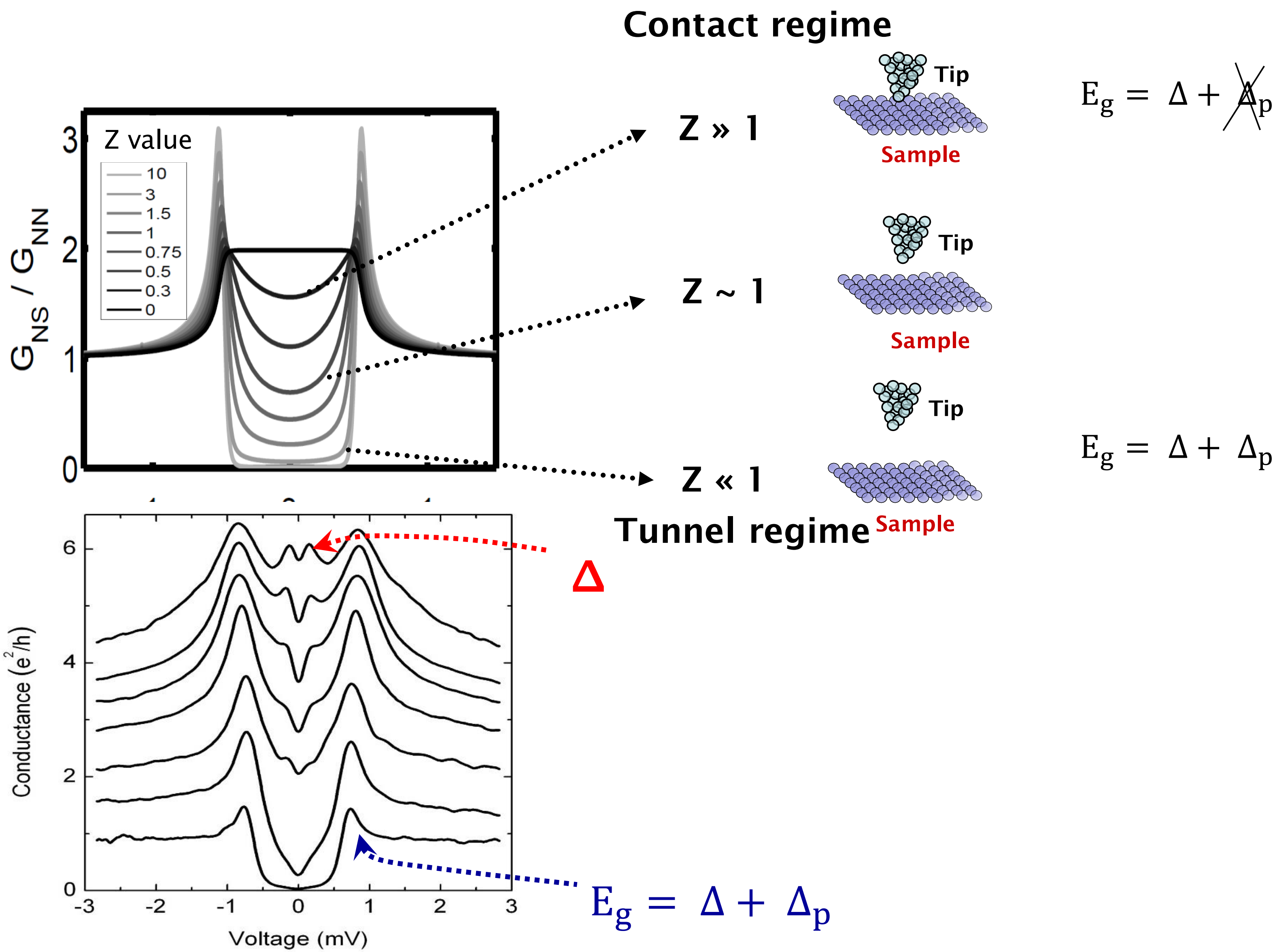
Contact regime



$E_g = \Delta + \cancel{\Delta_p}$

$E_g = \Delta + \Delta_p$

Close to the superconductor-insulator transition $E_g = \Delta + \Delta_p$



Disorder :

- Stong superconducting fluctuations above Tc
- Pseudogap due to preformed Cooper pairs

Disorder & Coulomb interaction :

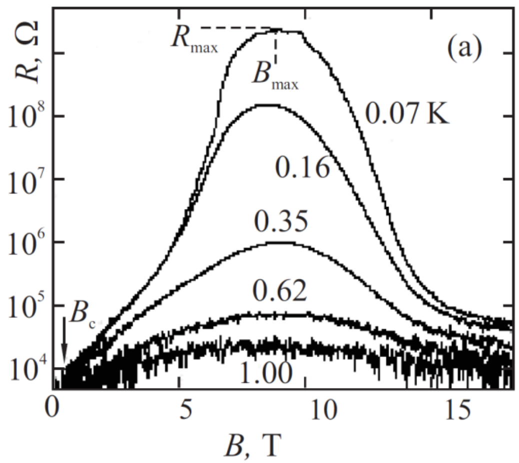
- Continuous decrease of Tc and Δ with disorder
- Keeps Δ/Tc ratio constant
- Spatial mesoscopic fluctuations of Tc

Disorder & Localization :

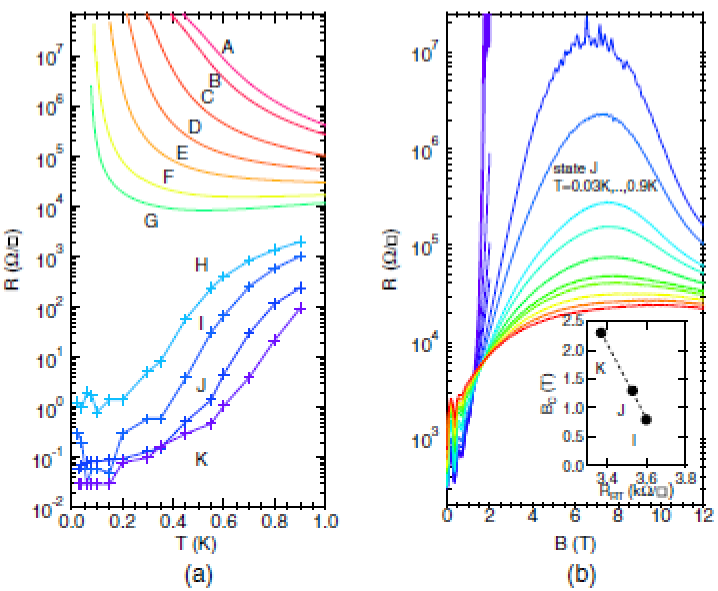
- Tc decreases faster than Δ with disorder : huge Δ/Tc ratio
- Parity gap
- Strong spatial fluctuations of Δ
- Localized Cooper pairs characterized by spectra without coherence peaks

Magnetic field studies through the SIT

InO_x



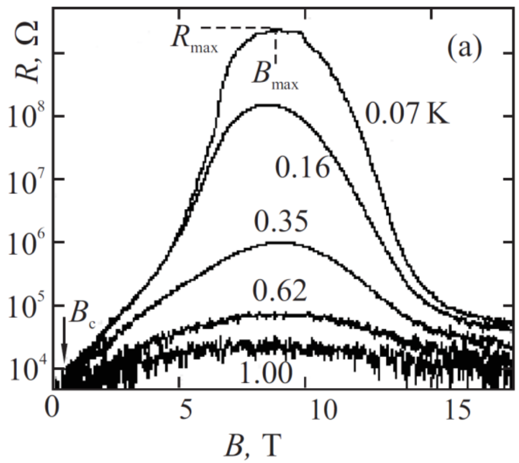
G. Sambandamurthy *et al., Phys. Rev. Lett.* ***92***, 107005, (2004)



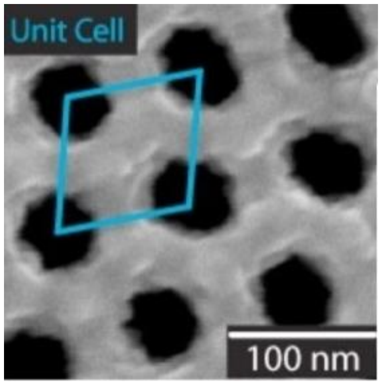
G. Kopnov *et al., Phys. Rev. Lett.* ***109***, 167002, (2012)

Magnetic field studies through the SIT

InO_x

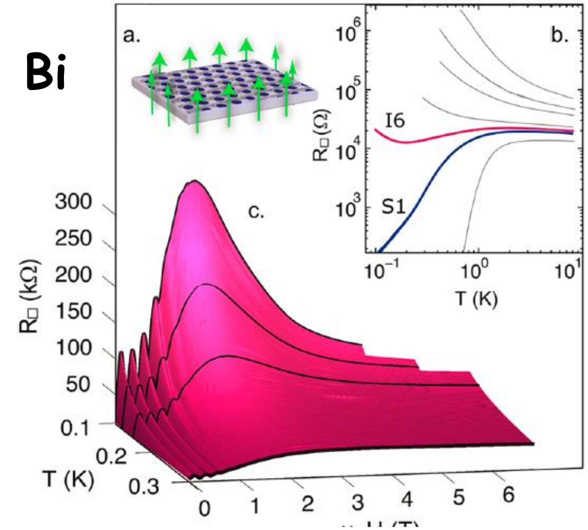


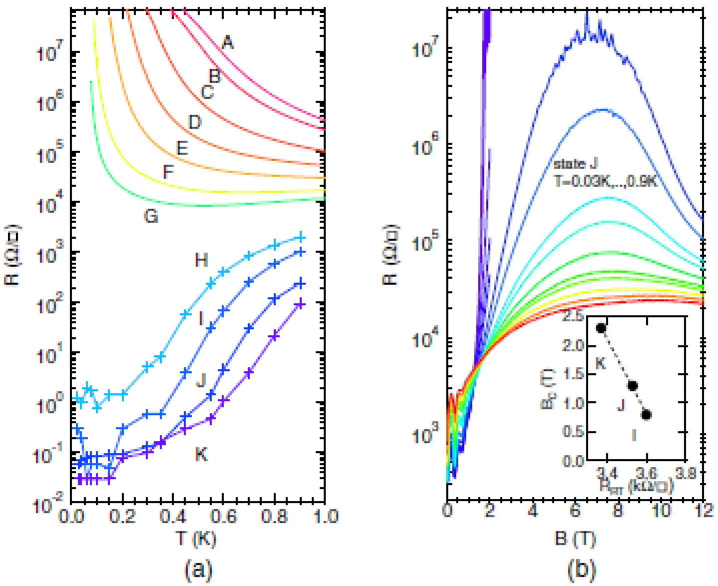
G. Sambandamurthy *et al., Phys. Rev. Lett.* ***92***, 107005, (2004)

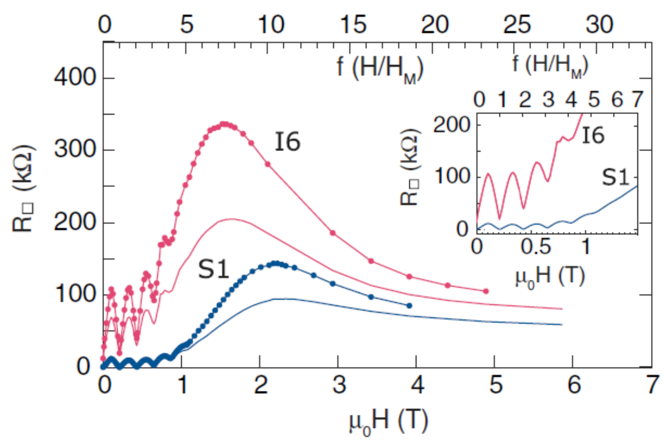


$$\phi = \frac{h}{2e}$$

Bi



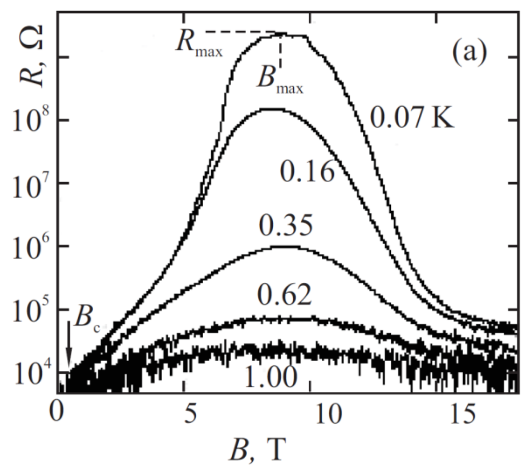


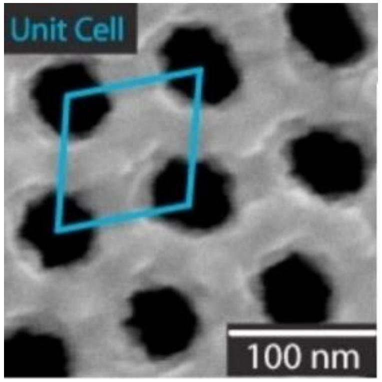


G. Kopnov *et al., Phys. Rev. Lett.* ***109***, 167002, (2012)

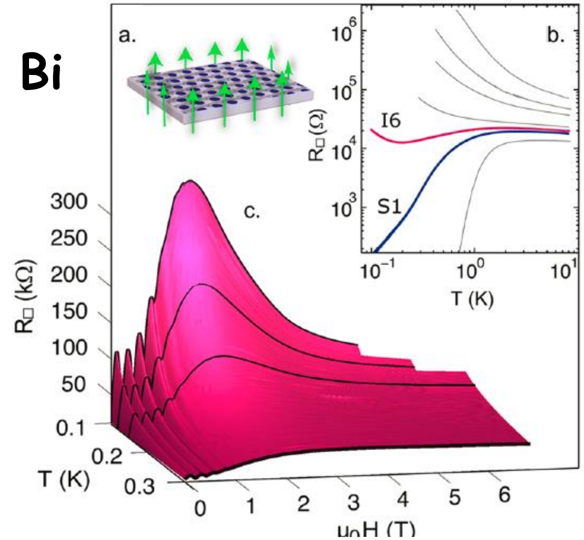
Stewart, Jr. *et al. , Science* **318**, 1273, (2007)
H.Q. Nguyen et al., *Phys. Rev. Lett*. **103**, 157001 (2009)

InO_x





$$\phi = \frac{h}{2e}$$



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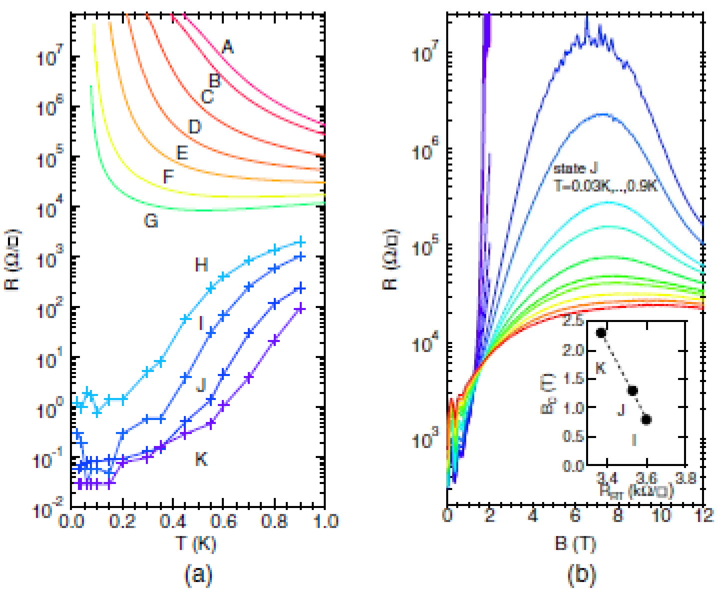

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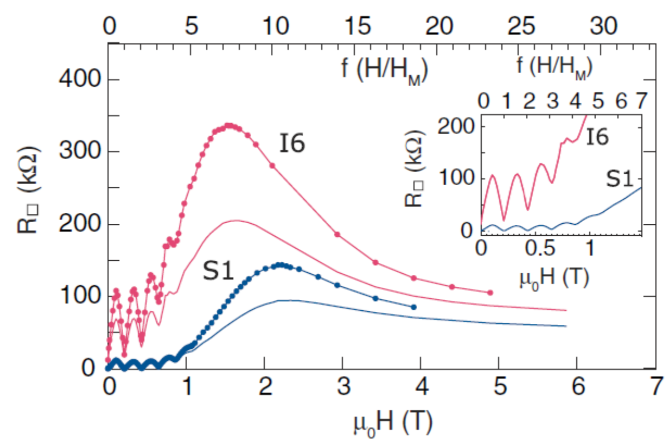

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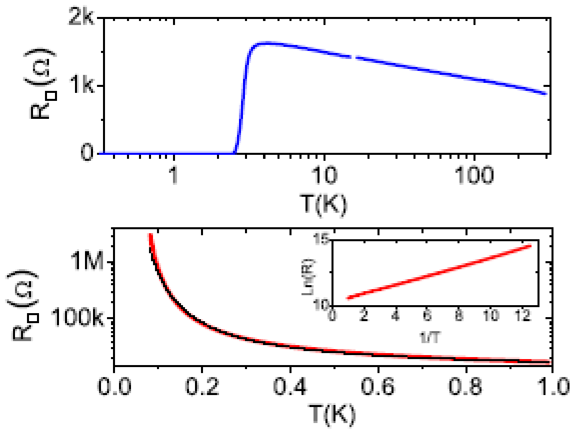
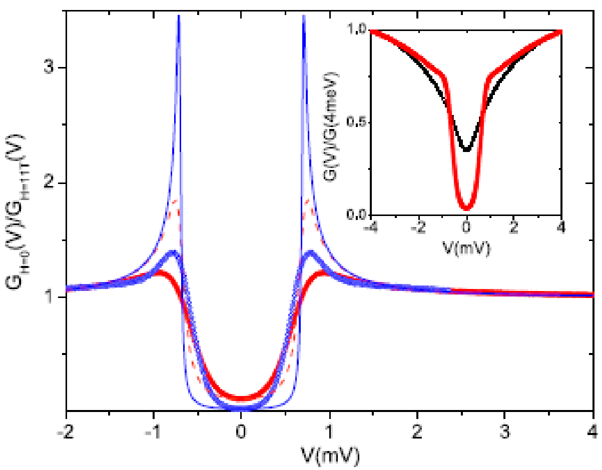
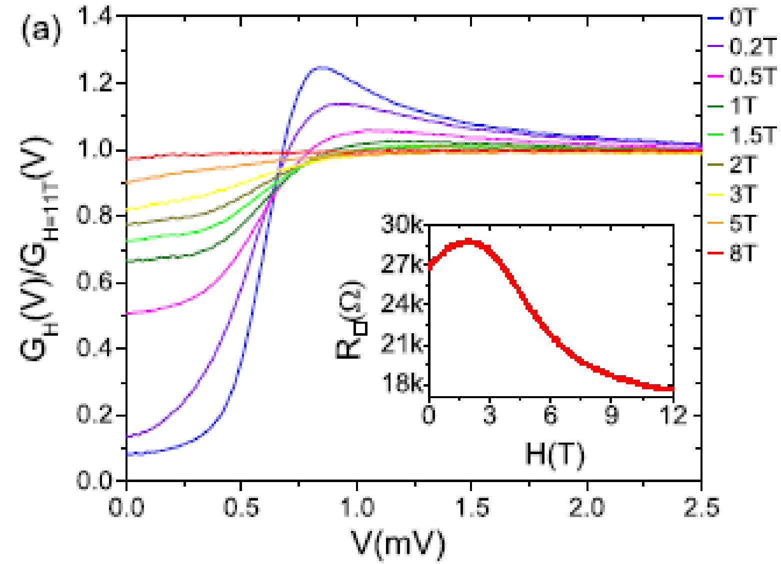
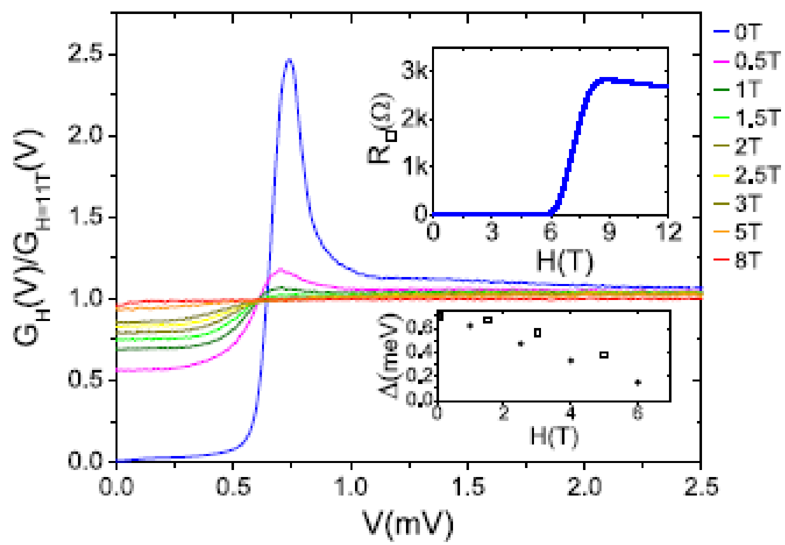
➢ Cooper pair insulator ?

Magnetic field studies through the SIT









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