Mesoscopic Superconductivity

Cargèse, 11/2016

H. Pothier

iramis.cea.fr/spec/Pres/Quantro

→ theses (under Publications)

Charging effects → P. Lafarge

Proximity, Usadel → S. Guéron

A. Anthore

H. le Sueur

Qubits → A. Cottet

A. Palacios-Laloy

Andreev bound states → L. Bretheau

C. Janvier

J-D Pillet

Mesoscopic Superconductivity

Assume you know superconductivity ... but have forgotten most of it!

Tu AM HP — I. Reminders on BCS theory

→ Cooper pairs. Gap

We PM CB — II. Small superconductors

$\Delta \to \xi = \frac{\hbar v_F}{L}$ or $\sqrt{\frac{\hbar D}{\Delta}}$

In a B field: London length λ_L for screening

What happens if $L < \lambda_L$? Role of ξ ?

Thu AM HP+CB — III. Mesoscopic transport

N conductors → Landauer formalism → LB formula QPC

N-S, S-S weak links ⇒ equivalent description?

→ ABS, Shiba states, Josephson effect, currents and supercurrents

Fri AM HP — IV. Charging effects

Small superconductors with respect to C : $\frac{(2e)^2}{2C} > \Delta$

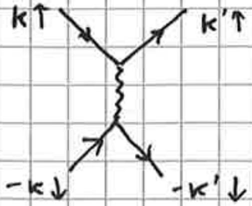
→ parity effects. Superconducting qubits

Sa PM CB — V. Proximity effect

Describe hybrid structure at $L \lesssim \frac{\hbar D}{E^2}$ for $E \lesssim \Delta$

From BCS to BdG : representations of superconductors

BCS (1957) : based on (phonon-mediated) attraction between Cooper pairs



$$H = \sum_{k,\sigma} \varepsilon_k \, c^+_{k\sigma} c_{k\sigma} - V \sum_{kk'} c^+_{k'\uparrow} c^+_{-k'\downarrow} c_{-k\downarrow} c_{k\uparrow}$$

$\varepsilon_k = \dfrac{\hbar^2 k^2}{2m}$

Mean-field theory in grand-canonical ensemble :
(fix μ, not N)

$$H - \mu N = \sum_{k,\sigma} \xi_k \, c^+_{k\sigma} c_{k\sigma} - V \left(\sum_k c^+_{k\uparrow} c^+_{-k\downarrow} \right) \times \left(\sum_k c_{-k\downarrow} c_{k\uparrow} \right)$$

$\xi_k = \varepsilon_k - \mu$: energies counted from Fermi L.

$$D \equiv \sum_k c_{-k\downarrow} c_{k\uparrow} = \langle D \rangle + \underbrace{(D - \langle D \rangle)}_{\equiv d \text{, assumed small}}$$

$$\to D^+ D = (\langle D^+ \rangle + d^+)(\langle D \rangle + d)$$

$$= \langle D^+ \rangle \langle D \rangle + \underbrace{d^+}_{D^+ - \langle D^+ \rangle} \langle D \rangle + \underbrace{d}_{D - \langle D \rangle} \langle D^+ \rangle + \cancel{d^+ d}$$

$$\simeq D^+ \langle D \rangle + D \langle D^+ \rangle - \langle D^+ \rangle \langle D \rangle$$

Directly : if x & y have small variations around their average,

$$xy = \underbrace{(x - \langle x \rangle)(y - \langle y \rangle)}_{\text{second order}} + x\langle y \rangle + \langle x \rangle y - \langle x \rangle \langle y \rangle$$

Define $\Delta = -V \langle D \rangle$

$\Rightarrow -V D^\dagger D \simeq \Delta D^\dagger + \Delta^* D + \frac{|\Delta|^2}{V}$

Transform* 1st term of $\mathcal{H} = H - \mu N$; for $\sigma = \downarrow$,

use $c^\dagger_{k\downarrow} c_{k\downarrow} = 1 - c_{k\downarrow} c^\dagger_{k\downarrow}$

and instead of $\sum_k$ do $\sum_{-k}$

*(Because D mixes $c_{k\uparrow}$ and $c_{-k\downarrow}$)

$$\Rightarrow \mathcal{H} = \sum_k \mathcal{H}_k + \frac{|\Delta|^2}{V}$$

where $\mathcal{H}_k = (c^\dagger_{k\uparrow}\ c_{-k\downarrow}) \begin{pmatrix} \xi_k & \Delta \\ \Delta^* & -\xi_k \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^\dagger_{-k\downarrow} \end{pmatrix} + \xi_k$

Quadratic form → can be diagonalized.

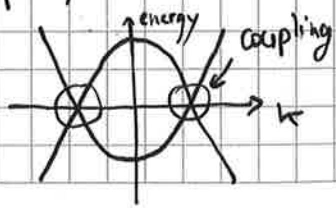
Get rid of spin indices; introduce "electron" and "holes" operators: $\begin{cases} e^\dagger_k \equiv c^\dagger_{k\uparrow} \\ h^\dagger_k \equiv c_{-k\downarrow} \end{cases}$

(forget k indices for a moment)

$$\mathcal{H}_k = (e^\dagger\ h^\dagger) \begin{pmatrix} \xi & \Delta \\ \Delta^* & -\xi \end{pmatrix} \begin{pmatrix} e \\ h \end{pmatrix} + \xi$$

Dispersion for e: ξ

for h: $-\xi$



e-h symmetry: solutions at $\pm E$ for each k.

Diagonal form $\mathcal{H}_k = (\gamma^\dagger_\oplus\ \gamma^\dagger_\ominus) \begin{pmatrix} E & 0 \\ 0 & -E \end{pmatrix} \begin{pmatrix} \gamma_\oplus \\ \gamma_\ominus \end{pmatrix} + \text{cst.}$

Trace: $+E^2 = +\xi^2 + |\Delta|^2$

$$\boxed{E = \sqrt{\xi^2 + |\Delta|^2}}$$

Assume $\Delta_s = \Delta e^{i\varphi}$

Check that $\begin{cases} \gamma_\oplus = u\, e + v\, e^{i\varphi} h \\ \gamma_\ominus = -v\, e^{-i\varphi} e + u\, h \end{cases}$

with $u = \sqrt{\frac{1}{2}\left(1 + \frac{\xi}{E}\right)}$; $v = \sqrt{\frac{1}{2}\left(1 - \frac{\xi}{E}\right)}$

Proof: $\gamma^\dagger_\oplus \gamma_\oplus = u^2 e^\dagger e + v^2 h^\dagger h + uv\,(e^{i\varphi} e^\dagger h + e^{-i\varphi} e h^\dagger)$

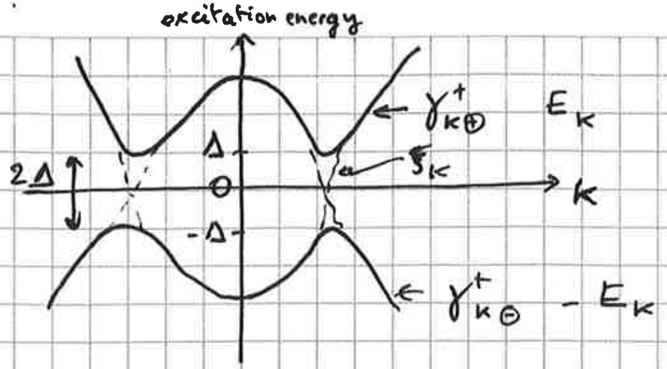
$\gamma^\dagger_\ominus \gamma_\ominus = v^2$ " $+ u^2$ " $-$ "

$\rightarrow \mathcal{H}_k = E\,(u^2 - v^2)(e^\dagger e - h^\dagger h) + 2Euv\,(\ "\ )$

$= \xi\,(\ "\ ) + \Delta\,(\ "\ )$

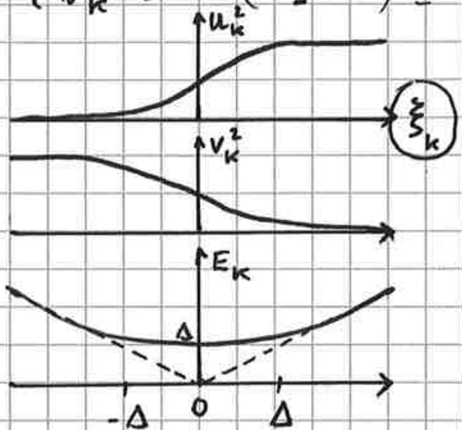
$\Rightarrow \begin{cases} u^2 - v^2 = \frac{\xi}{E} & \text{ok} \\ 2uv = \frac{\Delta}{E} & \text{ok} \end{cases}$

$\Rightarrow$ Excitations at $\begin{cases} +E_k & \text{created by } \gamma^\dagger_{k\oplus} \\ -E_k & \gamma^\dagger_{k\ominus} \end{cases}$



- At $k = k_F$ $\xi_k = 0$ $E = \Delta$

- $\begin{cases} u_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left(1 + \frac{\xi_k}{\sqrt{\xi_k^2 + |\Delta|^2}}\right) \\ v_k^2 = \quad (- \quad) = \quad (- \quad) \end{cases}$

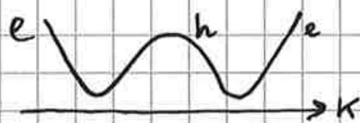


(v_k^2 resembles a Fermi function at $k_B T = \frac{\Delta}{2}$)

For $\xi_k \gg \Delta$ ($|k| > k_F$) $u_k \sim 1$ $v_k \sim 0$

$\rightarrow \begin{cases} \gamma_{k\oplus} \simeq e_k \text{ "electron"} \\ \gamma_{k\ominus} \simeq h_k \text{ "hole"} \end{cases}$

(upper half of top graph right and left)



(OK with curvature : $m_e > 0$, $m_h < 0$)

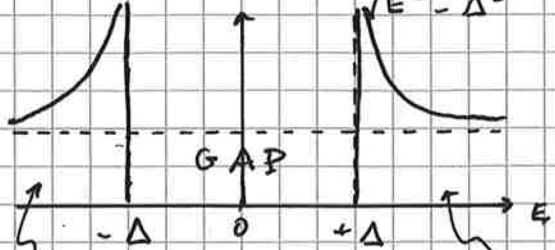
At $|\xi_k| \ll \Delta$ ($|E| \simeq \Delta$), $u_k^2 \simeq v_k^2 \simeq \frac{1}{2}$: equal weight of e & h.

- QP DOS : $n_s(E)\, dE = n(k)\, dk$

Fermi sphere (3D) : $n(k) \propto k^2$. But since $\Delta \ll E_F$, take $n(k) \simeq cst$ & $k \simeq k_F$

$$\frac{dE}{dk} = \frac{dE}{d\xi}\frac{d\xi}{dk} \simeq \frac{\xi}{\sqrt{\Delta^2 + \xi^2}} \frac{\hbar^2}{2m} k_F \propto \frac{\sqrt{E^2 - \Delta^2}}{|E|}$$

$$\Rightarrow n_s(E) = n_N \frac{|E|}{\sqrt{E^2 - \Delta^2}}$$



States filled at T=0

states empty at T=0

excitations created by $\gamma_{k\ominus}$ (no dagger!) : empty a state at $E < 0$ ⇒ energy cost $-E > 0$ = excitation

excitations by $\gamma^+_{k\oplus}$: occupy state at $E > 0$

- BCS ground state ?

All states at $E < 0$ filled :

$$|BCS\rangle = \prod_k \gamma^+_{k\ominus} |\text{vacuum}\rangle$$

$|\text{vaccuum}\rangle$ = no "electron", no "hole"

(electron → spin ↑ states unoccupied; hole → spin ↓ states all occupied!)

$= \prod_{k'} c^{+}_{k'\downarrow} |0\rangle$ (vacuum of electrons)

$$|BCS\rangle = \prod_k \left(-v_k e^{i\varphi} c^{+}_{k\uparrow} + u_k c_{-k\downarrow}\right) \prod_{k'} c^{+}_{k'\downarrow} |0\rangle$$

(gather terms k and $k' = -k$)

$$= \prod_k \left(-v_k e^{i\varphi} c^{+}_{k\uparrow} c^{+}_{-k\downarrow} + u_k \underbrace{c_{-k\downarrow} c^{+}_{-k\downarrow}}_{\text{applied to } |0\rangle : \mathbb{1}}\right) |0\rangle$$

$$= \prod_k \left(-v_k e^{i\varphi} c^{+}_{k\uparrow} c^{+}_{-k\downarrow} + u_k\right) |0\rangle$$

$$= \prod_k \left(-v_k e^{i\varphi} |\bullet\bullet\rangle_k + u_k |\circ\circ\rangle_k\right)$$

⚠ • Not e^- pairs but superpositions of pair states occupied & unoccupied.

• Product state on all ks, not $|k| < k_F$

• Phase φ is the same for all ks!

...

- Recover "normal" ground state: $\Delta = 0$

$$\Rightarrow \begin{cases} u_k = 0 & \text{for } \xi_k < 0 \quad & 1 \text{ for } \xi_k > 0 \\ v_k = 1 & \quad & 0 \end{cases}$$

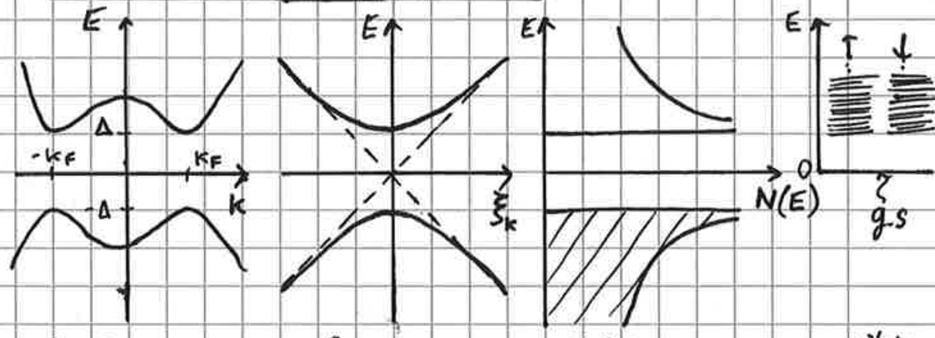
($|k| < k_F$; $|k| > k_F$)

$$\Rightarrow |N\rangle = \prod_{|k| < k_F} (-e^{i\varphi}) c^{+}_{k\uparrow} c^{+}_{-k\downarrow} \prod_{|k| > k_F} (+1) |0\rangle$$

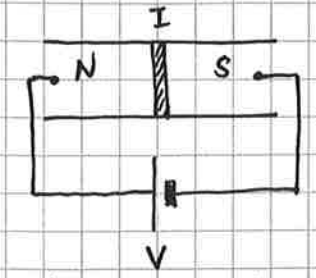
all states at $|k| < k_F$ are doubly occupied (↑↓)
(global phase not relevant)

- In $|BCS\rangle$, only states close to Fermi energy ($k \sim k_F$, $\frac{E}{\Delta}$ not too large) are different from what they are in $|N\rangle$.

■ Representations of a superconductor:



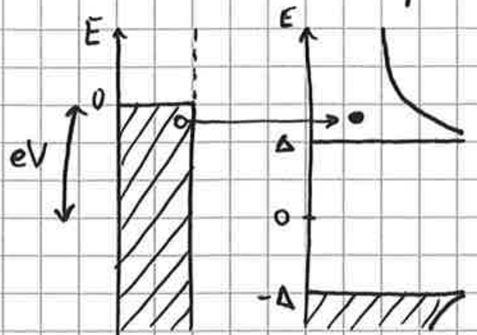
Current through NIS junctions $(T=0)$



(Tunnel hamiltonian:

$$H_t = t \sum_{k,\sigma} c_{kL\sigma}\, c^{+}_{kR\sigma} + h.c.$$)

Use semi-conductor representation: $\mu_L - \mu_R = eV$



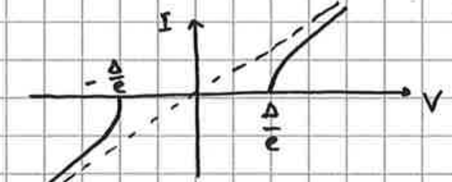
$I = 0$ for $|V| < \frac{\Delta}{e}$

$$I \propto \int_{\Delta}^{eV} n_s(E)\, dE$$

$$\Rightarrow \boxed{\frac{dI}{dV} = \frac{1}{R_t}\, n_s(eV)}$$

principle of tunneling spectroscopy from N-electrode

$$I(V) = \frac{1}{R_t}\sqrt{V^2 - \left(\frac{\Delta}{e}\right)^2}$$



(skip) Is this really correct? Excitations in S and in N are not of the same nature! Need to decompose $c^{+}_{kR\sigma}$ into $\gamma^{(+)}_{k\oplus}$ and $\gamma^{(+)}_{k\ominus}$ in S!

Cf p.3:
$$\begin{cases} \gamma^{+}_{k\oplus} = u_k\, c^{+}_{k\uparrow} + v_k\, e^{-i\varphi}\, c_{-k\downarrow} & \times u_k \\ \gamma^{+}_{k\ominus} = -v_k\, e^{i\varphi}\, c^{+}_{k\uparrow} + u_k\, c_{-k\downarrow} & \times v_k e^{-i\varphi} \end{cases}$$

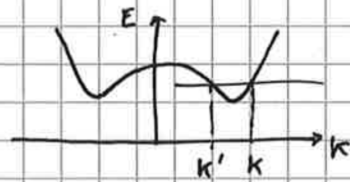
$$\Rightarrow c^{+}_{k\uparrow} = u_k\, \gamma^{+}_{k\oplus} - v_k\, e^{-i\varphi}\, \gamma^{+}_{k\ominus} \qquad (u_k^2 + v_k^2 = 1)$$

In $|BCS\rangle$, all states $\ominus$ are occupied:

$$c^{+}_{k\uparrow}|BCS\rangle = u_k\, \gamma^{+}_{k\oplus}|BCS\rangle$$

oops, shouldn't this coefficient enter in the calculation ?!

In fact, there is a compensation: at a given energy E, there is also a state on the other side of k_F:



This state has opposite ξ:

$$\xi_{k'} = -\xi_k$$

$$\Rightarrow u_{k'} = v_k$$

In the calculation of the current, 2 contributions at energy E combine:

$$c^{+}_{k\uparrow}|BCS\rangle = u_k\, \gamma^{+}_{k\oplus}|BCS\rangle$$

$$c^{+}_{k'\uparrow}|BCS\rangle = v_k\, \gamma^{+}_{k'\oplus}|BCS\rangle$$

In Fermi Golden Rule, contributions $|u_k|^2 + |v_k|^2 = 1$

$\Rightarrow$ altogether, we can forget about u_k & v_k!

(same compensation occurs of course for $c^{+}_{k\downarrow}$)

Calculation of $\Delta(T)$

cf p. 2 : $\Delta = +V \langle \sum_k c_{k\uparrow} c_{-k\downarrow} \rangle$

(take $\varphi = 0$) (inverted order $c_{-k\downarrow} c_{k\uparrow}$)

$$\begin{cases} c_{k\uparrow} = u_k \gamma_{k\oplus} - v_k \gamma_{k\ominus} \quad (p.9) \\ c_{-k\downarrow} = v_k \gamma^+_{k\oplus} + u_k \gamma^+_{k\ominus} \end{cases}$$

$$c_{k\uparrow} c_{-k\downarrow} = u_k v_k \left(\gamma_{k\oplus}\gamma^+_{k\oplus} - \gamma_{k\ominus}\gamma^+_{k\ominus} \right)$$

+ terms in $\gamma_{k\oplus}\gamma^+_{k\ominus}$ & $\gamma_{k\ominus}\gamma^+_{k\oplus}$ that give $\langle \dots \rangle = 0$

$$u_k v_k = \frac{1}{2}\sqrt{1 - \left(\frac{\xi_k}{E_k}\right)^2} = \frac{\Delta}{2E_k}$$

$$\langle \gamma_{k\oplus}\gamma^+_{k\oplus} \rangle = 1 - \langle \gamma^+_{k\oplus}\gamma_{k\oplus} \rangle = 1 - f(E_k)$$

$$\langle \gamma_{k\ominus}\gamma^+_{k\ominus} \rangle = f(E_k) \qquad (\text{in N, } f = \langle c^+ c \rangle)$$

since $\gamma^+_{k\ominus}$ and $\gamma_{k\ominus}$ create excitations in S

$$\Rightarrow \Delta = +V \sum_k \frac{\Delta}{2E_k}\left(1 - 2f(E_k)\right)$$

$$1 - 2f(E) = \text{th}\,\frac{\beta E}{2} \qquad \beta = \frac{1}{k_B T}$$

$$\sum_k \to n_N \int_\Delta^{+\infty} dE\, n_s(E) \qquad (\text{DOS is } n_N \cdot n_s(E))$$

Introduce $g = \frac{1}{2} n_N V$

$$1 = g \int_\Delta^{+\infty\ \Omega} dE \frac{1}{\sqrt{E^2 - \Delta^2}}\, \text{th}\,\frac{\beta E}{2}$$

(Ω ← remove log divergence : cut-off at Debye energy)

$\underline{T \to 0}$: $\Delta \to \Delta_0$ $\text{th}\,\frac{\beta E}{2} = 1$ for $E > \Delta$

$$1 = g \underbrace{\int_\Delta^\Omega \frac{dE}{\sqrt{E^2 - \Delta^2}}}_{\text{argch}\left(\frac{\Omega}{\Delta}\right)}$$

$$\underset{(\Omega \gg \Delta)}{\approx} \ln \frac{2\Omega}{\Delta}$$

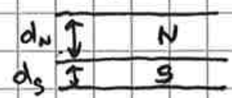
$$\Rightarrow \boxed{\Delta = 2\Omega\, e^{-1/g}}$$

Al : $\Delta = 180\ \mu eV = 2.1\ k_B K$ ($\neq T_c$!)

$\Omega = 433\ k_B K \to g \simeq 0.17$

$e^{-1/g} \simeq 0.0024$

Bilayer with perfect interface & thicknesses $\ll \xi$:



$$g_{eff} = \frac{1}{2} n_N \frac{d_S}{d_S + d_N} V \quad (n \text{ equal})$$

If $d_S = d_N$, $g_{eff} = \frac{1}{2} g_S$

$$\to \Delta^* = 2\Omega e^{-2/g_S} = \Delta e^{-1/g_S} \ll \Delta \ !$$

$\underline{T \to T_c}$: $\Delta \to 0$ $1 = g \int_0^\Omega \frac{dE}{E}\, \text{th}\,\frac{\beta E}{2}$

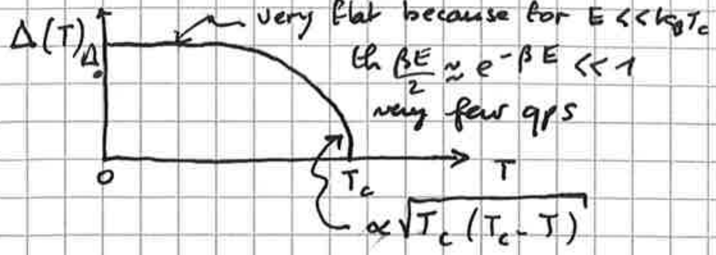
$$= g \ln\left(\frac{2\gamma}{\pi}\Omega\beta\right) \qquad \gamma : \text{Euler's constant}$$

Compare with eq. obtained at $T=0$: $1 = g \ln \frac{2\Omega}{\Delta_0}$

Ratio $\rightarrow \frac{\cancel{2}\cancel{\Omega}}{\Delta_0} = \frac{\cancel{2}\gamma}{\pi} \cancel{\Omega} \frac{1}{k_B T_c}$

$\rightarrow \frac{\Delta_0}{k_B T_c} = \frac{\pi}{\gamma} \simeq 1.76$ Al: $\Delta_0 = 2.1 k_B K$, $T_c = 1.2 K$

$T \neq 0$:



$\Delta\left(\frac{T_c}{2}\right) = 0.96\,\Delta_0$ characteristic of mean field theory

$\Delta(0.9T_c) = \frac{\Delta_0}{2}$

From BCS to BdG equation

Cf p. 2 : in 2nd quantization,

$$\mathcal{H} = \sum_k \begin{pmatrix} e_k^+ & h_k^+ \end{pmatrix} \begin{pmatrix} \xi_k & \Delta \\ \Delta^* & -\xi_k \end{pmatrix} \begin{pmatrix} e_k \\ h_k \end{pmatrix}$$

When $\mathcal{H}$ contains a term that depends on position (like $\vec{A}(r)$, or any non-translation invariant geometry) need to solve Schrödinger eq. in real-space representation with

$\psi(x) = \begin{pmatrix} a(x) \\ b(x) \end{pmatrix}$ 2-component wavefunction with $a(x)$: electron amplitude, $b(x)$: hole amplitude

(N.B. If spin matters, need $a_\uparrow, a_\downarrow, b_\uparrow, b_\downarrow$)

BdG eq.: $\begin{pmatrix} H_1 & \Delta \\ \Delta^* & -H_1^* \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = E \begin{pmatrix} a \\ b \end{pmatrix}$

where H_1, Δ, a, b are position-dpd^t.

- if $H_1 = \frac{p^2}{2m} - \mu$, recover previous solutions by taking $\begin{cases} a(x) = a_k e^{ikx} \\ b(x) = b_k e^{ikx} \end{cases}$

$$\rightarrow \begin{pmatrix} \xi_k & \Delta \\ \Delta^* & -\xi_k \end{pmatrix} \begin{pmatrix} a_k \\ b_k \end{pmatrix} = E_k \begin{pmatrix} a_k \\ b_k \end{pmatrix}$$

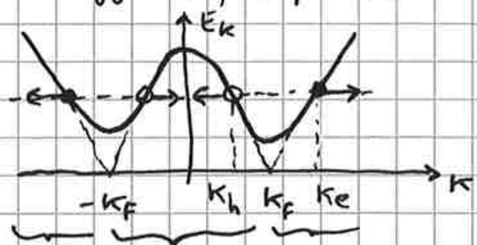
coeffs of e_k & h_k in $\gamma^+_{k\oplus}$ $\qquad\qquad$ $\gamma^+_{k\ominus}$

→ solutions $\begin{pmatrix} a_k \\ b_k \end{pmatrix} = \begin{pmatrix} u_k \\ v_k e^{-i\varphi} \end{pmatrix}$ or $\begin{pmatrix} -v_k e^{i\varphi} \\ +u_k \end{pmatrix}$

energy $+E_k$ $\qquad$ energy $-E_k$

$$(E_k^2 = |\Delta|^2 + \xi_k^2)$$

at a given energy E_k, 4 possible states :



all of the form $\begin{pmatrix} u_k \\ v_k e^{-i\varphi} \end{pmatrix}$ $\qquad (E_k > 0)$

Group velocity $\frac{\partial E}{\partial k}$: at k_e, •→

at k_h, ←• $\qquad$ etc.

In a normal metal, $\Delta = 0 \to E_k = |\xi_k|$

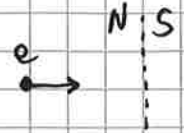
(p.3) $u_k^2 = \frac{1}{2}(1 + \text{sgn}(\xi_k)) = 1$ for e^-, 0 for h

$v_k^2 = \frac{1}{2}(1 - \text{sgn}(\xi_k)) = 0$ for e^-, 1 for h

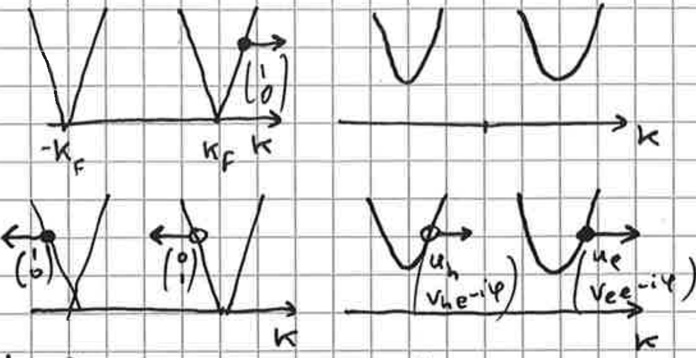
e^- : $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad$ h : $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\qquad$ ok !

Andreev Reflection

• Ballistic NS interface



what happens to an e^- at energy $E > 0$?



u_e : at k_e

u_h : at k_h

amplitudes $\quad b \quad a \quad d \quad c$

(notations of BTK)

Match wavefunctions at NS interface ;

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 1 \\ 0 \end{pmatrix} + a\begin{pmatrix} 0 \\ 1 \end{pmatrix} = d\begin{pmatrix} u_h \\ v_h e^{-i\varphi} \end{pmatrix} + c\begin{pmatrix} u_e \\ v_e e^{-i\varphi} \end{pmatrix} \quad \begin{matrix} (1) \\ (2) \end{matrix}$$

because $\Psi_N(x) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ik_e x} + b\begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ik_e x}$

$$+ a\begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ik_h x}$$

$$\Psi_S(x) = d\begin{pmatrix} u_h \\ v_h e^{-i\varphi} \end{pmatrix} e^{ik_h^S x} + c\begin{pmatrix} u_e \\ v_e e^{-i\varphi} \end{pmatrix} e^{ik_e^S x}$$

Match derivative of ψ at $x=0$:

$$\begin{pmatrix}1\\0\end{pmatrix} k_e^* - b\begin{pmatrix}1\\0\end{pmatrix} k_e + a\begin{pmatrix}0\\1\end{pmatrix} k_h$$

$$= d\begin{pmatrix}u_h\\ v_h e^{-i\varphi}\end{pmatrix}(-k_h^S) + c\begin{pmatrix}u_e\\ v_e e^{-i\varphi}\end{pmatrix} k_e^S$$

Andreev approximation : $k_{e,h}^{N,S} \approx k_F$!

(3)
(4) → $\begin{pmatrix}1\\0\end{pmatrix} - b\begin{pmatrix}1\\0\end{pmatrix} + a\begin{pmatrix}0\\1\end{pmatrix} = -d\begin{pmatrix}u_h\\ v_h e^{-i\varphi}\end{pmatrix} + c\begin{pmatrix}u_e\\ v_e e^{-i\varphi}\end{pmatrix}$

(2)-(4) : $d=0 \to a = c\, v_e e^{-i\varphi}$

(1)+(3) : $2 = 2c\,u_e \to c = \frac{1}{u_e}$ $\to a = \frac{v_e}{u_e} e^{-i\varphi}$

(1)-(3) : $b=0$

Comments : $b=0$, $a\neq 0$: reflection as a hole = AR

$d=0$, $c\neq 0$: transmission as quasi-e^-

Amplitude of reflected hole is $a = \frac{v_e}{u_e} e^{-i\varphi}$

$|a| = \sqrt{\frac{E-\xi_s^e}{E+\xi_s^e}}$ ⚠ in N, E = energy of incident e

in S, $E = \sqrt{\Delta^2+\xi_s^2}$

$\xi_s = \sqrt{E^2-\Delta^2}$

- if $E>\Delta$, no pb, $\xi_s>0$: ∃ propagating quasi-e^- at energy E

$$|a| = \sqrt{\frac{E-\xi_s}{E+\xi_s}} = \frac{E-\xi_s}{\sqrt{E^2-\xi_s^2}} = \frac{E-\sqrt{E^2-\Delta^2}}{\Delta}$$

when $E \gg \Delta$, $|a|\to 0$, $c\to 1$: more & more transmitted as quasi-e^- because resembles e^- !

- if $E<\Delta$, $\xi_s \in i\mathbb{R}$ $\quad \xi_s = \frac{\hbar^2 k_s^2}{2m}$

→ k_s has imaginary part : decays on $\frac{\xi}{\sqrt{1-\left(\frac{E}{\Delta}\right)^2}}$: evanescent

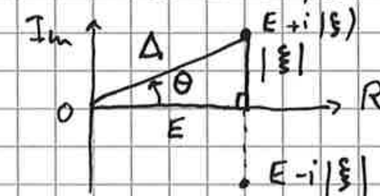
$$a = \sqrt{\frac{E-\xi_s}{E+\xi_s}}\, e^{-i\varphi} = \underbrace{\sqrt{\frac{E+\xi_s^*}{E+\xi_s}}}_{\text{modulus } 1} e^{-i\varphi}$$

$|a|=1$
total refl. as hole

Phase acquired at reflection : $\mathrm{Arg}(a)$

$$= \frac{1}{2}\mathrm{Arg}\frac{E-\xi}{E+\xi} - \varphi$$

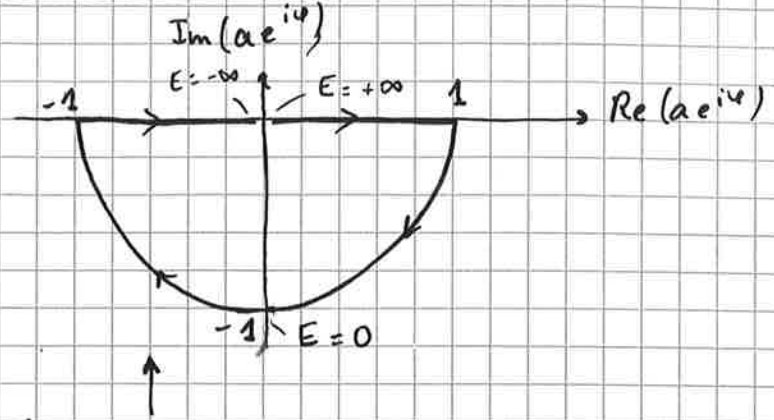
$\xi = i|\xi| \quad \to \quad E^2 = \Delta^2 - |\xi|^2$



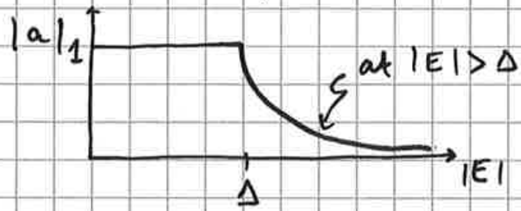
$\mathrm{Arg}(E\pm\xi) = \pm\theta \quad \to \quad \frac{1}{2}\mathrm{Arg}\frac{E-\xi}{E+\xi} = -\theta = -\mathrm{Arccos}\frac{E}{\Delta}$

⇒ phase acquired during A.R. is $\boxed{-\varphi - \arccos\frac{E}{\Delta}}$

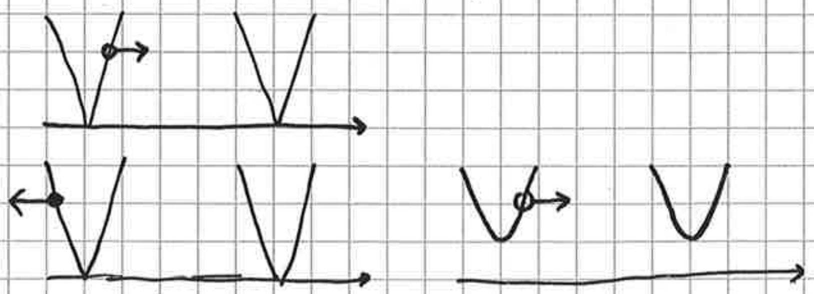
$$\boxed{a = e^{-i\left(\varphi + \arccos\frac{E}{\Delta}\right)}} \quad (|E|<\Delta)$$



(we did calculation for $E>0$, but at $E<0$ get this side)



• Repeat same calculation for incident hole at $E>0$



$$\Psi_N(x) = b\begin{pmatrix}1\\0\end{pmatrix}e^{-ik_e x} + \begin{pmatrix}0\\1\end{pmatrix}e^{-ik_h x}$$

$$\Psi_S(x) = d\begin{pmatrix}u_h\\v_h e^{-i\varphi}\end{pmatrix}e^{-ik_h^S x}$$

Continuity of Ψ, Ψ', $k^{e,h}_{N,S} \simeq k_F$

$$\rightarrow \begin{cases} b = d\,u_h \\ 1 = d\,v_h e^{-i\varphi} \end{cases} \rightarrow b = \frac{u_h}{v_h}e^{i\varphi} = \frac{v_e}{u_e}e^{i\varphi}$$

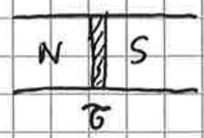
$$\rightarrow \boxed{b = e^{i\left(\varphi - \arccos\frac{E}{\Delta}\right)}} \quad \text{for } |E|<\Delta$$

⇒ same coefficient as for $e^- \rightarrow h$, but for sgn(φ)

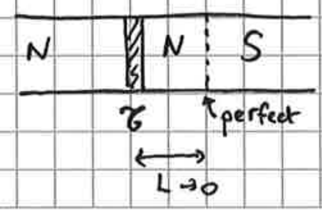
* Imperfect interface : BTK model to calculate current through NIS junction (PRB 25, 4515 (1982))

2 methods : • in BTK paper, introduce barrier $V\delta(x)$ in H_1 and solve BdG eq. → introduce $Z = \frac{V}{\hbar v_F}$ which sets the discontinuity of Ψ at barrier.

• use the result at $V=0$, and model



by

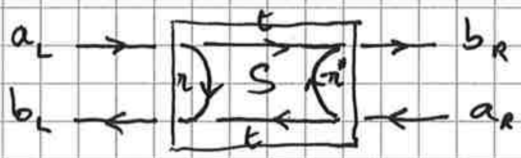


τ : transmission proba. for e^- : $\tau = \frac{1}{1+Z^2}$

$\tau = |t|^2$ and $S_h = S_e^*$

$$S_e = e^{i\eta}\begin{pmatrix} r & t \\ t & -r^* \end{pmatrix} \quad t \in \mathbb{R},\ r \in \mathbb{C},\ \text{take } \eta = 0$$

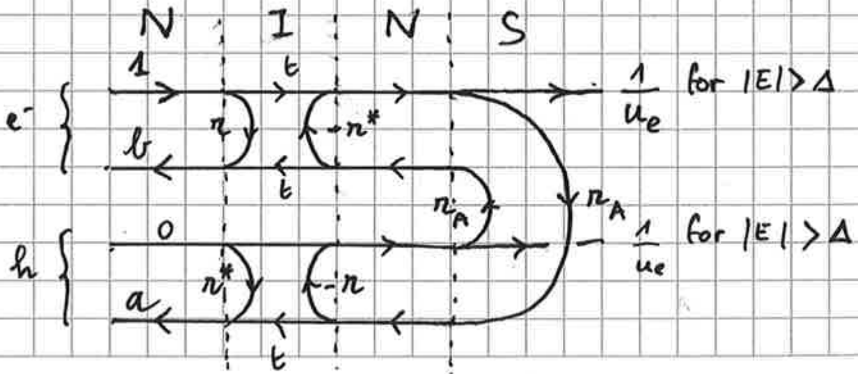
S_e relates outgoing waves to incoming waves



$$\begin{pmatrix} b_L \\ b_R \end{pmatrix} = S \begin{pmatrix} a_L \\ a_R \end{pmatrix}$$

Back to NIS : assume $\varphi = 0$ because when a single S, φ has no incidence (vanishes if kept here)

$\Rightarrow$ call r_A the AR for e^- & h

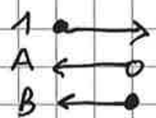


$$a = t^2 r_A \left(1 + |r|^2 r_A^2 + \dots\right) = \frac{t^2 r_A}{1 - |r|^2 r_A^2} = \frac{\tau r_A}{1-(1-\tau) r_A^2}$$

$$b = r - r t^2 r_A^2 \left(1 + |r^2| r_A^2 + \dots\right) = r\left(1 - \frac{\tau r_A^2}{1-(1-\tau) r_A^2}\right) = \sqrt{1-\tau}\,\frac{1 - r_A^2}{1-(1-\tau) r_A^2}$$

Rmq : at $|E| = \Delta$, $r_A = \pm 1 \rightarrow a = \pm 1$:
always perfect AR even if $\tau \neq 1$:
Fabry-Perot-like

Proba to be reflected as h is $A = |a|^2$
as e^- is $B = |b|^2$



$1 + A - B$ e^- flows in left electrode,
each contribute by $G_0 = \frac{2e^2}{h}$

at $T=0$: $I = G_0 \int_0^{eV} \frac{dE}{e}\left(1 + A(E) - B(E)\right)$

$$\frac{dI}{dV} = G_0\left(1 + A(eV) - B(eV)\right)$$

$E \gg \Delta \quad A=0 \quad B = 1-\tau \rightarrow \frac{dI}{dV} = G_0 \tau$ ok

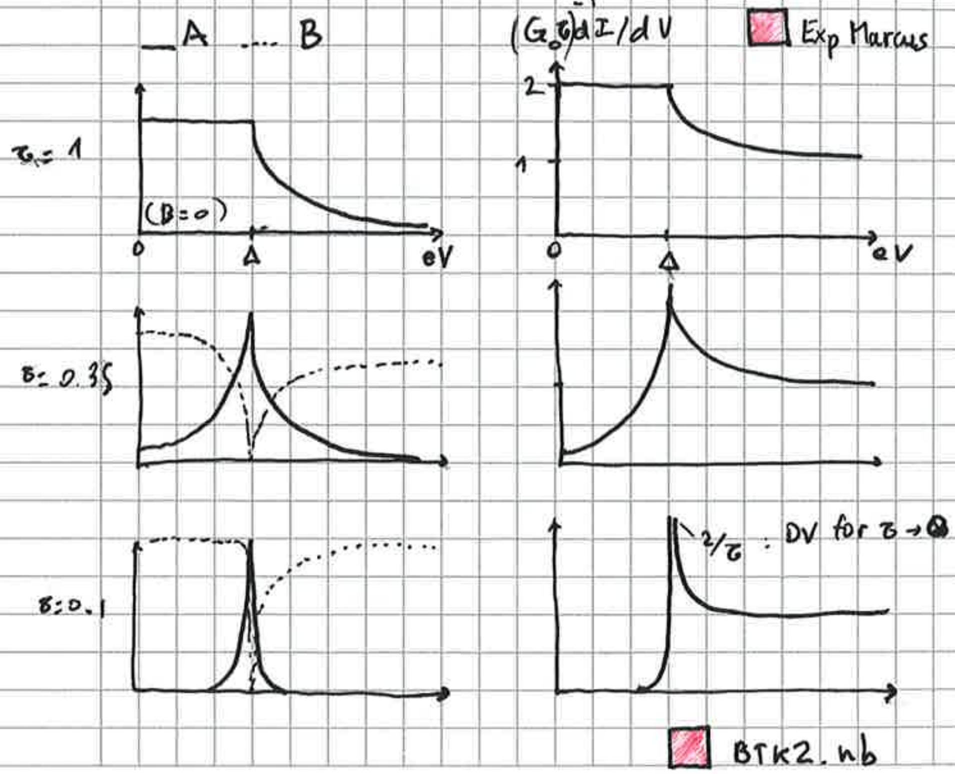
$E < \Delta$ nothing transmitted in S $\rightarrow B = 1 - A$

$\Rightarrow \frac{dI}{dV} = 2 G_0 A(eV)$

for each e^- sent, 1 h reflected

→ 1 pair transmitted in S (we just discussed excitations in S !)

⇒ increase of conductance by factor 2 compared to NIN when $\tau = 1$



Exercice : show that when $\tau \to 0$ $\frac{1}{G_0 \tau} \frac{dI}{dV} = \frac{E}{\sqrt{E^2-\Delta^2}} = n_s(E)$

$A \propto \tau^2 \quad B \propto 1 - \tau \rightarrow \frac{dI}{dV} = G_0 (1-B)$

$B = (1-\tau)\left(\frac{1-r_A^2}{1-(1-\tau) r_A^2}\right)^2$ let $\alpha = r_A^2$

$= (1-\tau)\left(\frac{1}{1-\frac{\alpha}{1-\alpha}\tau}\right)^2 = (1-\tau)\left(1 - \frac{2\alpha}{1-\alpha}\tau\right)$

$= 1 - \tau\left(1 + \frac{2\alpha}{1-\alpha}\right) = 1 - \tau \frac{1+\alpha}{1-\alpha}$

$1 - B = \tau \frac{1+\alpha}{1-\alpha}$

$\alpha = \left(\varepsilon - \sqrt{\varepsilon^2-1}\right)^2$ with $\varepsilon = \frac{E}{\Delta}$

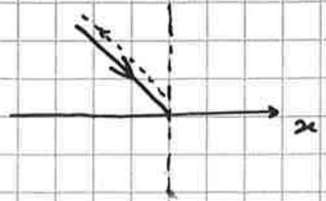
$(\varepsilon + \sqrt{\ })\alpha = (\varepsilon + \sqrt{\ })(\varepsilon - \sqrt{\ })(\varepsilon - \sqrt{\ })$

$= (\varepsilon^2 - (\varepsilon^2 - 1))(\varepsilon - \sqrt{\ })$

$= \varepsilon - \sqrt{\ }$

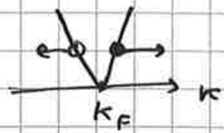
$\Rightarrow \alpha = \frac{\varepsilon - \sqrt{\ }}{\varepsilon + \sqrt{\ }}$ $\quad 1 + \alpha = \frac{2\varepsilon}{\varepsilon + \sqrt{\ }}$, $\quad 1 - \alpha = \frac{2\sqrt{\ }}{\varepsilon + \sqrt{\ }}$ $\Big\} \quad \frac{1+\alpha}{1-\alpha} = \frac{\varepsilon}{\sqrt{\varepsilon^2-1}}$ ok !

Remarks on AR : at 3D, a retroreflection



along x, $k_e^\perp \simeq k_h^\perp$

(Andreev approx)



In transverse direction, $k_{//}$ is conserved.

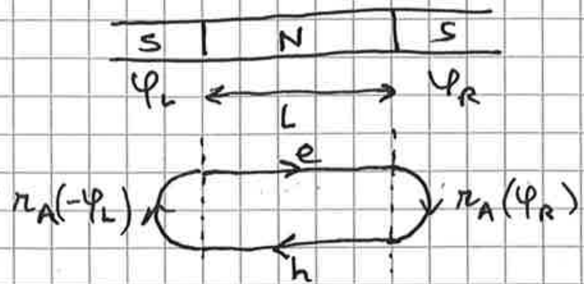
⇒ total $\vec{k}$ is conserved, but at same $\vec{k}$ group velocity of e^- & h are opposite → retroreflection

This is not the case if Δ is not $\ll E_F$ because k_e & k_h can be then significantly ≠. In graphene, A.R. can be at an angle, specular, or even forbidden at certain angles: cf Beenakker, PRL 97, 067007 (2006)

What BTK misses : in many situations, backscattering in N region → multiple points where AR can occur → constructive interference increasing greatly the total proba of AR → see later

From AR to ABS & Josephson effect

1-D SNS junction, first without scattering in N, & perfect interfaces



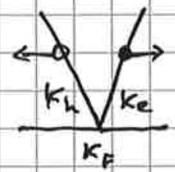
$\delta = \varphi_L - \varphi_R$

e^- at E

Phase acquired . at 2 AR : $-2\arccos\frac{E}{\Delta} - \varphi_R + \varphi_L$

. while crossing N : $(k_e - k_h)L$

$= 2(k_e - k_F)L$



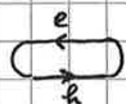
$E = \frac{\hbar^2}{2m}(k_e^2 - k_F^2) \simeq \frac{\hbar^2}{m} k_F (k_e - k_F) = \hbar v_F (k_e - k_F)$

$\to 2(k_e - k_F)L = \frac{2E}{\hbar v_F} L = 2 \frac{\Delta}{\hbar v_F} \frac{E}{\Delta} L = \frac{2}{\xi}\varepsilon$

with $\varepsilon = \frac{E}{\Delta}$; $\xi = \frac{\hbar v_F}{\Delta}$ superc. coherence length (⚠ before, ξ was an energy!)

→ bound states when $\boxed{-2\arccos\varepsilon + \delta + \frac{2}{\xi}\varepsilon \equiv 0\ [2\pi]}$

Same calc for

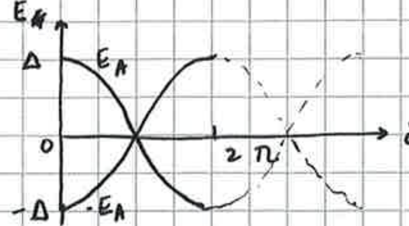


⇒ $+2\arccos\varepsilon - \delta + \frac{2}{\xi}\varepsilon \equiv 0\ [2\pi]$ (only sign of δ changes)

$\begin{cases} 2\arccos\varepsilon = u \to \varepsilon \\ 2\arccos\varepsilon = u + 2\pi \to -\varepsilon \end{cases}$

- **Short junction limit : $L \ll \xi$**

$2 \arccos \varepsilon = \delta\,[2\pi] \rightarrow \varepsilon = \pm \cos\frac{\delta}{2} = \frac{\pm E_A}{\Delta}$ Andreev energy

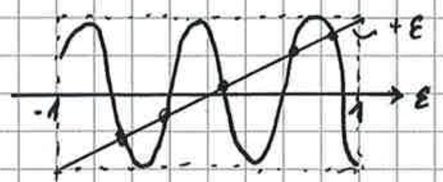


- States at $|E| < \Delta \rightarrow$ localized on scale defined by evanescent tails in S : $\frac{\xi}{\sqrt{1 - E_A^2/\Delta^2}}$
- Superpositions of e & h : like Cooper pairs in BCS $\rightsquigarrow$ localized Cooper pair
- At $T = 0$, only state at $E < 0$ occupied

- **Long junction : $L > \xi$**

$\arccos \varepsilon = \left(\frac{\delta}{2} + \frac{L}{\xi}\varepsilon\right)[2\pi]$

$\pm\varepsilon = \cos\left(\frac{\delta}{2} + \frac{L}{\xi}\varepsilon\right)$



solutions $\simeq$ nb of zeros of $\cos\left(\frac{L}{\xi}\varepsilon\right)$ for $-1 \leq \varepsilon \leq 1$

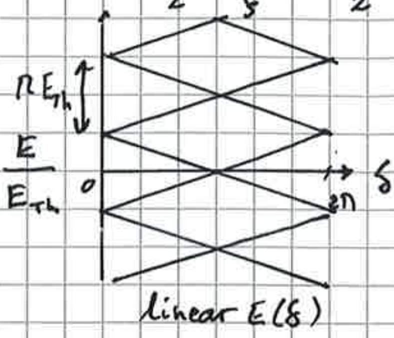
$\times 2\ (\pm\varepsilon) \rightarrow \frac{2L/\xi}{2\pi} \times 2 = \frac{2}{\pi}\frac{L}{\xi}$

If $\frac{L}{\xi} \gg 1$, # of ABS $\gg 1$. Around $\varepsilon \simeq 0$,

since $\varepsilon = \cos(\ldots)$, $\cos(\ldots) \simeq 0 \rightarrow \frac{\pm\delta}{2} + \frac{L}{\xi}\varepsilon = \frac{\pi}{2} + n\pi$

$\Rightarrow \frac{L}{\xi}\varepsilon = \frac{\pi}{2} \pm \frac{\delta}{2} + n\pi$

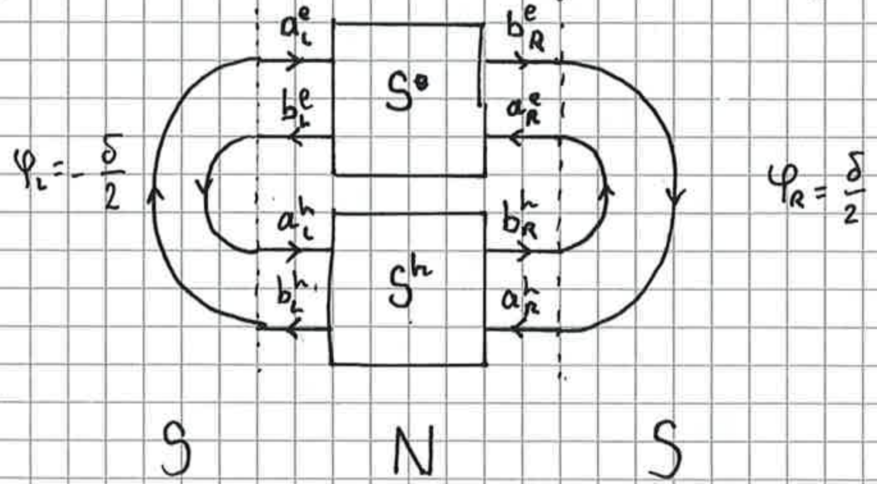
$\frac{L}{\xi}\varepsilon = \frac{L}{\hbar v_F/\Delta}\frac{E}{\Delta} = \frac{E}{E_{Th}}$



at $\delta = 0$, gap $\frac{\pi}{2}E_{Th}$
at $\delta = \pi$, gap $= 0$
} general features

Solutions for various L/ξ.

▶ Now assume $\tau \neq 1$ & short junction $L \ll \xi$



$$\begin{pmatrix} b_L^e \\ b_R^e \end{pmatrix} = S^e \begin{pmatrix} a_L^e \\ a_R^e \end{pmatrix}$$

$$\begin{pmatrix} a_L^e \\ a_R^e \end{pmatrix} = \begin{pmatrix} e^{i\varphi_L} & 0 \\ 0 & e^{i\varphi_R} \end{pmatrix} |r_A(\varepsilon)| \begin{pmatrix} b_L^h \\ b_R^h \end{pmatrix} = R_A \begin{pmatrix} b_L^h \\ b_R^h \end{pmatrix}$$

$$\begin{pmatrix} b_L^h \\ b_R^h \end{pmatrix} = S^h \begin{pmatrix} a_L^h \\ a_R^h \end{pmatrix}$$

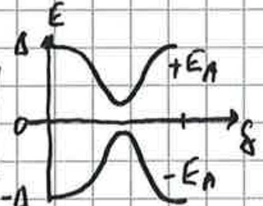
$$\begin{pmatrix} a_L^h \\ a_R^h \end{pmatrix} = R_A^* \begin{pmatrix} b_L^e \\ b_R^e \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} b_L^e \\ b_R^e \end{pmatrix} = S^e R_A S^h R_A^* \begin{pmatrix} b_L^e \\ b_R^e \end{pmatrix}$$

$\rightarrow$ dét $(1 - S^e R_A S^h R_A^*) = 0$

$\Rightarrow -2 + 2\varepsilon^2 + \tau - \tau \cos\delta = 0$

$$\Rightarrow \boxed{E = \pm \Delta \sqrt{1 - \tau \sin^2\left(\frac{\delta}{2}\right)}}$$



At $\delta = \pi$, gap $= 2\Delta\sqrt{1-\tau}$

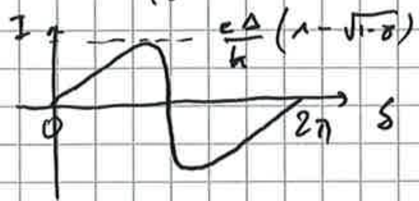
Link to Josephson effect:

$$I = \frac{1}{\varphi_0} \frac{\partial E}{\partial \delta}$$

For $\frac{L}{\xi} = 0$, only ABS carry supercurrent (otherwise also states in continuum have a δ-dependent energy)

At $T=0$, only state at $-E_A$ is occupied

$$\rightarrow I = -\frac{1}{\varphi_0}\frac{\partial E_A}{\partial \delta} = \frac{e\Delta}{2\hbar} \tau \frac{\sin\delta}{\sqrt{1 - \tau \sin^2\frac{\delta}{2}}}$$



MLDR

$\frac{e\Delta}{\hbar} \sim 50$ nA for Al

In tunnel junctions: $\tau \ll 1 \rightarrow I = \frac{e\Delta}{2\hbar} \tau \sin\delta$ for each channel

In normal state, $G = \frac{2e^2}{h}\tau = R_t^{-1}$ tunnel resistance

$$\Rightarrow I_0 = \frac{\pi}{4} \frac{2\Delta}{eR_t} \; ; \quad I = I_0 \sin\delta$$

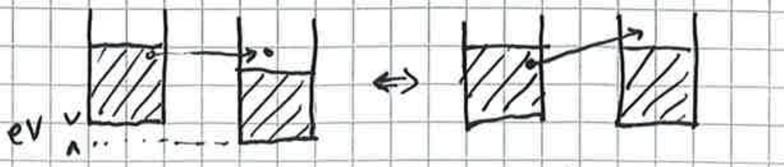
↑ Ambegaokar-Baratoff relation

↑ Josephson relation

Ballistic junctions $\tau = 1$

Exp'ts with Bi nanowires

Multiple Andreev Reflections (MAR)

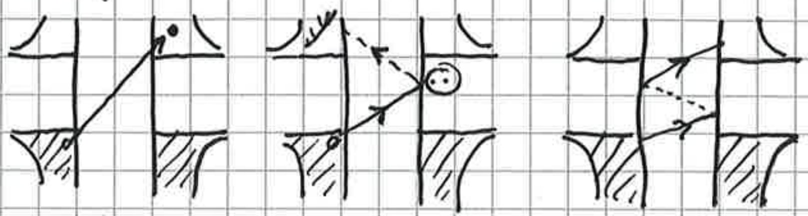


(1) shift μ

(2) slope in arrow → energy gain for e^-

(2) more adapted when AR occurs, because need to describe e^- & h, which gain same energy in opposite directions.

SNS junctions:



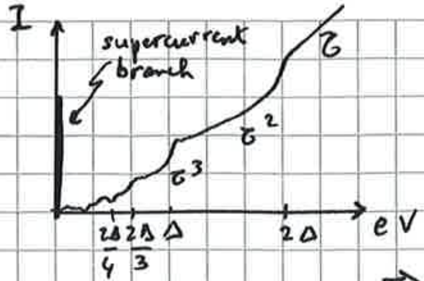
$eV \gg 2\Delta$ — Proba for AR at $E > \Delta$ small: absorbed as e^-

$\Delta < eV < 2\Delta$ — $e^- \to h \to$ absorbed as quasi-h

$\frac{2\Delta}{3} < eV < \frac{2\Delta}{2}$ — two AR

... for $\frac{2\Delta}{n+1} < eV < \frac{2\Delta}{n}$, need at least n AR to reach available states → proba $\propto \tau^{n+1}$

⇒ cusps in I-V at $eV = \frac{2\Delta}{n}$



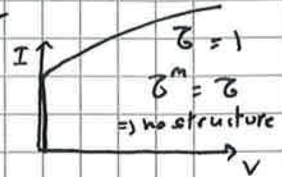
contains all moments of $\{\tau_i\}$

→ very non-linear I-V

$I(\tau, V)$ known, tabulated

⇒ decompose $I(V) = \sum_{\tau_i} I(\tau_i, V)$

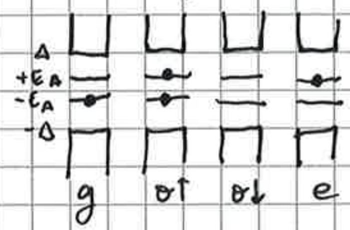
& find $\{\tau_i\}$ & # of channels N



Semi-c structure: $N \sim \frac{\text{area}}{(\lambda_F/2)^2}$

atomic contacts, $N \sim$ chemical valency of atom

- I-V fits in atomic contacts & InAs nanowires
- Spectroscopy of ABS



μw spectro

tunneling spectro for extended systems (CNT)

Shiba states: Christophe

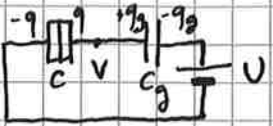
Parity effects in Superconducting islands

⚠ Charge on capacitor = continuous variable

charge on isolated electrode = quantized (Millikan) in units of e

S electrode: quantitation in units of 2e ?

1) N state : single electron box



Total energy (including work performed by source):

$$\mathcal{E} = \frac{q^2}{2C} + \frac{q_g^2}{2C_g} + q_g U \qquad \left(\frac{d\mathcal{E}}{dq} = 0 \to \text{sign OK}\right)$$

$$= \frac{q^2}{2C} + \frac{(q_g + C_g U)^2 - (C_g U)^2}{2C_g}$$

Use $V = \frac{q}{C} = \frac{q_g}{C_g} + U = \frac{(q+q_g) + C_g U}{(C + C_g)}$, with $q+q_g = Q$ and $C + C_g = C_\Sigma$

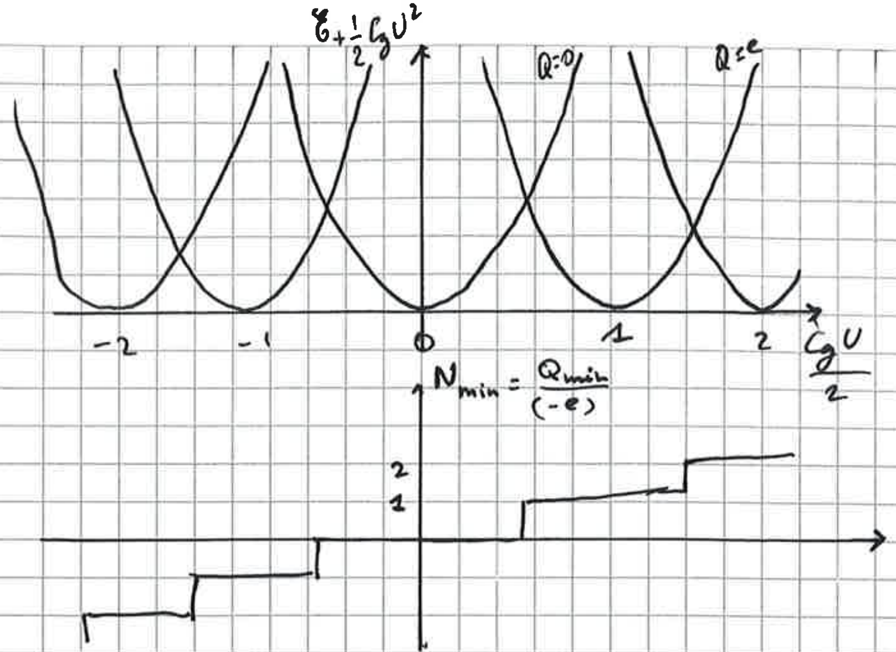
if $\frac{a}{b} = \frac{c}{d}$, then $= \frac{a+c}{b+d}$

$$\Rightarrow \begin{cases} q = \dfrac{C}{C_\Sigma}\left(Q + C_g U\right) \\ q_g + C_g U = \dfrac{C_g}{C_\Sigma}\left(Q + C_g U\right) \end{cases}$$

$$\Rightarrow \mathcal{E} = \frac{C}{2C_\Sigma^2}\left(Q + C_g U\right)^2 + \frac{C_g}{2C_\Sigma^2}\left(Q + C_g U\right)^2 - \frac{1}{2} C_g U^2$$

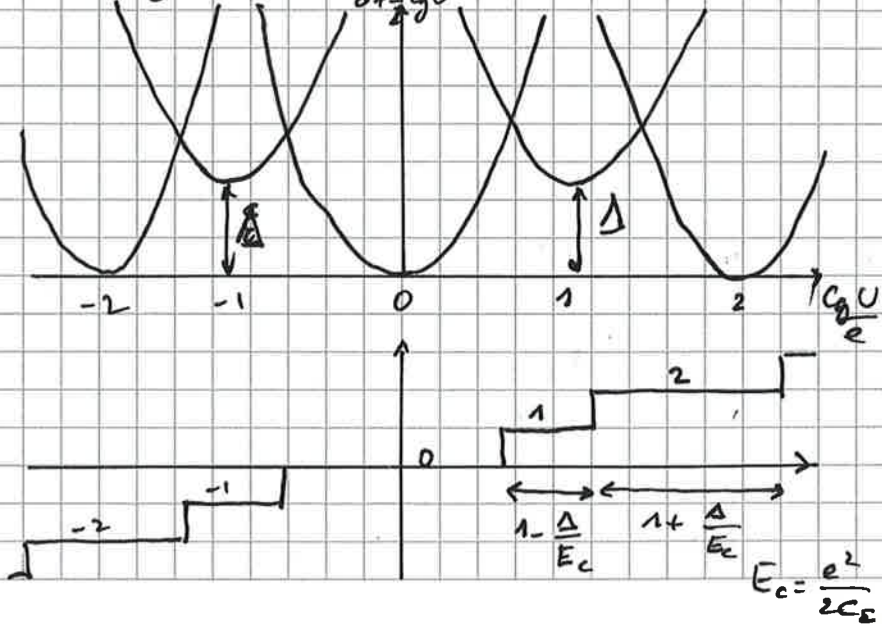
$$= \frac{1}{2C_\Sigma}\left(Q + C_g U\right)^2 - \frac{1}{2} C_g U^2$$

← indpdt of Q



2) S state in island only

When $\frac{Q}{e}$ is odd, additional qp at energy $\geq \Delta$



Crossing of parabola at $Q = 0$ and $Q = 2e$:

$$(C_g U)^2 = (2e + C_g U)^2$$

$\rightarrow$ $C_g U = e$ of course

then $\mathcal{E} + \frac{1}{2} C_g U^2 = \frac{e^2}{2 C_\Sigma} = E_c$

$\Rightarrow$ - at $T = 0$, Q always even if $\Delta > E_c$

- at $T \neq 0$, requires $\Delta - k_B T \ln \mathcal{N} > E_c$

with $\mathcal{N} = \underbrace{\nu \mathcal{V} \Delta}_{\text{nb of states at energy} < \Delta \text{ in island}} \sqrt{\frac{2\pi k_B T}{\Delta}}$ Exp

(because many states are available if Q is odd $\rightarrow$ need to compare F_{odd} & F_{even})

NB: usually $C \gg C_g$; $C_\Sigma \sim C$

If junction $(100\ \text{nm})^2$, $C \sim 1\ \text{fF}$, $E_c \sim 1\text{K} \sim \Delta$

$\rightarrow$ not easy to get even parity always $\forall N_g$!

3) Cooper pair box : all S

At $T \ll \Delta$, no qp (in principle...) $\Rightarrow$ odd states not accessible

$\Rightarrow$ like single electron box, with $e \rightarrow 2e$

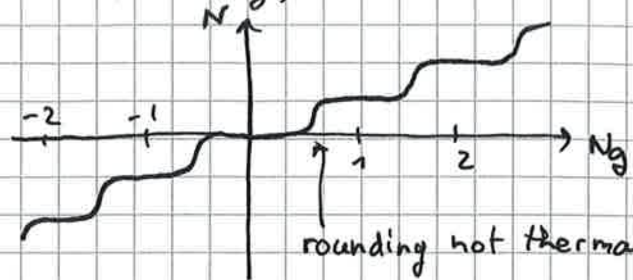
$$\Rightarrow E = \mathcal{E} + \frac{1}{2} C_g U^2$$

$$= \frac{1}{2C_\Sigma} (Q + C_g U)^2 \qquad Q = N(-2e)$$

$$N_g = \frac{C_g U}{2e}$$

$$E_c = \frac{(2e)^2}{2C_\Sigma}$$

$$= E_c (N - N_g)^2$$



rounding not thermal !

because states N & N+1 are coupled by J. effect:

$$H = E_c (\hat{N} - N_g)^2 - E_J \cos\hat{\theta}$$

↑ phase of island

$[\hat{\theta}, \hat{N}] = i$ $\quad \hat{\theta}$ & $\hat{N}$ are conjugate variables

In a JJ, in general: $\hat{\theta}$ phase across junction

$\hat{N}$ nb of pairs that passed through the junction

Current: $I = 2e\dot{N} = \frac{2e}{\hbar}\frac{\partial H}{\partial \theta} = \frac{2e}{\hbar} E_J \sin\theta$

$= I_0 \sin\theta \quad \left(E_J = I_0 \frac{\hbar}{2e}\right)$

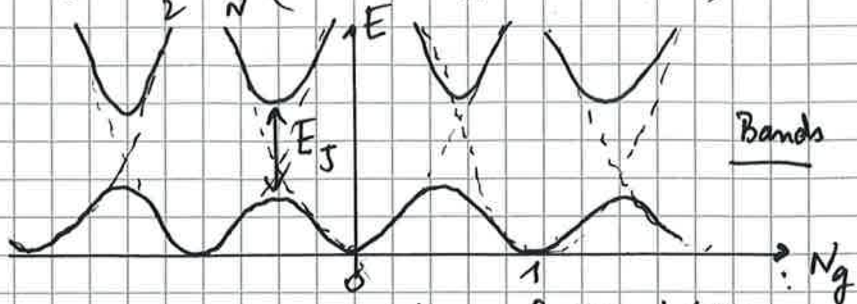
Voltage: $V = \frac{\hbar}{2e}\dot{\theta} = \varphi_0 \dot{\theta}$

$$\cos\hat{\theta} = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$$

↳ translation operator of N by 1

$$\left(e^{\frac{i}{\hbar} a \hat{p}} |x\rangle = |x+a\rangle\right)$$

$$\cos\hat{\theta} = \frac{1}{2}\sum_N \left(|N-1\rangle\langle N| + |N+1\rangle\langle N|\right)$$



→ rounding of steps between N & N+1.

At $N_g = \frac{1}{2}$, $\frac{1}{\sqrt{2}}(|N=0\rangle \pm |N=1\rangle)$

(if $\frac{E_J}{E_c} \ll 1$, otherwise higher states matter !)

→ first superconducting Qubits: manipulate with N_g superposition of $|N=0\rangle$ and $|N=1\rangle$

⇝ Nakamura, Quantronium

In the limit $E_J \gg E_c$, bands become flat

→ insensitive to N_g noise → better qubits :

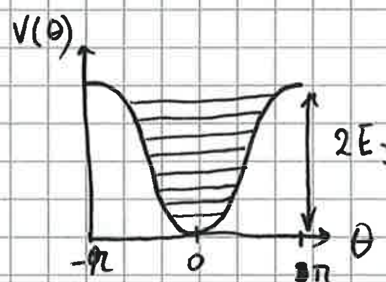
" transmon "

Then instead of considering coupling of parabola by E_J, better to consider potential $-E_J \cos\delta$ and "kinetic" term $E_c \hat{N}^2$ (in a first step, neglect N_g since expect little N_g-dependence of energy levels) :

$$H = E_c \hat{N}^2 - E_J \cos\hat{\theta}$$

$$H = \frac{\hat{p}^2}{2m} - V \cos\hat{x}$$

$E_J \gg E_c$ → large mass → state localized in potential well



Many state in well

Low energy states : $\theta \approx 0$ $\quad \cos\theta \approx 1 - \frac{\theta^2}{2} + \frac{\theta^4}{24}$

$$H \approx \underbrace{E_c \hat{N}^2 + \frac{E_J}{2}\hat{\theta}^2}_{\text{Harmonic oscillator}} - \frac{E_J}{24}\hat{\theta}^4 + cst$$

Harmonic oscillator ⇒ $\hbar\omega_0 \left(\hat{n} + \frac{1}{2}\right)$

$$E_c \hat{N}^2 + \frac{E_J}{2}\hat{\theta}^2 = \sqrt{\frac{E_J E_c}{2}}\left(\hat{P}^2 + \hat{X}^2\right) \text{ with } \begin{cases} \hat{X}^2 = \sqrt{\frac{2E_c}{E_J}}\,\hat{N}^2 \\ \hat{P}^2 = \sqrt{\frac{E_J}{2E_c}}\,\hat{\theta}^2 \end{cases}$$

$$\hat{a} = \frac{1}{\sqrt{2}}(\hat{X} + i\hat{P}) \Rightarrow \hat{P}^2 + \hat{X}^2 = 2a^\dagger a + 1$$

$$\Rightarrow H = \hbar\omega_0 (a^\dagger a + 1/2)$$

with $\hbar\omega_0 = \sqrt{2 E_J E_c}$

$$\omega_0 = \frac{1}{\sqrt{L_J C}} \qquad L_J = \frac{\varphi_0}{I_0} \text{ Josephson inductance}$$

$$-E_J \cos\theta \approx cte + \frac{\theta^2}{2L_J}$$

Effect of $\frac{\theta^4}{24}$: anharmonicity

$$\Rightarrow E_m - E_{m-1} = \hbar\omega_0 - m\frac{E_c}{4}$$ (reduces because of $V(\theta) < \frac{\theta^2}{2L_J}$)

Compare anharmonicity with level spacing :

$$\frac{E_c/4}{\hbar\omega_0} = \frac{E_c}{4}\frac{1}{\sqrt{2E_J E_c}} = \frac{1}{4}\sqrt{\frac{E_c}{2E_J}} \ll 1$$

Anharmonicity ↗ with E_c because $\frac{\hbar\omega_0}{E_J} \propto \sqrt{\frac{E_c}{E_J}}$: nb of states in well ↓ , levels "feel" more rapidely top of potential & feel anharmonicity

N_g-dependance (Koch & al, PRA 76, 042319 (2007))

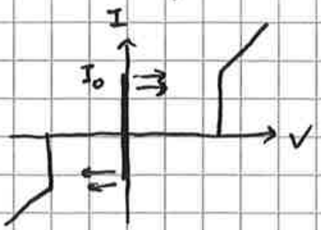
$$E_m(N_g) \approx E_m\left(N_g = \frac{1}{4}\right) - \frac{\varepsilon_m}{2}\cos(2\pi N_g)$$

$$\varepsilon_m \propto E_c \exp\left[-\frac{8E_J}{E_c}\right] \lll E_c$$

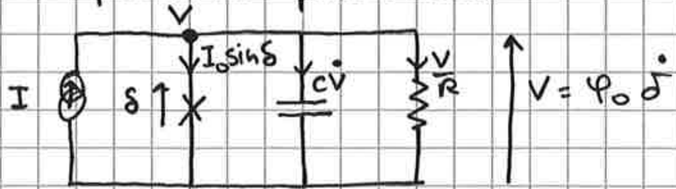
⚠ Anharmonicity is essential if one wants to be able to use only 2 levels !

RCSJ model

Measure I_{sw} of JJ → observe dispersion : why ?



Describe biasing circuit : junction capacitance (+parasitic), parallel impedance : simplified into



$$I = C\dot{V} + \frac{V}{R} + I_0 \sin\delta$$

$$= C\varphi_0 \ddot{\delta} + \frac{\varphi_0}{R}\dot{\delta} + I_0 \sin\delta$$

$$s = \frac{I}{I_0} = \frac{C\varphi_0}{I_0}\ddot{\delta} + \frac{\varphi_0}{RI_0}\dot{\delta} + \sin\delta$$

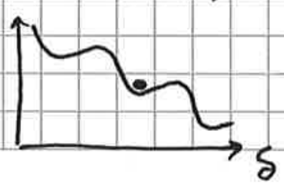
of the form $M\ddot{\delta} = -\lambda\dot{\delta} - \frac{dU(\delta)}{d\delta}$

mass $\frac{C\varphi_0}{I_0}$; friction $\frac{\varphi_0}{RI_0}$; potential

$$U(\delta) = \int (\sin\delta - s)\, d\delta$$

$$= -\cos\delta - s\delta$$

tilted washboard potential



δ : position of fictitious particle

$\dot\delta$: velocity $\propto$ Voltage V

$\sin\delta \propto$ current through JJ

s : tilt of potential : at $s=1$, no local minimum

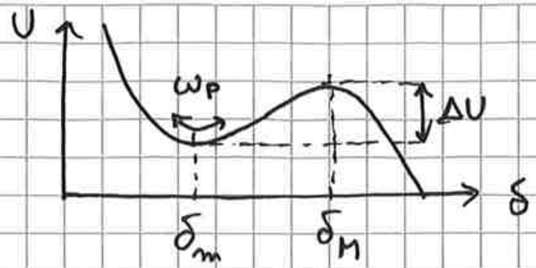
• $s<1$, $T=0$: particle trapped in local minimum : $\begin{cases} \dot\delta = 0 \rightarrow V=0 \\ \delta = \arcsin s \ [2\pi] \rightarrow I \neq 0 \end{cases}$ → Josephson branch

At $T=0$, leave minimum only at $s=1$: escape at $I = I_0$, neglecting quantum fluctuations.

• $T\neq 0$: causes noise (Johnson-Nyquist) across R ⇒ fluctuations of I → of s

⇒ possibility to escape at $\bar{s}<1$

Occurs if $kT \approx$ barrier height



$\delta_m = \arcsin s \qquad \delta_M = \pi - \arcsin s$

$\Delta U = -s(\delta_M - \delta_m) - (\cos\delta_M - \cos\delta_m)$

$\cos\delta_M = -\cos\delta_m = -\sqrt{1-s^2}$

$\Rightarrow \Delta U = -s(\pi - 2\arcsin s) + 2\sqrt{1-s^2}$

$\approx \frac{4\sqrt{2}}{3}(1-s)^{3/2}$ (exact at $s=1$, 1% error at $s=0.8$, 6% $s=0$)

Energy units : multiply initial eq. by φ_0 : $E_J = I_0\varphi_0$ is the natural energy unit

$\Rightarrow \Delta U = \frac{4\sqrt{2}}{3} E_J (1-s)^{3/2}$

• Thermal escape : Arrhenius law : $\Gamma_{sw} \propto e^{-\frac{\Delta U}{kT}}$

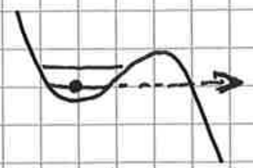
Prefactor = "attempt frequency" = freq of oscillations in potential well minimum $\frac{\omega_p}{2\pi}$ (exact calc : P. Hänggi, Rev. Mod. Phys. 62, 251 (1990))

$\omega_p = \omega_p^\circ (1-s^2)^{1/4}$ $\omega_p^\circ = \sqrt{\frac{I_0}{C\varphi_0}} = \frac{1}{\sqrt{L_J C}}$

$\rightarrow \Gamma_{th}(s) = \frac{\omega_p(s)}{2\pi} \exp\left(-\frac{\Delta U(s)}{kT}\right)$

- Quantum escape:

For $T < T_{co} = \frac{\hbar\omega_p}{2\pi k_B}$, particle in ground state of well oscillations. Then escape dominated by tunneling through barrier:



WKB calc: replace T by $\frac{5}{36}\frac{\hbar\omega_p}{k_B}$

$\Rightarrow \Gamma_{MQT} = A e^{-B}$ with $\begin{cases} A = \frac{\omega_p}{2\pi}\sqrt{120\pi B} \\ B = \frac{36}{5}\frac{\Delta U(s)}{\hbar\omega_p(s)} \end{cases}$

- After escape?

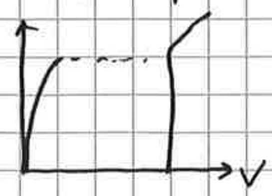
Depends on friction $\lambda = \frac{\varphi_0}{RI_0} = \frac{1}{Q\omega_p^\circ}$

$Q = RC\omega_p^\circ$ quality factor

If $Q \gg 1$, little friction → velocity in next well → accelerates → $V \neq 0$ till produces QPs

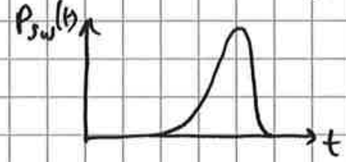
(Escape if $s = \frac{4}{\pi Q}$)

If Q small, retraps in next well: phase diffusion till velocity sufficient



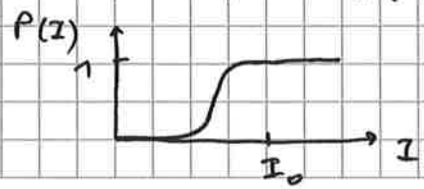
- Measuring switching

a) Ramps



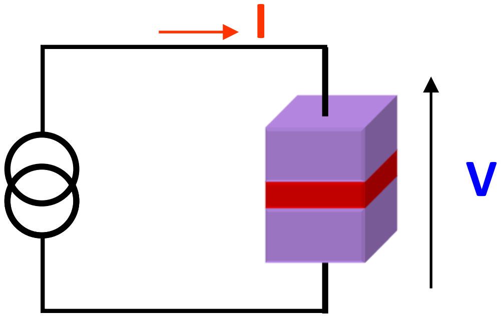
depends on $\frac{dI}{dt}$

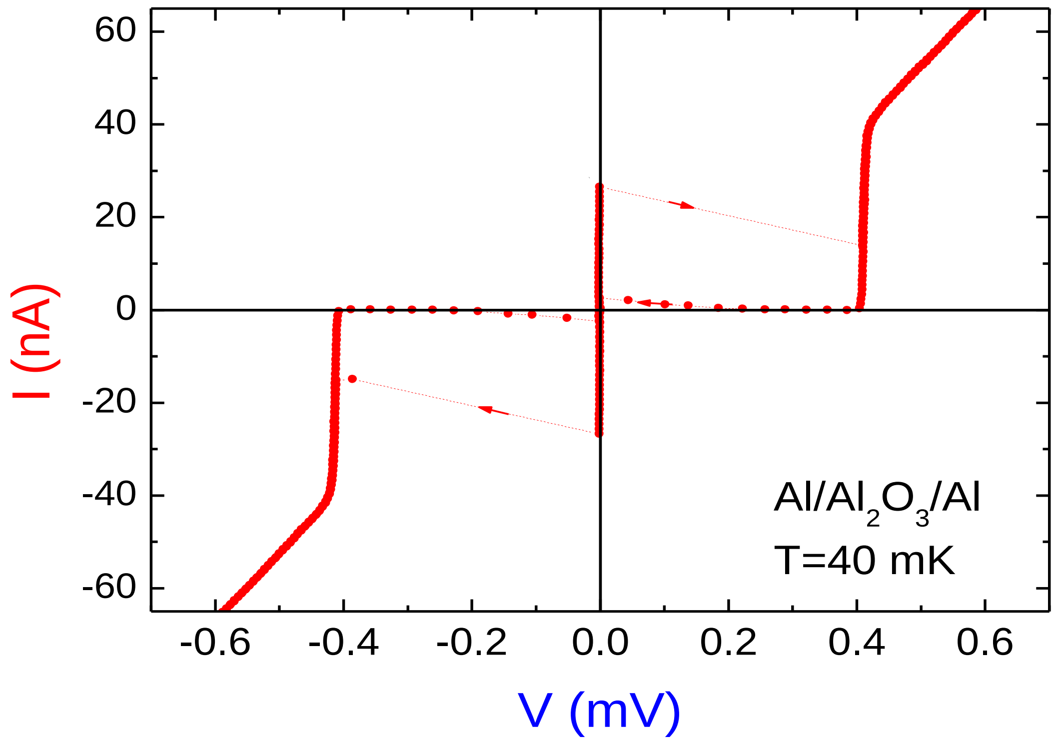
b) Pulses



$P_{sw} = 1 - e^{-\Gamma\tau}$

THE JOSEPHSON JUNCTION





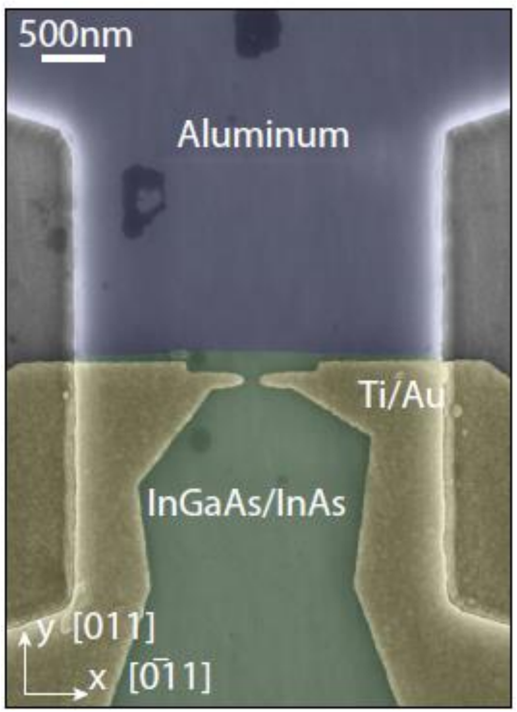
B.T.K.

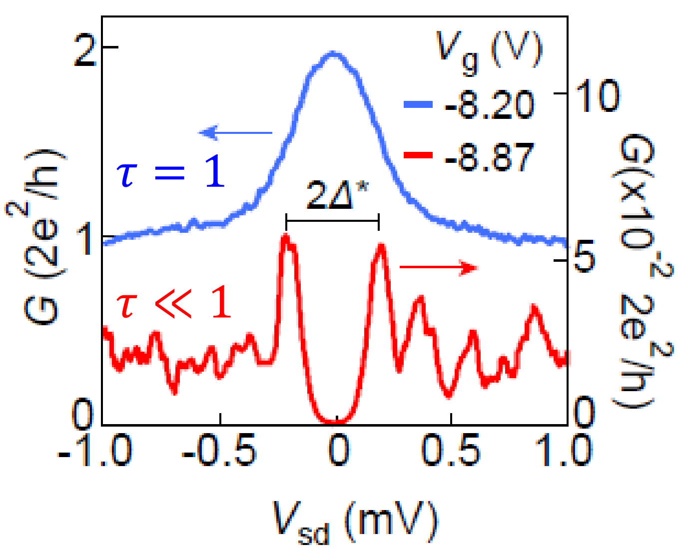
Conductance doubling: experiments

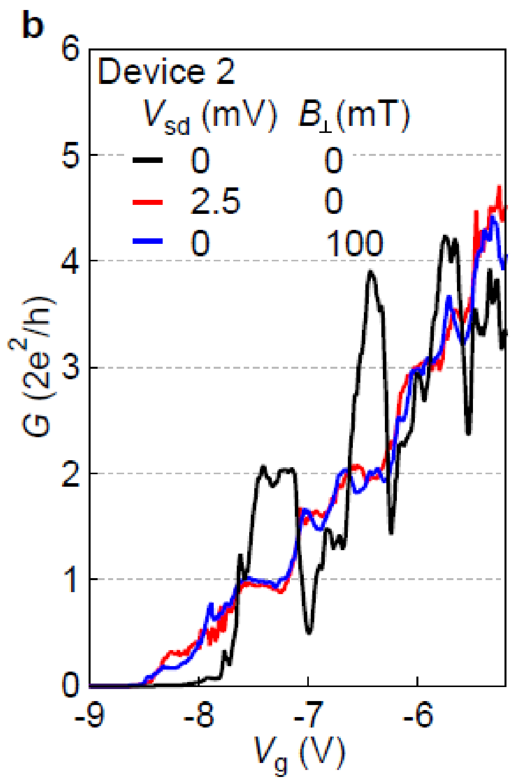
Quantized conductance doubling and hard gap in a two-dimensional semiconductor-superconductor heterostructure

M. Kjaergaard,[1] F. Nichele,[1] H. J. Suominen,[1] M. P. Nowak,[2,3,4] M. Wimmer,[2,3] A. R. Akhmerov,[2] J. A. Folk,[5,6] K. Flensberg,[1] J. Shabani,[7,*] C. J. Palmstrøm,[7] and C. M. Marcus[1]

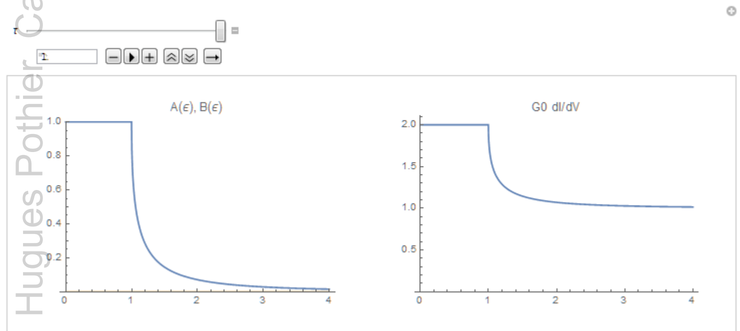
arXiv:1603.01852v1 Nat. Commun. 7, 12841 (2016)

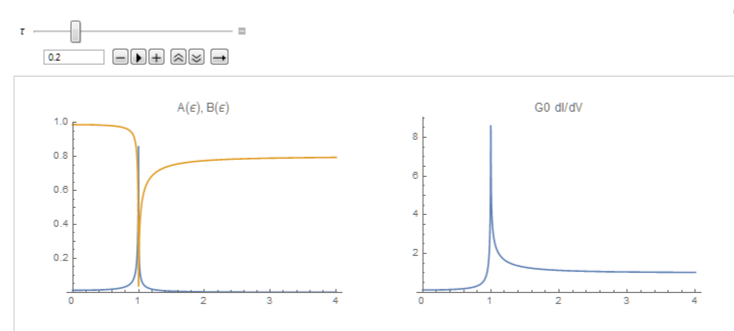


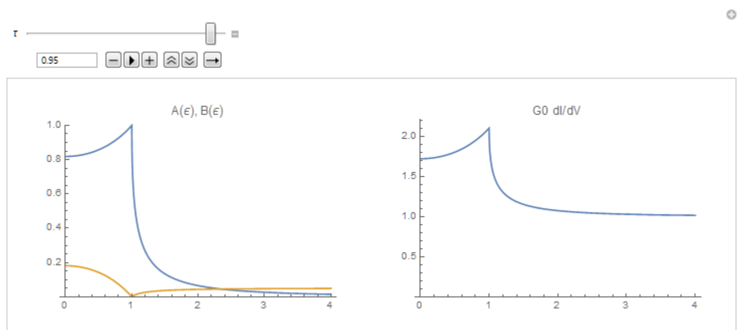


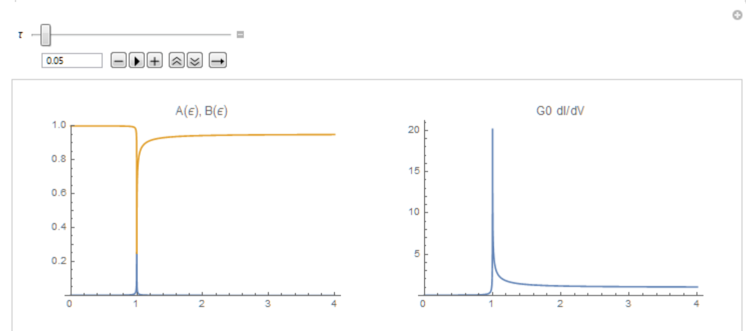


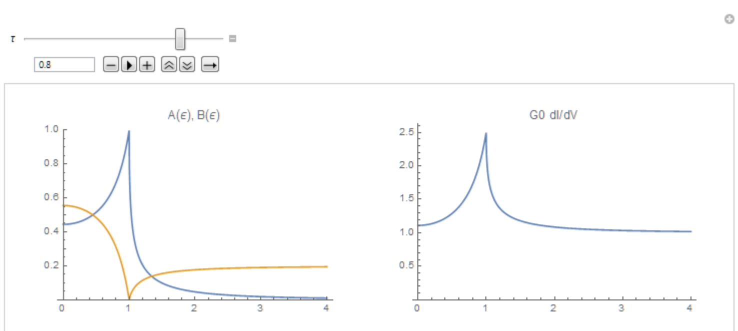
BTK: conductance of ballistic NIS junction







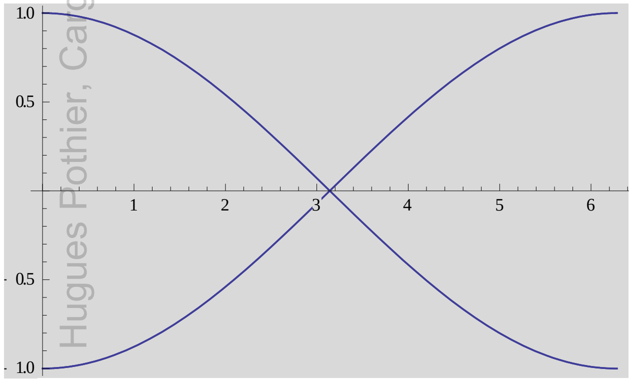




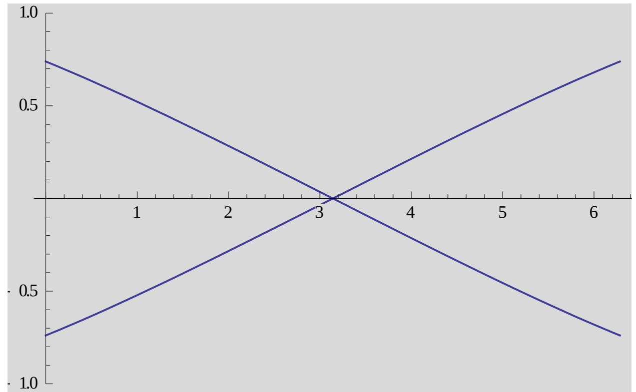
ABS: long junction limit

ABS

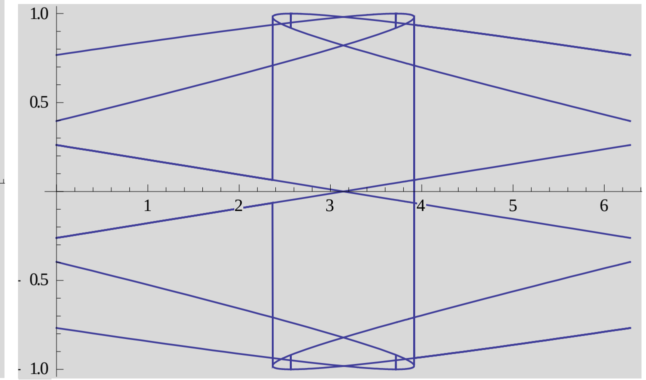
L/ξ=0



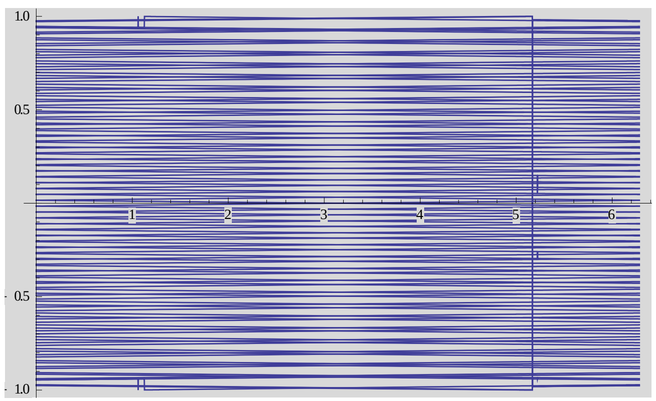
L/ξ=1

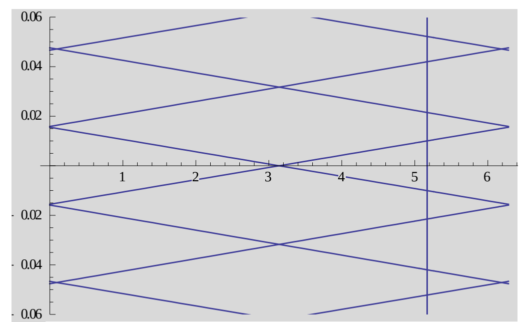


L/ξ=5



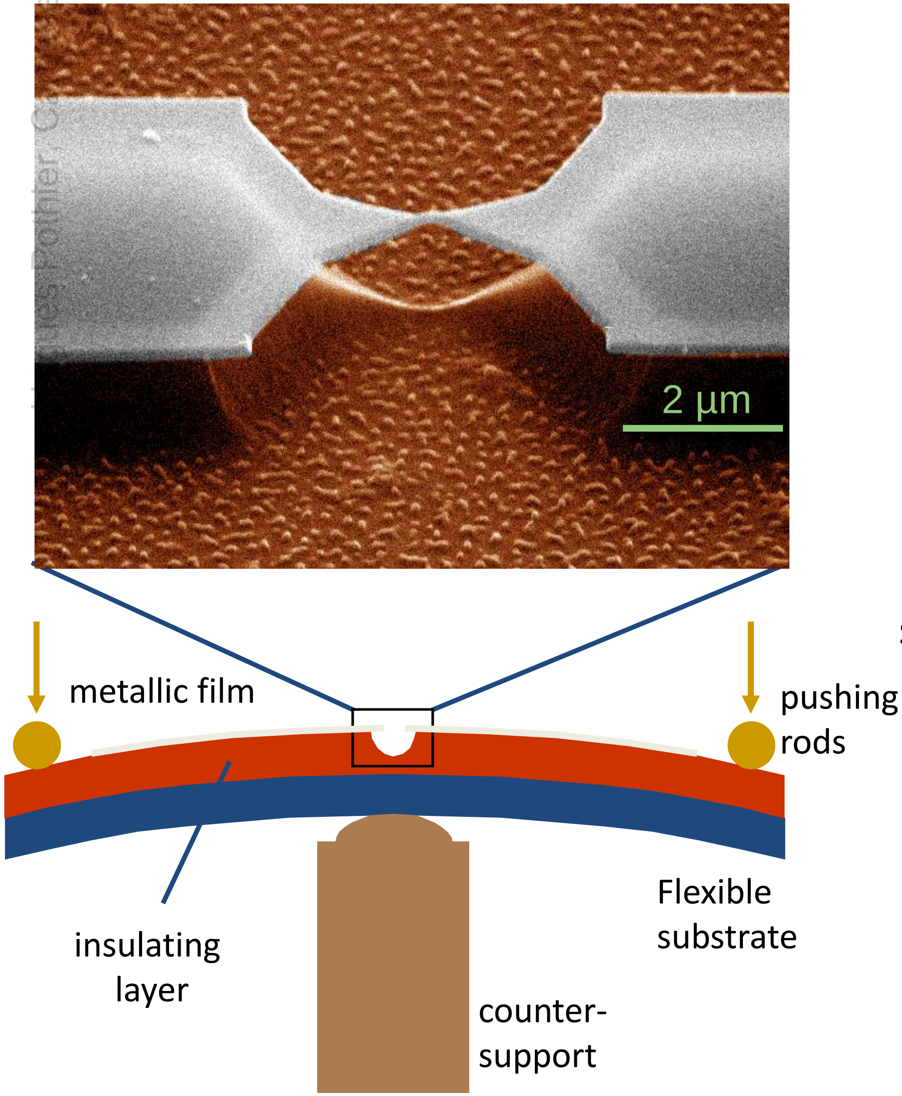
L/ξ=100

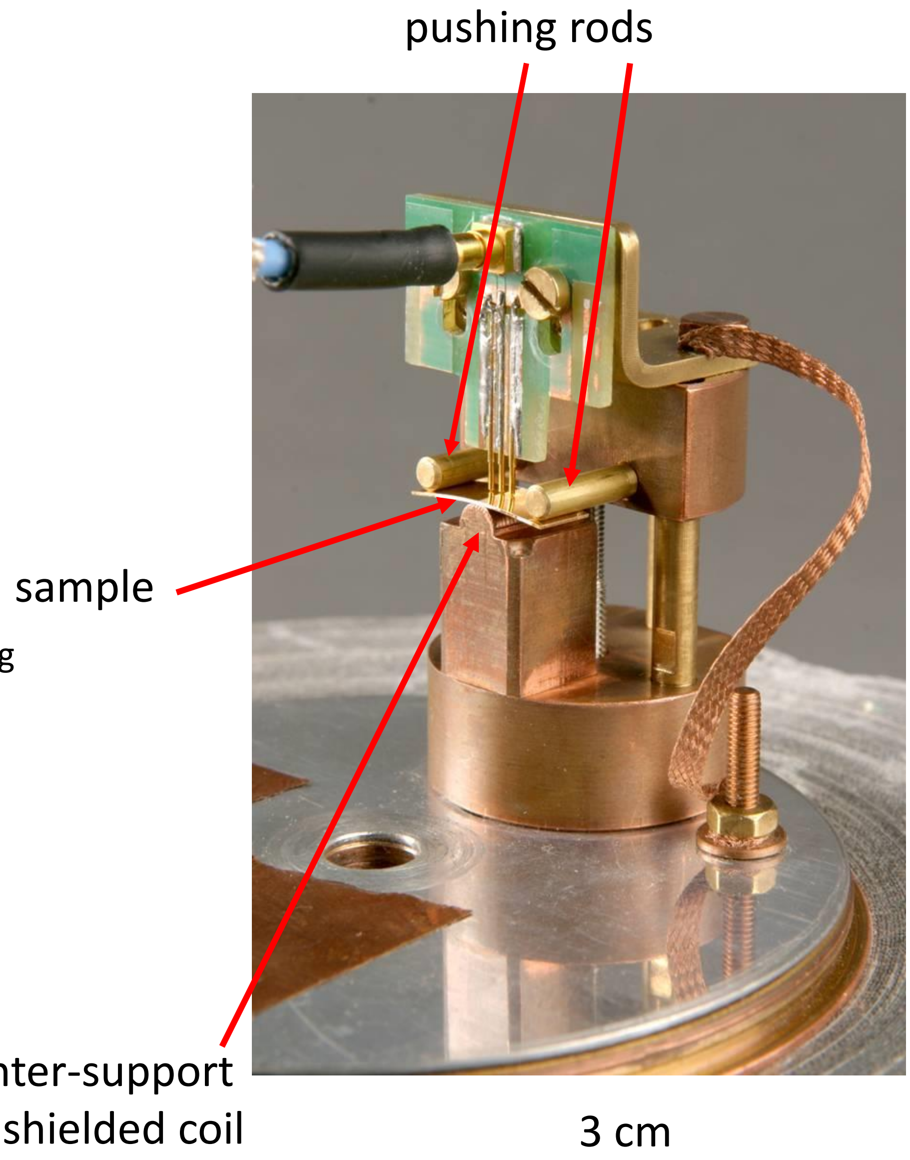




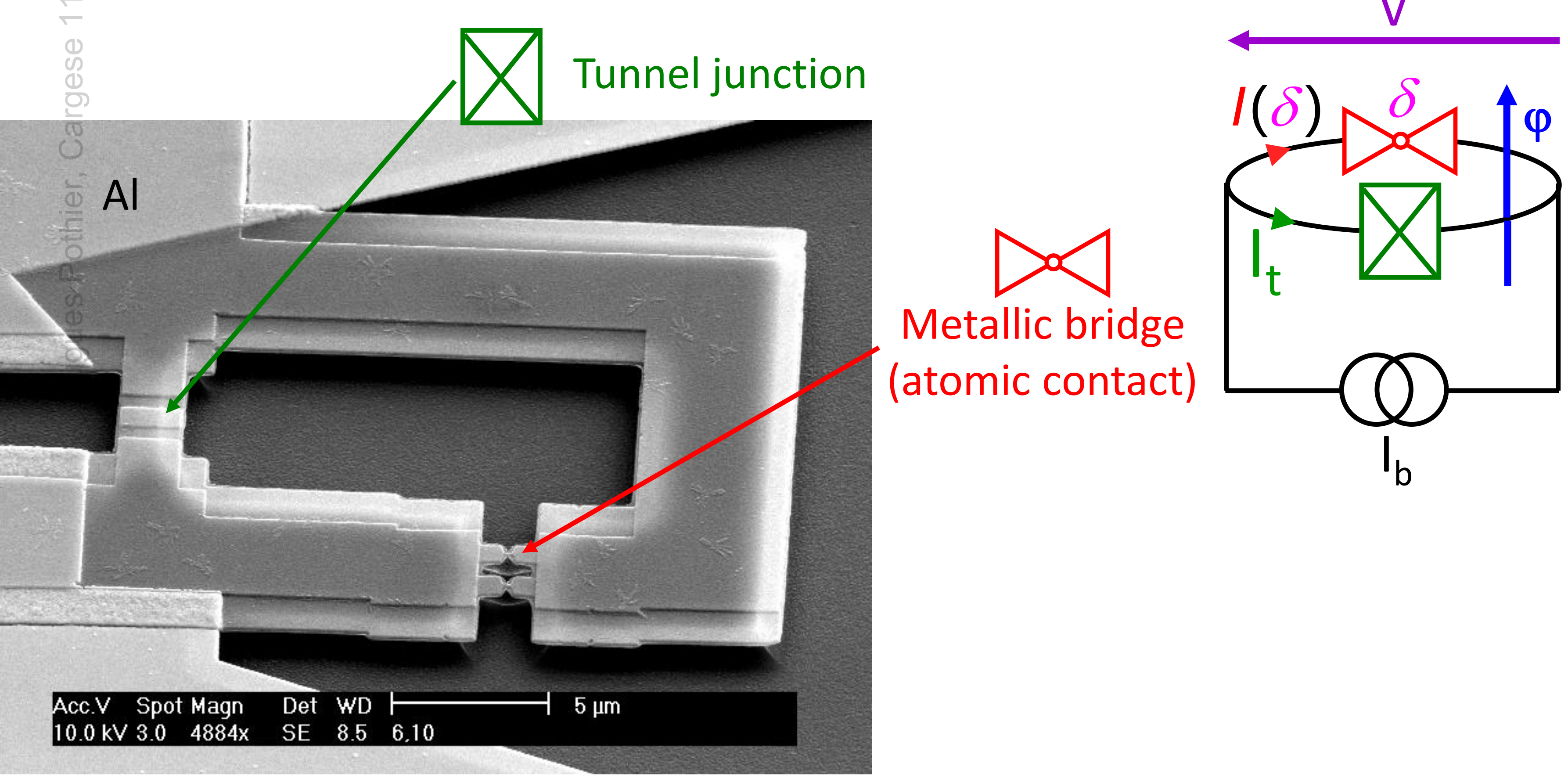
Atomic contacts

Atomic contacts



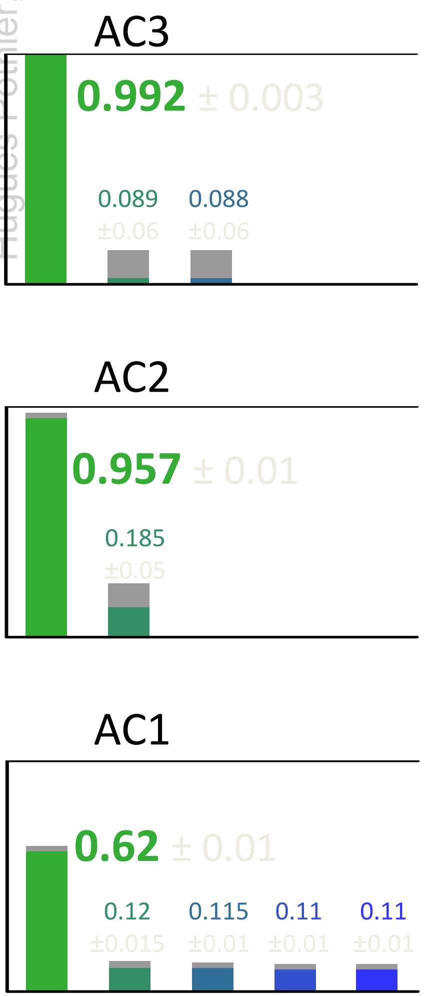


Measurement of the current-phase relation



$30\ \text{nA} \approx I_{max} \ll I_t^0 \approx 0.7\ \mu\text{A}$

Measurement of the current-phase relation



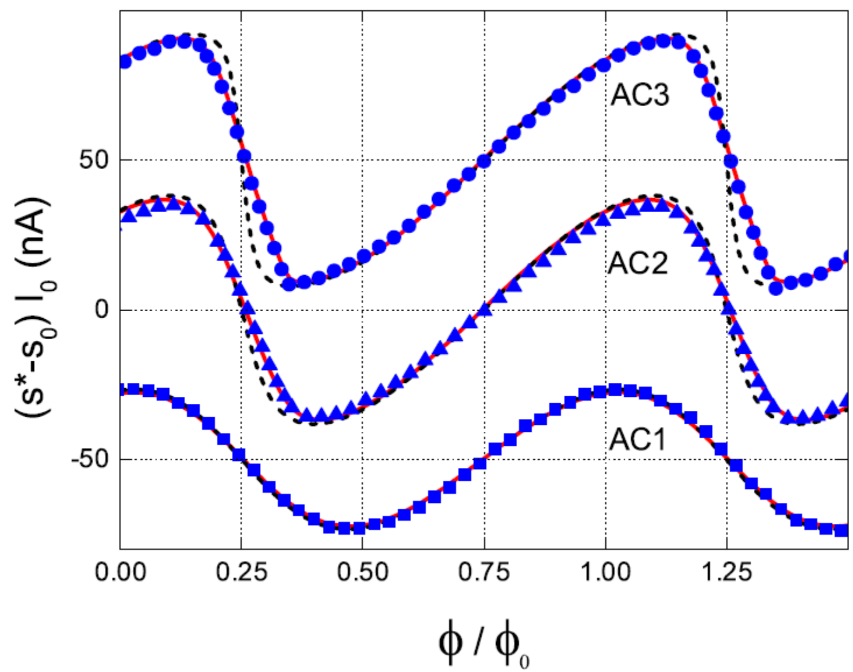
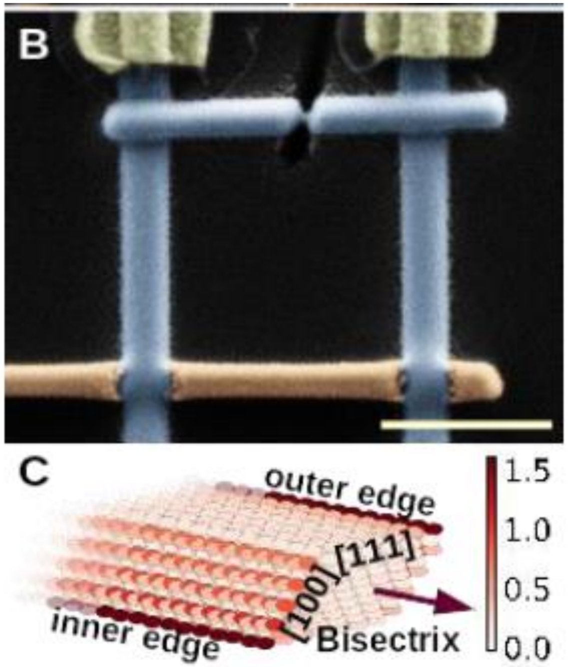
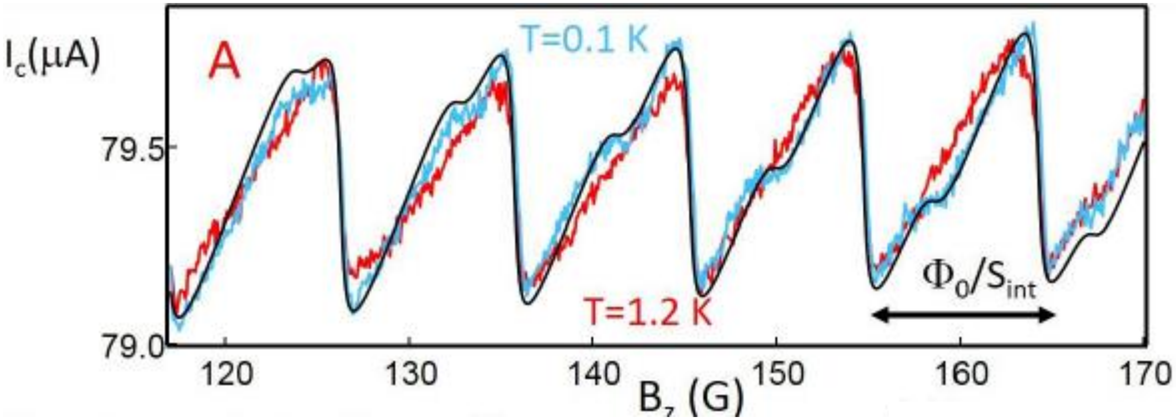


FIG. 3 (color online). Symbols: measured switching current $[s^*(\varphi) - s_0]I_0$ as a function of applied flux ϕ/ϕ_0, for the three SQUIDS corresponding to the contacts of Fig. 2. Curves AC3 and AC1 shifted for clarity. Dashed curves: predicted ground state current-phase relation $I^-_{\{\tau_i\}}(\delta)$. Full lines: predictions of resistively shunted SQUID theory at $T_{\mathrm{esc}} = 130$ mK on the basis of Eq. (1). The transmission sets indicated in Fig. 2 caption have been used for both theories.

Della Rocca *et al.*, PRL 2007

Current-phase relation at τ=1





A. Murani et al., arXiv:1609.04848

MAR

N. Agraït et al. / Physics Reports 377 (2003) 81–279

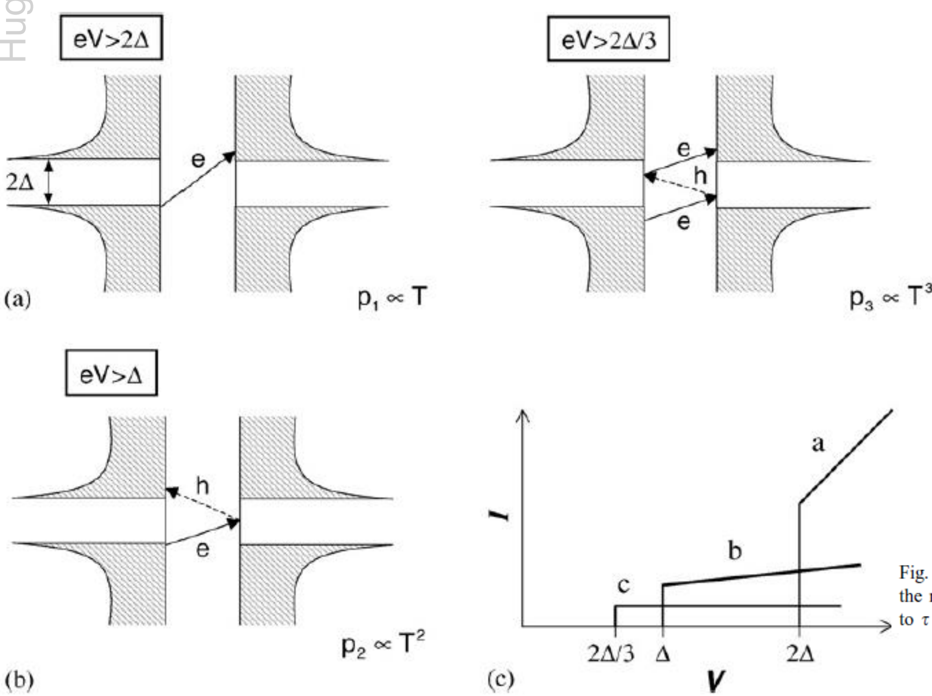


Fig. 23. Schematic explanation of the subgap structure in superconducting contacts.

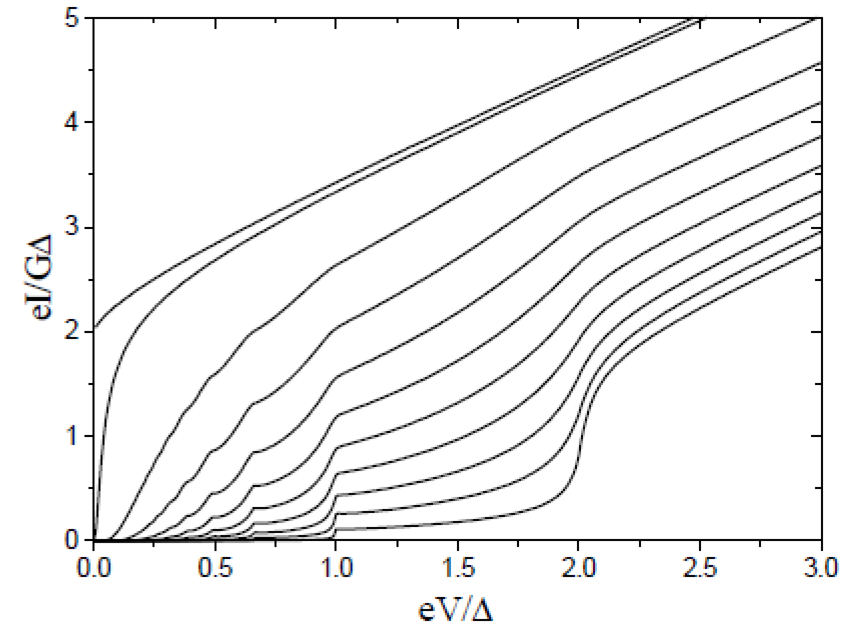
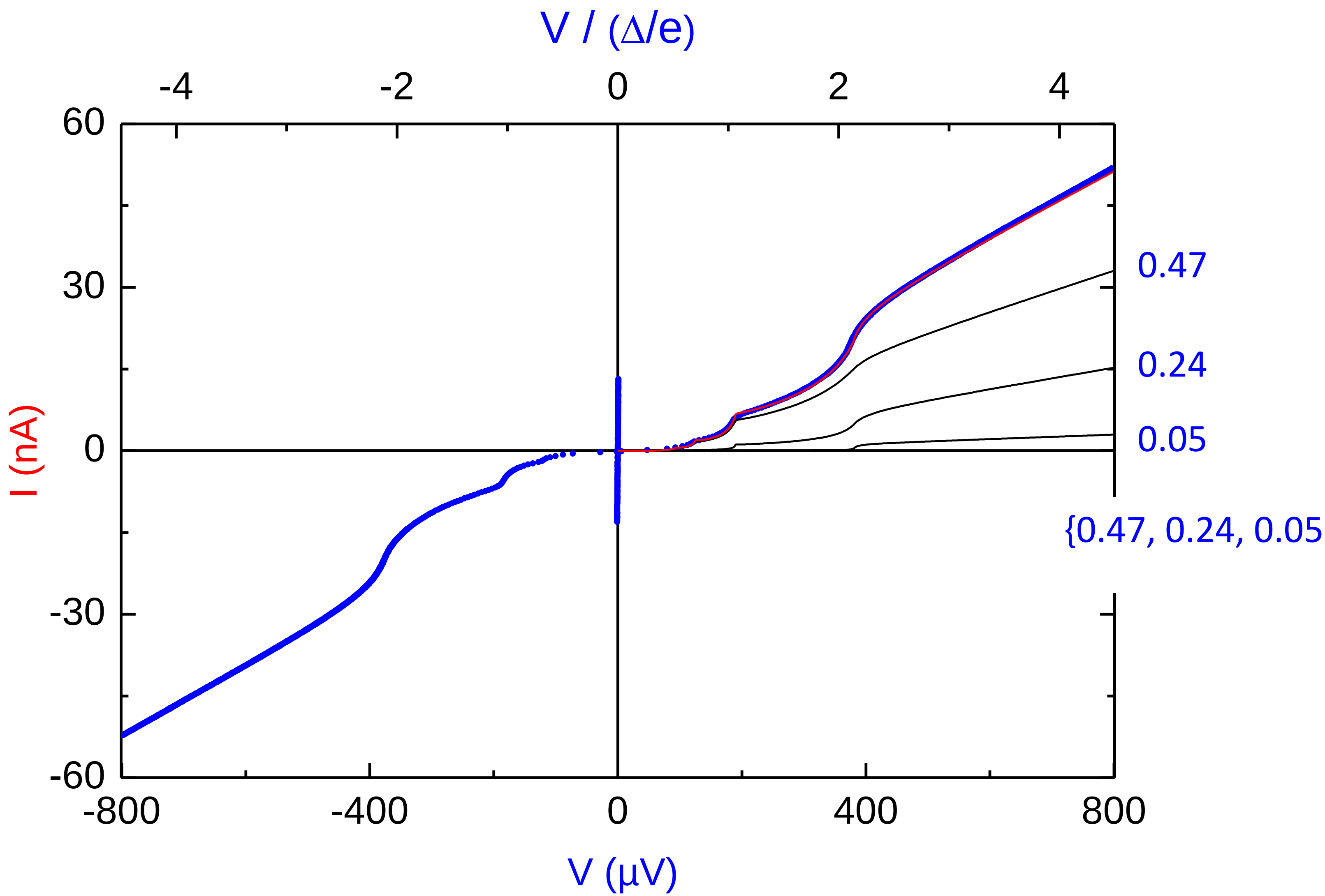


Fig. 24. The zero-temperature dc component of the current in a single mode superconducting contact, where the values of the normal transmission increase from $\tau = 0.1$ in the lower curve by increments of 0.1. The upper two curves correspond to $\tau = 0.99$ and 1. Reprinted with permission from [41]. © 1996 American Physical Society.

DETERMINATION OF TRANSMISSIONS



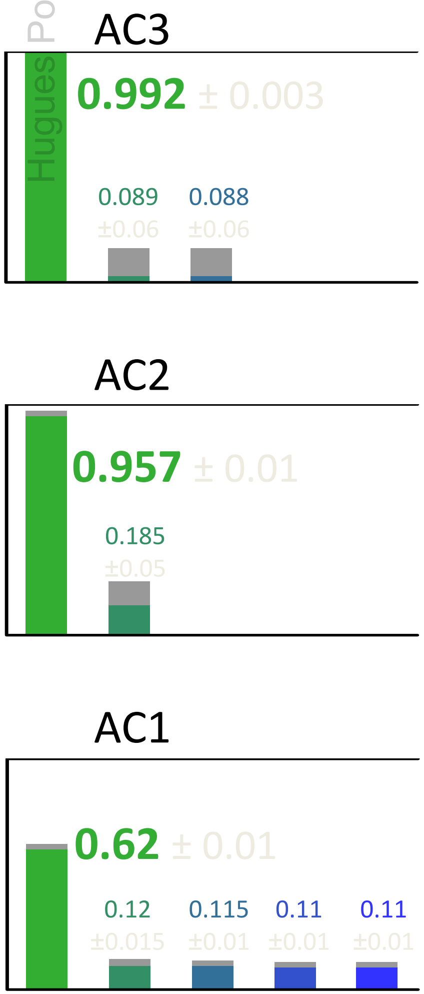
I-V characteristic ⟹ $\{\tau_1, \ldots, \tau_N\}$

Scheer *et al.* PRL 1997

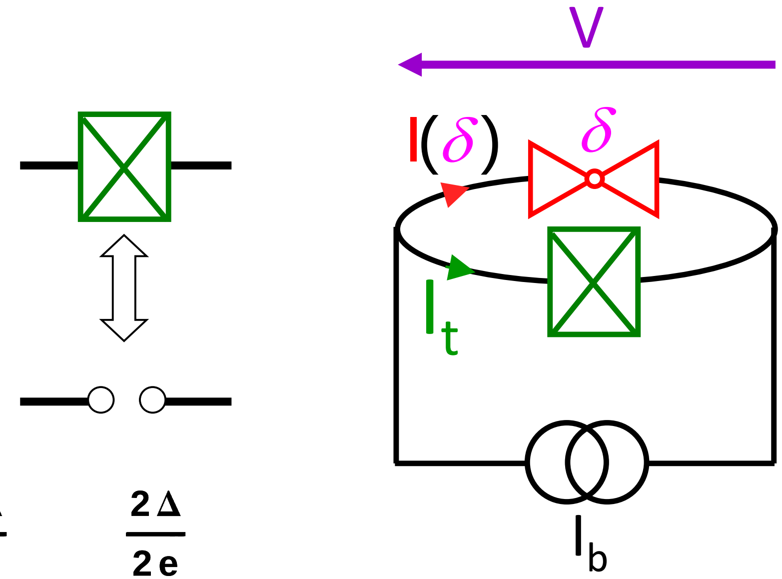
Determination of transmissions $\{\tau_1,\ldots,\tau_M\}$

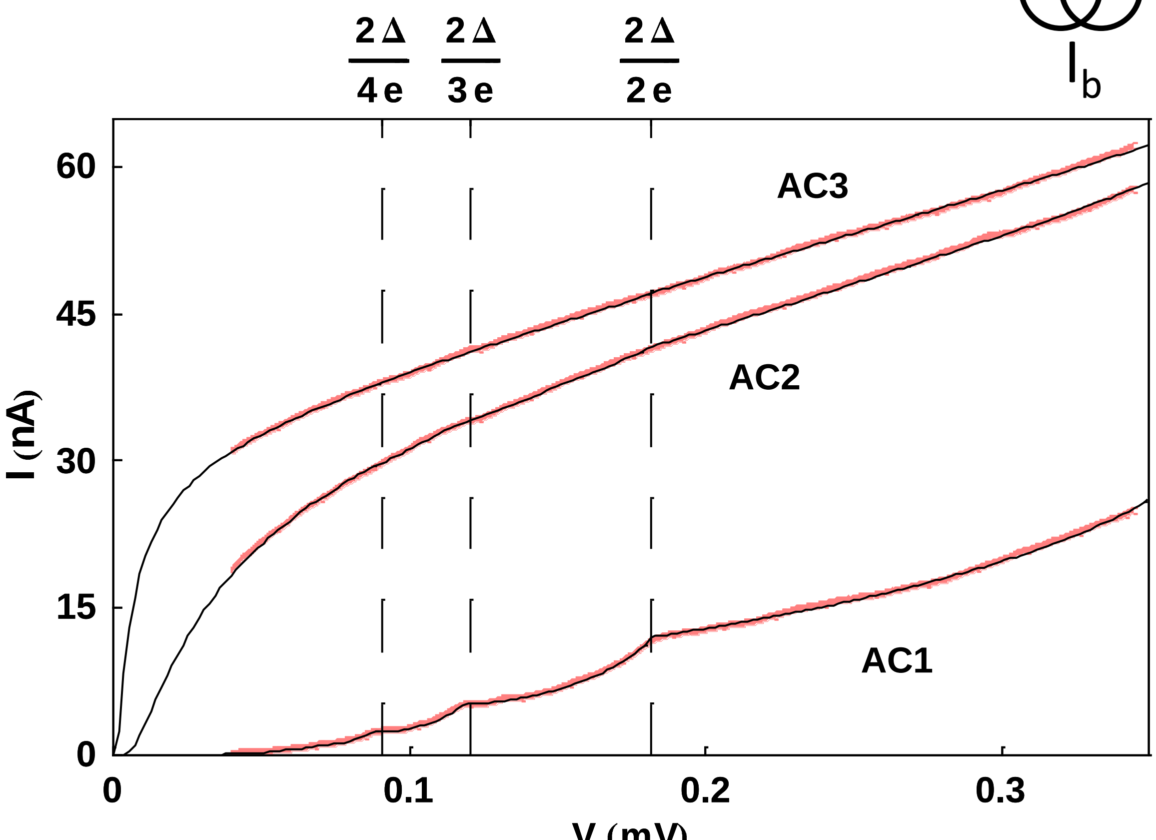
method: Scheer *et al.* 1997

Transmissions



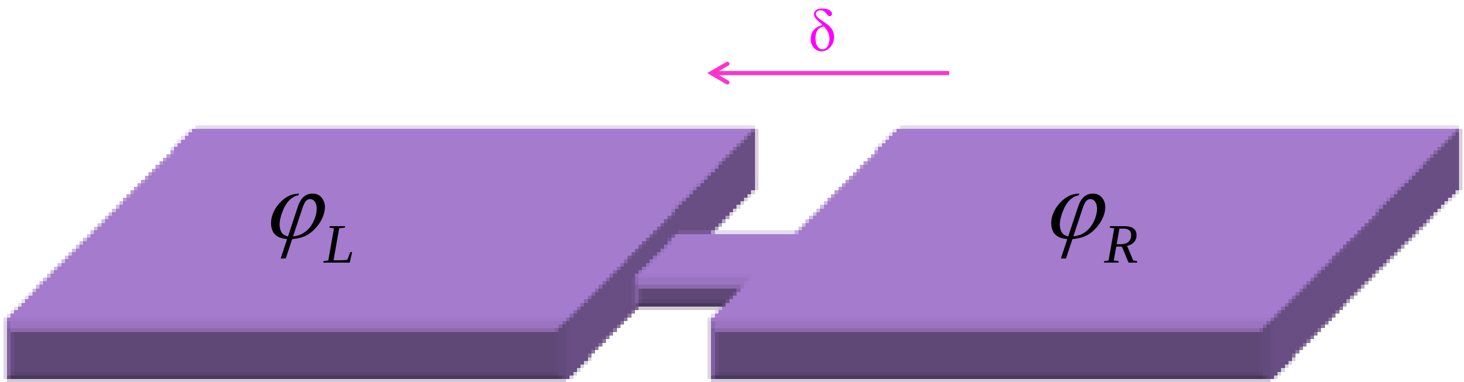
Measure I(V)





ELEMENTARY WEAK LINK

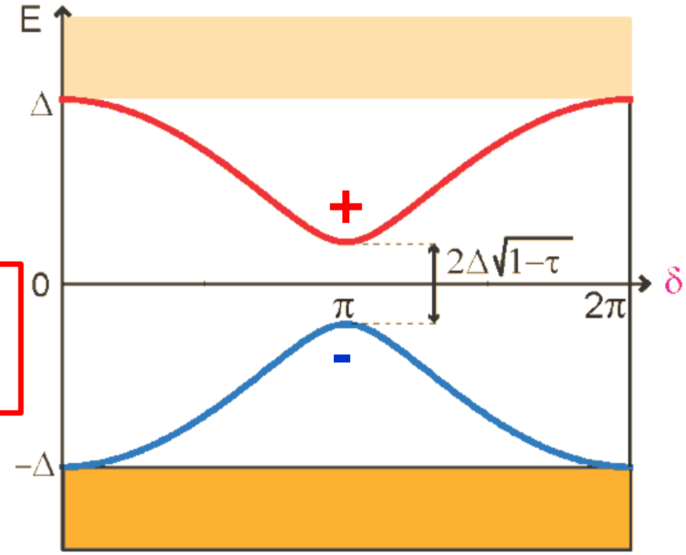
τ<1



Bound states: $E = \pm E_A$

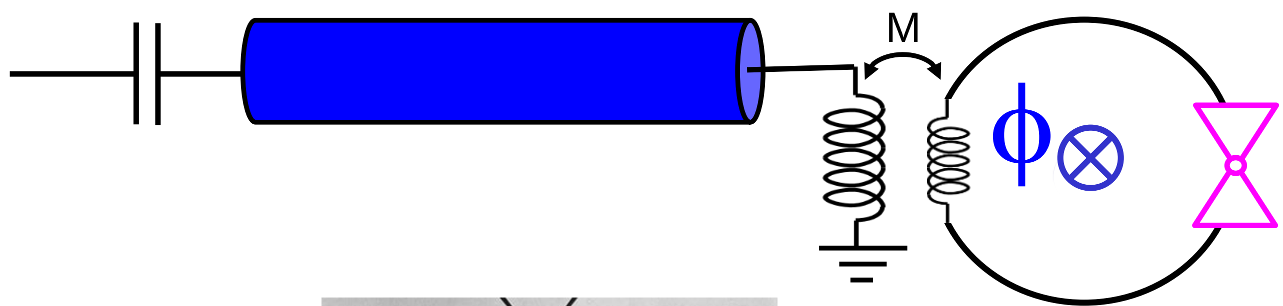
$$E_A = \Delta\sqrt{1-\tau\sin^2(\delta/2)}$$

Beenakker 1991

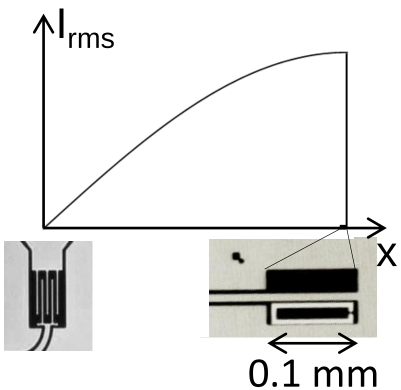


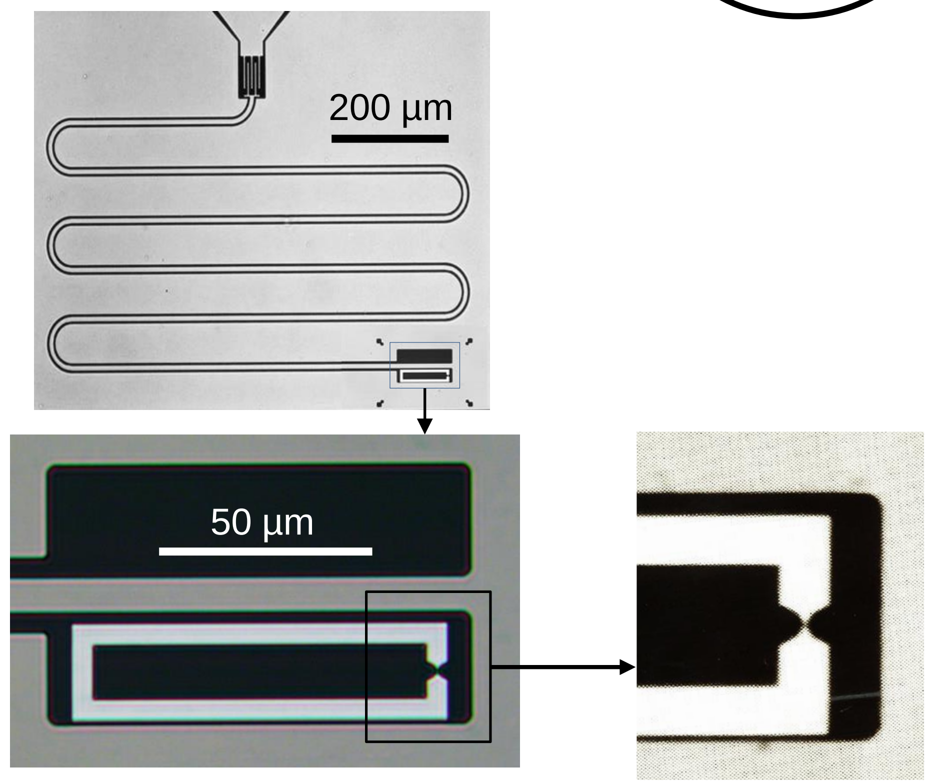
Hugues Pothier, Cargese 11/2016

ATOMIC SQUID COUPLED TO A COPLANAR MICROWAVE RESONATOR



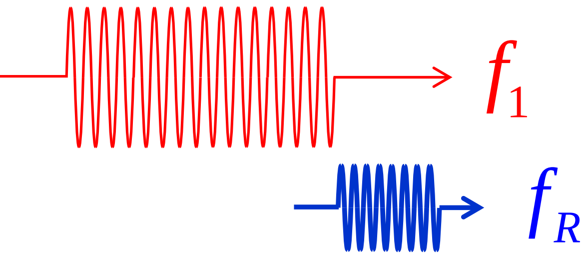
λ / 4 on Kapton at 10GHz

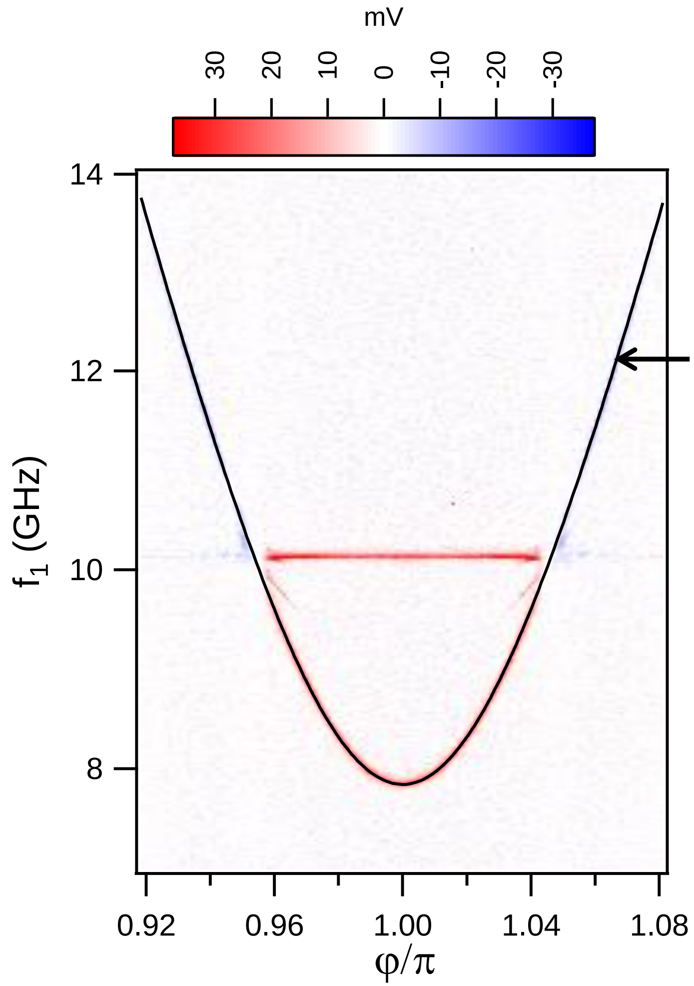




Janvier *et al.*, Science **349**, 1199 (2015)

SPECTROSCOPY OF ANDREEV TRANSITION

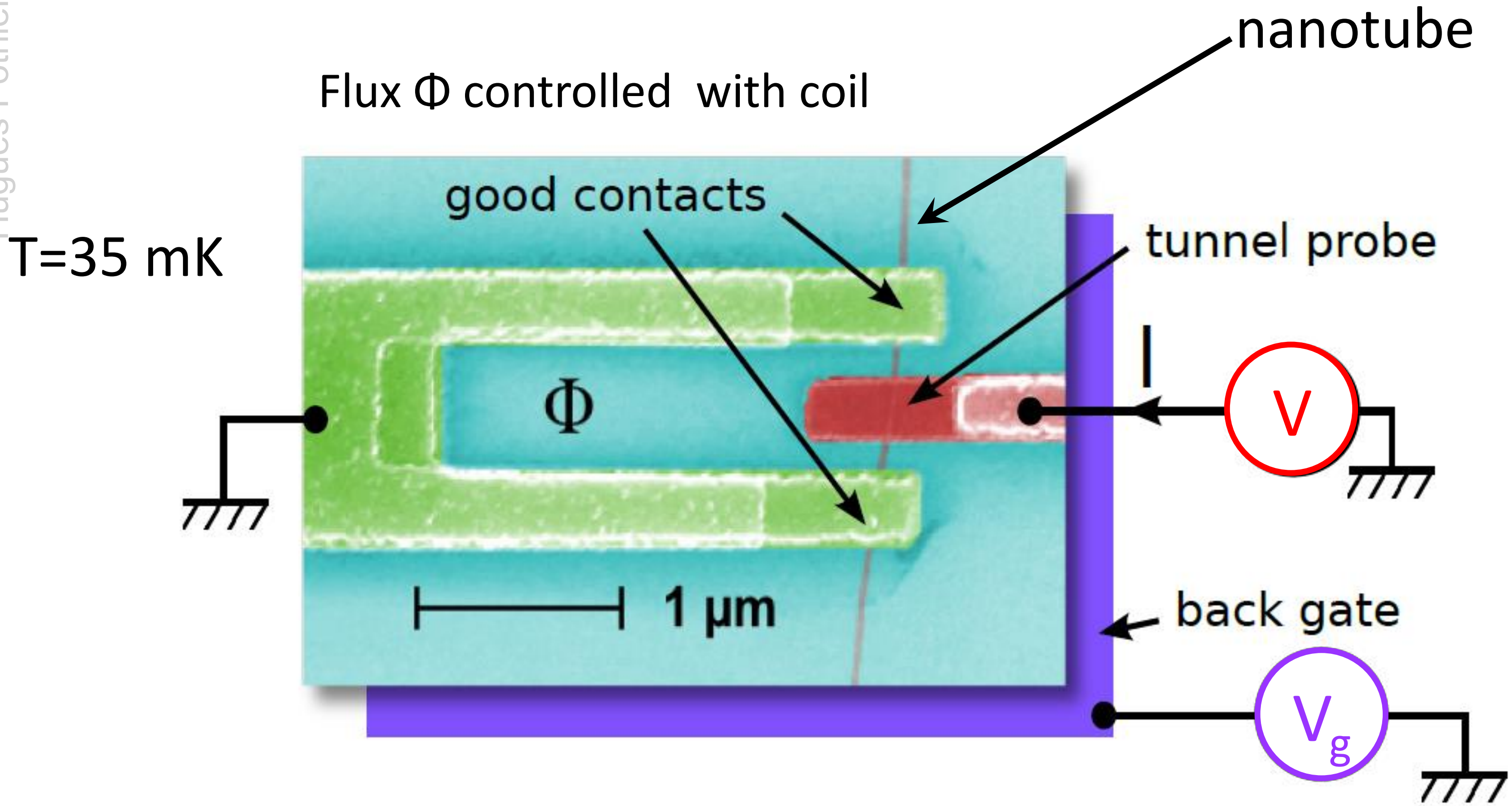




$$2E_A = 2\Delta\sqrt{1-\tau\sin^2(\varphi/2)}$$

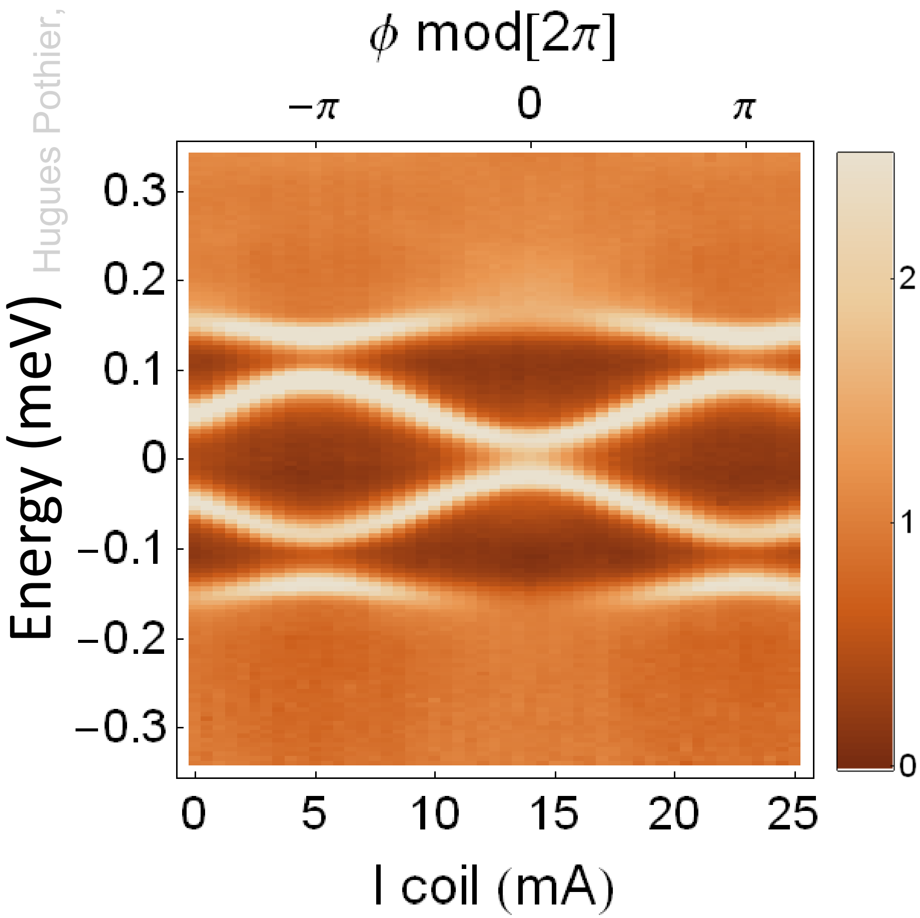
$$\tau = 0.99217$$

TUNNEL SPECTROSCOPY OF ABS IN CARBON NANOTUBES

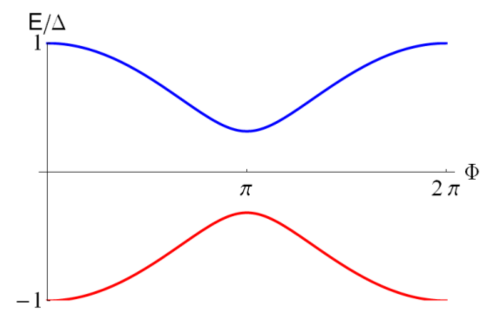


Pillet et al., Nature Physics **6**, 965 (2010)

TDOS flux dependence



Short single one dimensional channel:



Pillet et al., Nature Physics **6**, 965 (2010)

PARITY EFFECTS

Superconducting box

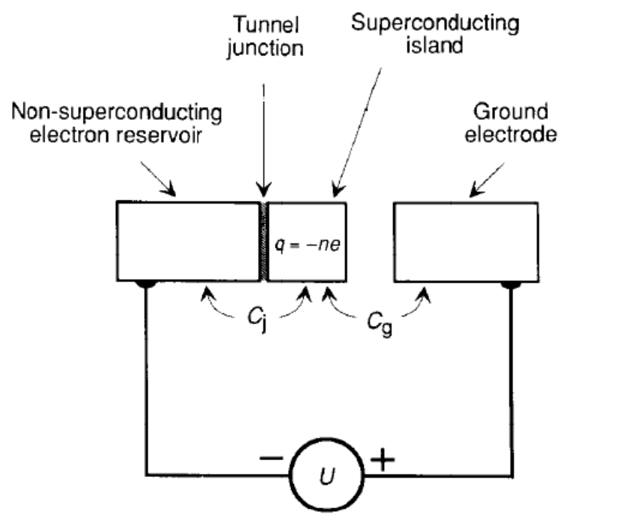
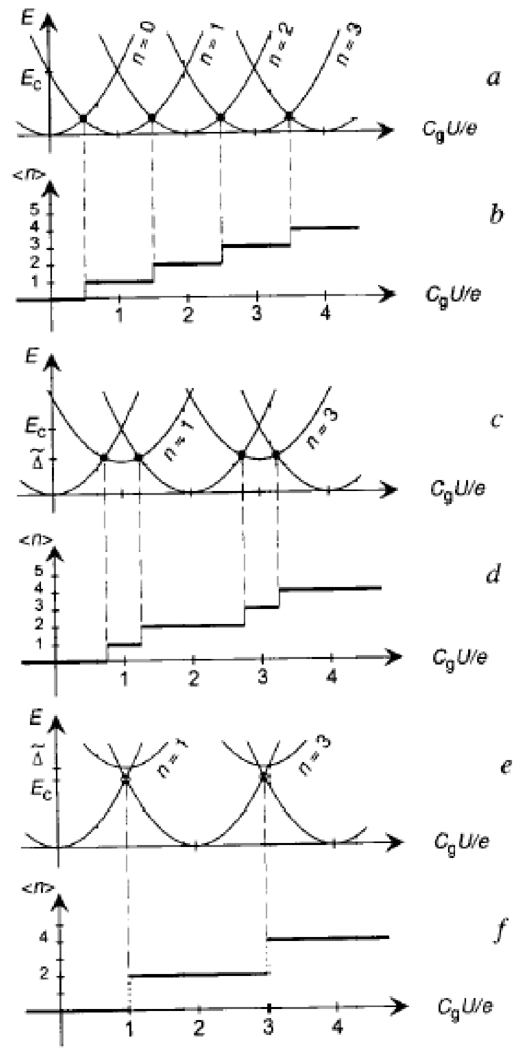


FIG. 1 Schematic diagram of the experiment. The superconducting island is a 30 × 110 × 2,260 nm Al strip containing ~10[9] atoms.



Two-elect
of the ch
superconductor

P. Lafarge, P. Joyez, D. Esteve, C. Urbina & M. H. Devoret

Service de Physique de l'État Condensé, CEA-Saclay, F-91191 Gif-sur-Yvette, France

NATURE · VOL 365 · 30 SEPTEMBER 1993

Superconducting box

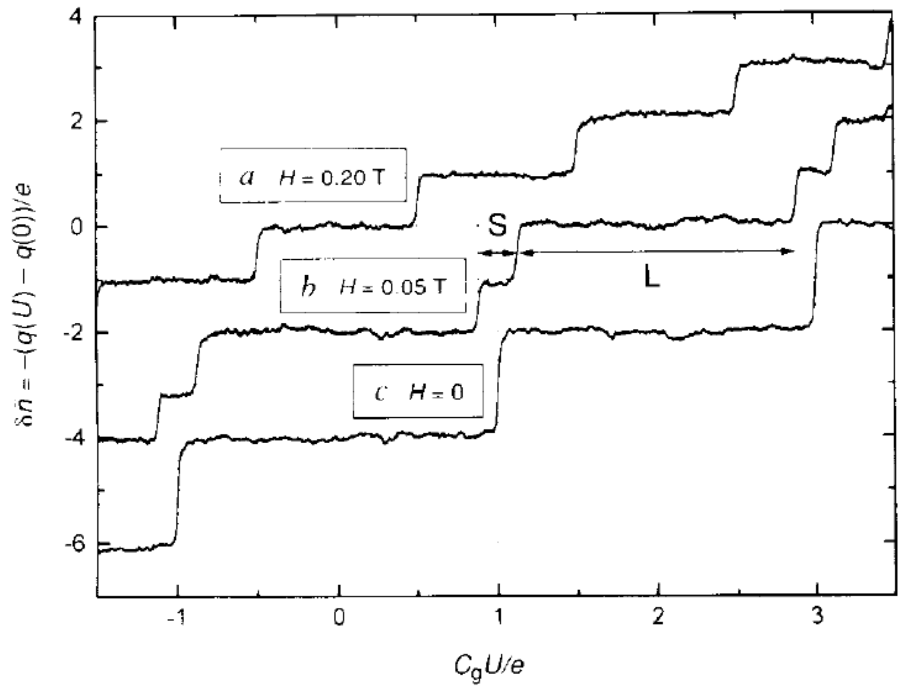
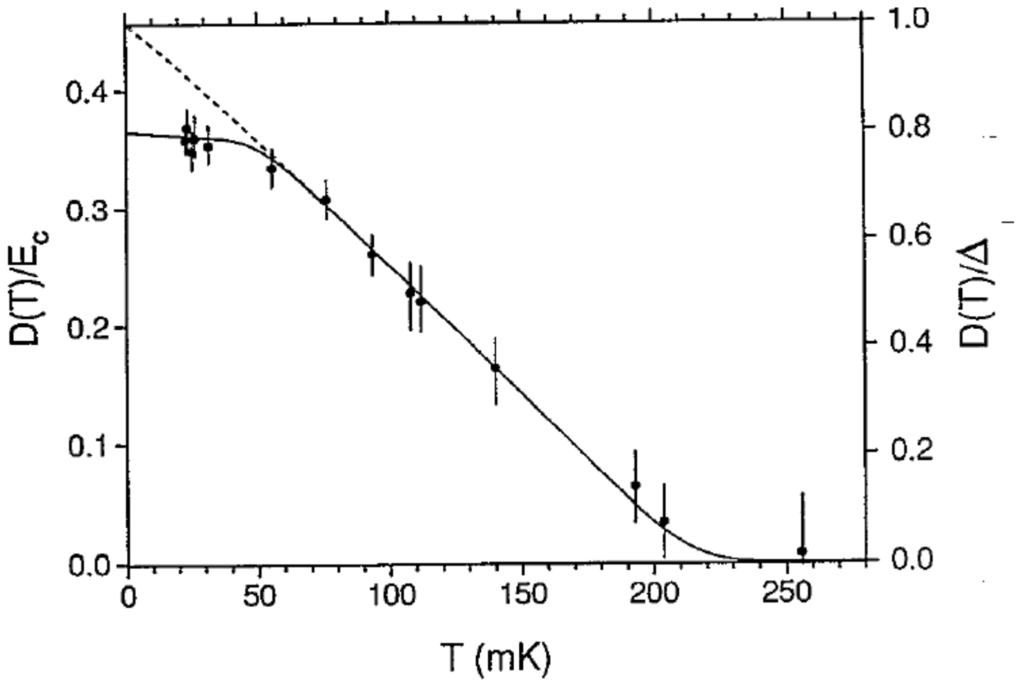


FIG. 2 Variations of the average value $\bar{q}$, in units of e, with the polarization $C_g U/e$, at $T = 28$ mK, for three values of the magnetic field H applied to the sample. *a*, Non-superconducting island. *b* and *c*, Superconducting island. For clarity, *b* and *c* have been offset vertically by 2 and 4 units, respectively. The letters L and S refer to the long and short steps, respectively.

Measured free energy difference D(T)



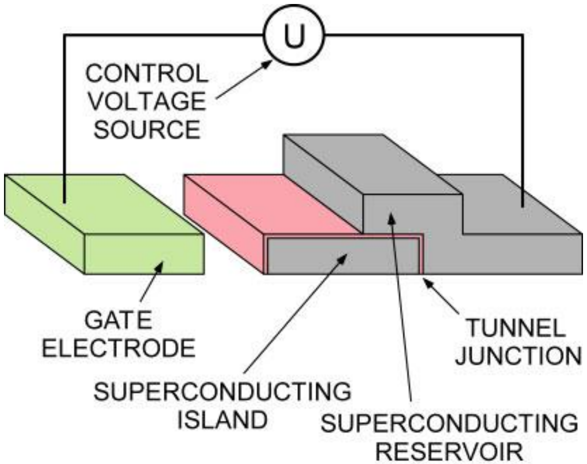
VOLUME 70, NUMBER 7 PHYSICAL REVIEW LETTERS 15 FEBRUARY 1993

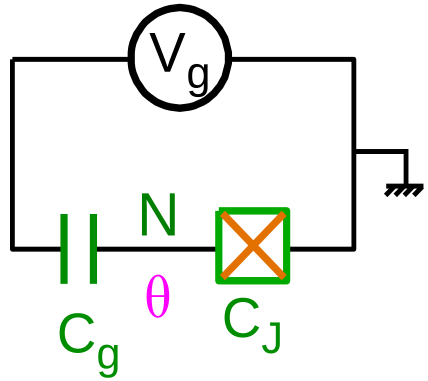
Measurement of the Even-Odd Free-Energy Difference of an Isolated Superconductor

P. Lafarge, P. Joyez, D. Esteve, C. Urbina, and M. H. Devoret

COOPER PAIR BOX

The single Cooper pair box





1 degree of freedom:

$$\left[\hat{\theta}, \hat{N}\right] = i$$

2 characteristic energies:

Hamiltonian:

$$\hat{H} = E_C(\hat{N} - N_g)^2 - E_J \cos\hat{\theta}$$

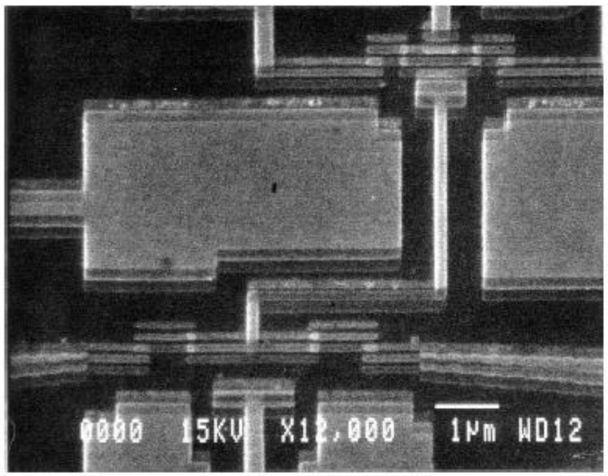
$$E_C = \frac{(2e)^2}{2(C_g + C_J)}$$

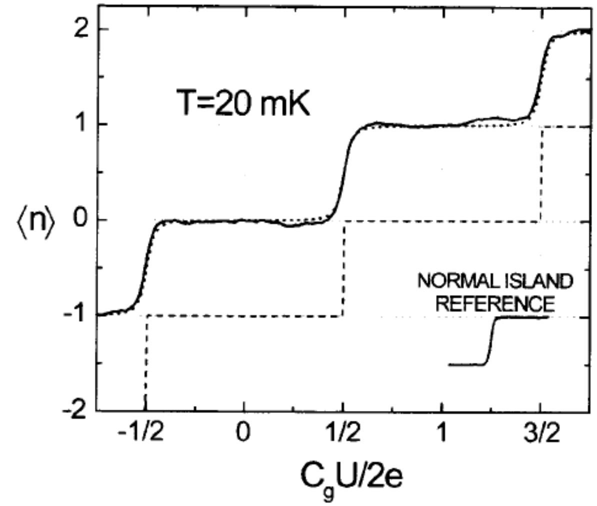
$$N_g = C_g V_g / 2e$$

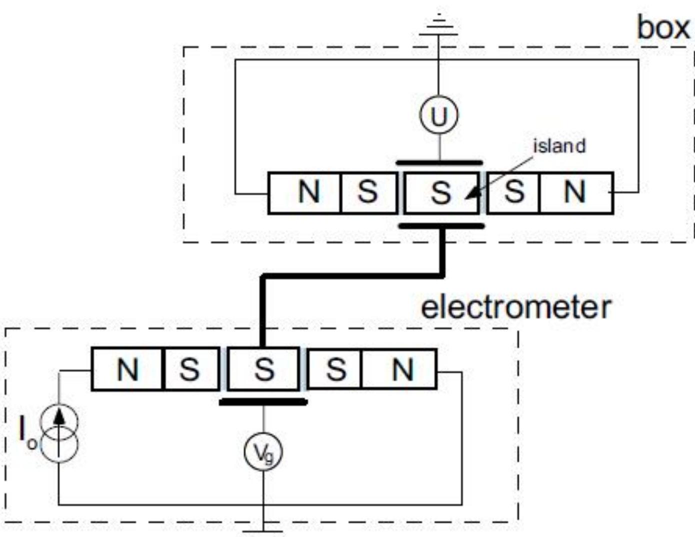
reduced gate charge

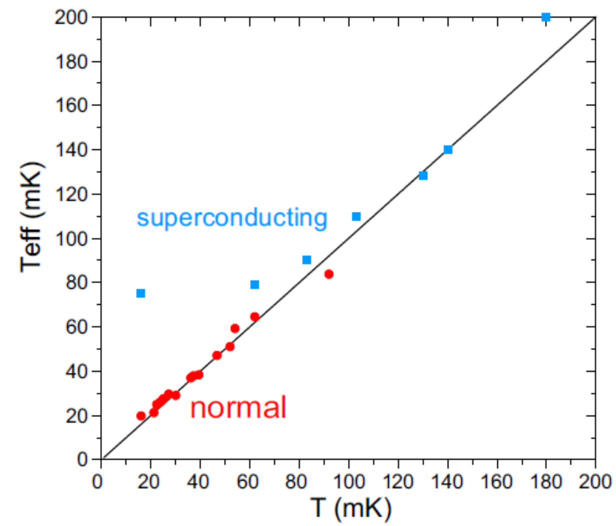
$$E_J = \frac{h\Delta}{8e^2 R_t}$$

Measuring the Single Cooper Pair Box



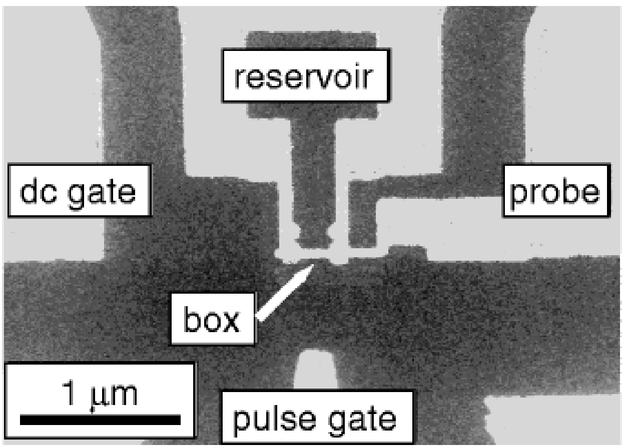


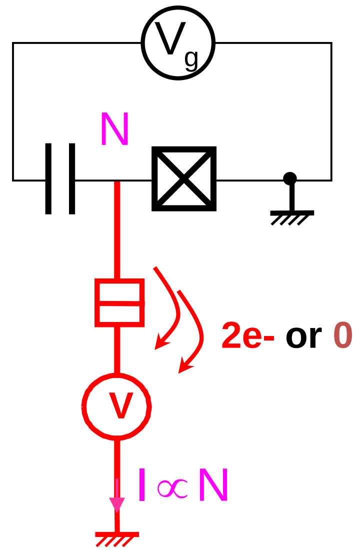


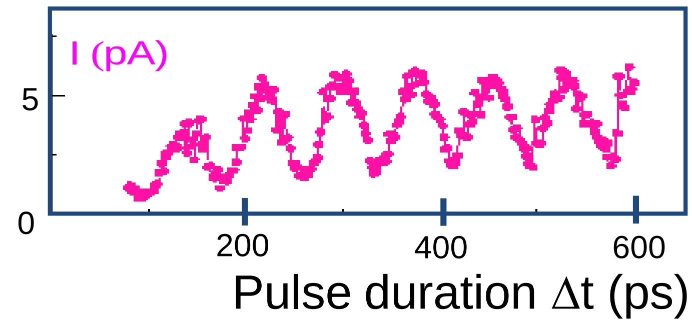


V. Bouchiat *et al.*, Physica Scripta **76**, 165 (1998).

Manipulating the charge state in a Cooper pair box







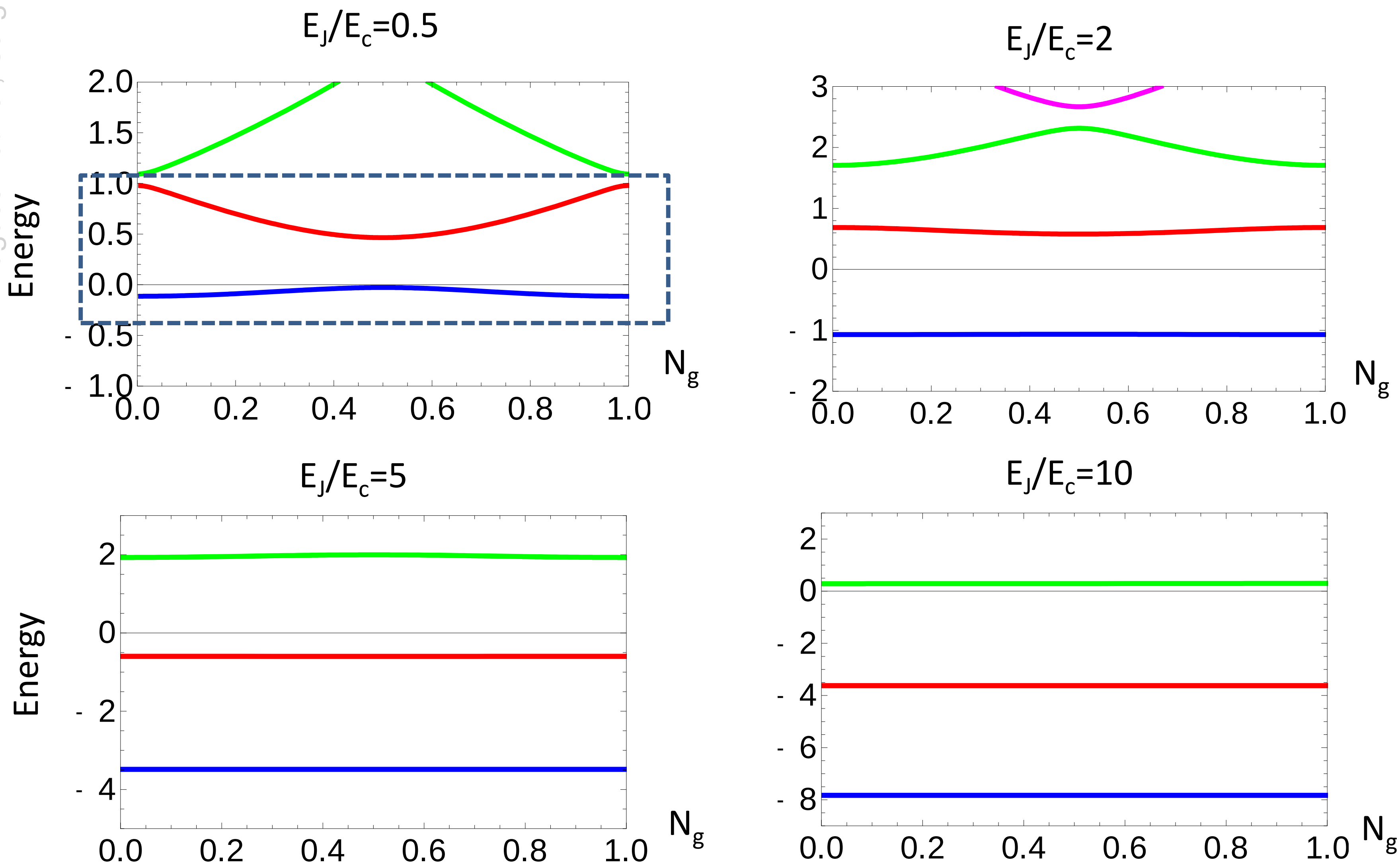
but poor coherence,
& no single-shot readout

Coherent control of macroscopic quantum states in a single-Cooper-pair box

Y. Nakamura*, Yu. A. Pashkin† & J. S. Tsai*

** NEC Fundamental Research Laboratories, Tsukuba, Ibaraki 305-8051, Japan*
† CREST, Japan Science and Technology Corporation (JST), Kawaguchi, Saitama 332-0012, Japan

From $E_J(\Phi) < E_C$ to $E_J(\Phi) > E_C$



TRANSMON QUBIT